Abstract

Due to Girard's (sometimes called Waring's) formula the sum of the $r-$th power of the zeros of every one variable polynomial of degree N, $P_N(x)$, can be given explicitly in terms of the coefficients of the monic $\tilde{P}_N(x)$ polynomial. This formula is closely related to a known $N-1$ variable generalisation of Chebyshev's polynomials of the first kind, $T_r^{(N-1)}$. The generating function of these power sums (or moments) is known to involve the logarithmic derivative of the considered polynomial. This entails a simple formula for the Stieltjes transform of the distribution of zeros. Perron-Stieltjes inversion can be used to find this distribution, *e.g.* for $N \to \infty$.

Classical orthogonal polynomials are taken as examples. The results for ordinary Chebyshev $T_N(x)$ and $U_N(x)$ polynomials are presented in detail. This will correct a statement about power sums of zeros of Chebyshev's $T-$polynomials found in the literature. For the various cases (Jacobi, Laguerre, Hermite) these moment generating functions provide solutions to certain Riccati equations.

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