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Proposal for an entry on Karl H e u n in 'The MacTutor of Mathematics'

Short Biography of Karl Heun, based on [vRent 02] and [vRent 95]

Karl Heun was born on April 3, 1859 in Wiesbaden, Germany, and died on January 10, 1929 in Karlsruhe, Germany.

After his school visits in Wiesbaden he began his study in mathematics and philosophy in 1878 in Göttingen (with Ernst Schering, Alfred Enneper, Hermann Amandus Schwarz). After two years he moved to Halle to study with Eduard Heine who, in 1861, had published his famous book on spherical harmonics. The first part of this book appeared in second edition in 1878. Heun stayed in Halle only between April and October 1880, probably due to the deteriorating health of Heine who died in 1881.

He returned to Göttingen and started with his thesis which was inspired by Heine and supervised by the astronomer Ernst Schering. His 1881 Dr. phil. thesis bears the title "Die Kugelfunctionen und Laméschen Functionen als Determinanten". After his doctorate he worked as an instructor at an agricultural winter school in Wehlau (eastern Prussia). This led to his qualification as teacher for secondary schools in Prussia.

Heun then moved to England and taught for two and a half years (1883-1885) at the Public School in Uppingham. In 1885/86 he complemented his studies in London.

His München habilitation thesis from July 1886 is entitled "Über lineare Differentialgleichungen zweiter Ordnung, deren Lösungen durch den Kettenbruchalgorithmus verknüpft sind." From 1886 to 1889 he lectured at the University of München on topics like: the theory of rational functions and their integrals, the theory of linear differential equations, introduction to the theory of linear substitutions and the general theory of differential equations. The seminal work from his time at München is "Zur Theorie der Riemann'schen Functionen zweiter Ordnung mit Vier Verzweigungspunkten", published in Mathematische Annalen 31 (1889) pp. 161-179.

Due to lack of financial support he left München and worked again as a teacher in Berlin from 1890 to 1902. At this time he became known in Germany, mainly due to his 1899 address at the München meeting of the DMV (Deutsche-Mathematiker-Vereinigung) whose complete version was published as "Die kinetischen Probleme der wissenschaftlichen Technik" in Jahresberichte d. Deutschen Math.-Ver. $\underline{9}$ 2. Abt. (1900) pp. 1-123.

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In 1900 Heun was honoured with the title professor, and in 1902 he was nominated as first candidate for the vacant chair in technical mechanics at Technische Hochschule Karlsruhe. He had been recommended by Felix Klein. Heun accepted this offer and stayed in Karlsruhe until his death in 1929.

In 1883 Heun had married the widow Henriette Jatho (née Bock) with two children Alfred and Paul Jatho; together they had a son Howard (1884-1902) and a daughter Charlotte (1891-1948).

In 1912 he was honoured with the title 'Geheimer Hofrat'. Technische Hochschule Berlin-Charlottenburg awarded him a doctor honoris causa (Dr. h.c.) in 1921.

His assistants in Karlsruhe were Georg Hamel (from 1902 to 1905), Max Winkelmann (from 1905 to 1911) and Fritz Noether (from 1911 to 1918). Kurt von Sanden was his student from 1905 to 1909, and he became Heun's successor in 1923.

Heun did not recover from a stroke he suffered in 1921 and retired in 1922.

The Heun equation is a second order linear differential equation of the Fuchsian type with four singular points. It generalizes the hypergeometric differential equation which has three singular points, and is used today in mathematical physics, *e.g.*, in the context of integrable systems.

$$\frac{d^2\,y}{dz^2} \;+\; \left(\frac{\gamma}{z} \;+\; \frac{\delta}{z-1} \;+\; \frac{\varepsilon}{z-a}\right)\frac{d\,y}{dz} \;+\; \frac{\alpha\,\beta\,z\,-\,q}{z\,(z-1)(z-a)}\,y \;=\; 0 \;.$$

Some publications of Karl Heun

- "Die Kugelfunctionen und Laméschen Functionen als Determinanten", Innaugural Dissertation, Göttingen 1881.

- "Untersuchungen über die Gaußsche Quadraturmethode". Wissenschaftliche Beilage zum Programm der Ersten Höheren Bürgerschule zu Berlin. Ostern 1892. Gaertner, Berlin 1892 Heft 19.

- "Zur Theorie der Riemann'schen Functionen zweiter Ordnung mit Vier Verzweigungspunkten", Math. Ann. $\underline{31}$ (1889) 161-179.

- " Neue Methode zur approximativen Integration der Differentialgleichungen mit einer unabhängigen Variablen", Zeitschr. f. Math. u. Physik $\underline{45}$ (1900) 23-28.

- "Die kinematischen Probleme der wissenschaftlichen Technik", Jber. dt. Math. Vereinig. $\underline{9}$ 2. Abt. (1900) 1-123.

- "Lehrbuch der Mechanik", I. Teil: Kinematik, Leipzig, G.J.Göschen'sche Verlagshandlung, 1906.

- Ansätze und allgemeine Methoden der Systemmechanik. Encyclopädie der mathematischen Wissenschaften, Bd.IV,1 Teil 2 Heft 3, Leipzig, 1913, 148 pp.

- **Some publications on Heun and the Heun equation**

[Ish-Suo 03] A. Ishkhanyan and K.-A. Suominen, New solutions of Heun's general equation, J. Phys. A: Math. Gen. 2003 36 L81-L85, http://www.iop.org/EJ/abstract/0305-4470/36/5/101,

[Mai 05] Robert S. Maier, On reducing the Heun equation to the hypergeometric equation, J. Differential Equations 213 (2005) 171-203, http://arxiv.org/abs/math.CA/0203264

[vRent 95] M. v. Renteln, "Karl Heun - his life and his scientific work", in [Ronv 95], p. XVII-XX, with a photograph of Heun on p. XVI.

[vRent 02] M. von Renteln: Die Mathematiker an der TH Karlsruhe (1825-1945), Druckerei Ernst Grässer, Karlsruhe, 2. Auflage 2002, pp. 173-186.

[Ronv 95] A. Ronveaux (ed.): Heun's Differential Equation, Oxford University Press, 1995

[Smir 02] A. O. Smirnov, Elliptic solitons and Heun's equation, CRM Proceedings and Lecture Notes, 2002, v.32, p.287-305. http://arXiv.org/abs/math/0109149

[Take 03] K. Takemura, The Heun equation and the Calogero-Sutherland system I: the Bethe Ansatz method, Communications in Mathematical Physics 235 467 (2003)

[Take 04] K. Takemura, The Heun equation and the Calogero-Sutherland system IV: the Hermite-Kritchever Ansatz, http://arxiv.org/abs/math.CA/0406141

[Val 06] G. Valent, Heun functions versus elliptic functions, http://arxiv.org/abs/math-ph/0512006

[Wiki 05] http://en.wikipedia.org/wiki/Heun's_ equation

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