5. QUANTIZING BRANES

· POLYAKOV ACTION [SK 2.2]

· THE NAMBU–GOTO ACTION

$$S_{NG} = \int \sqrt{g}\; d^{\dim+1}\xi \qquad g_{\alpha\beta} = \frac{\partial Y^\mu}{\partial \xi^\alpha}\frac{\partial Y_\mu}{\partial \xi^\beta}$$

EMBEDDS ALL ITS DYNAMICS IN THE SQUARE ROOT $\sqrt{g}$, WHICH POSES PROBLEMS FOR QUANTIZATION.

· WE CONSIDER THE POLYAKOV ACTION

$$S_P = \int \sqrt{(-1)^{\dim+1}\det h}\; h^{\alpha\beta}\,\partial_\alpha Y^\mu\,\partial_\beta Y_\mu\; d^{\dim+1}\xi$$

WHERE h IS A 2-TENSOR VARIABLE THAT ACTS AS A METRIC ON THE ξ SPACE. DIFFERENTLY FROM g, IT IS NOT DETERMINED BY THE EVOLUTION OF Y AND ξ, BUT WILL OBEY THE EULER–LAGRANGE EQUATIONS AS

$$\frac{\delta S_P}{\delta h_{\gamma\delta}} = \sqrt{\mathcal{H}}\left[\partial_\gamma Y^\mu\,\partial_\delta Y_\mu - \frac{1}{2}h_{\gamma\delta}\,h^{\alpha\beta}\,\partial_\alpha Y^\mu\,\partial_\beta Y_\mu\right] = 0$$

$$\Downarrow$$

$$\underline{\underline{h_{\alpha\beta} = \partial_\alpha Y^\mu\,\partial_\beta Y_\mu}}$$

WHICH, SUBSTITUTED BACK INTO THE POLYAKOV ACTION, GIVES

$$S_P\Big|_{h=\partial Y\partial Y} = \int \sqrt{g}\; d^{\dim+1}\xi = S_{NG}$$

· WE INTRODUCED AN AUXILIARY D.O.F. (h) WHICH, ON THE EOM, REDUCES TO THE USUAL METRIC, FOR CLASSICAL DYNAMICS.

· THE POLYAKOV ACTION IS LINEAR IN Y^μ, WHICH ARE THE DOF THAT WE WANT TO QUANTIZE.

· THE POLYAKOV ACTION IN MINKOWSKI SPACETIME IS INVARIANT UNDER
  - $ISO(1, D-1)$ FOR Y
  - REPARAMETRIZATIONS FOR ξ
  - WEYL TRANSFORMATIONS: $\delta X = 0 \;\wedge\; \delta h = 2\Lambda h$

$$\delta\left(\sqrt{\mathcal{H}}\;h^{\alpha\beta}\right) = \sqrt{\mathcal{H}\,(1 + h^{\bar\alpha\bar\beta}h_{\bar\beta\bar\alpha}2\Lambda)}\;h^{\alpha\beta}\left[(2\Lambda)^{-1}+1\right]$$
$$= \sqrt{\mathcal{H}}\;h^{\alpha\beta}\left[1 + (\dim+1)\Lambda - 2\Lambda\right]$$

IF $\dim = 2$ THEN THE ACTION IS WEYL-INVARIANT $\longrightarrow$ 1-BRANES

· THANKS TO WEYL INVARIANCE, WE CAN ALWAYS REDUCE THE METRIC TO THE FLAT ONE, GLOBALLY. THIS IS CALLED THE CONFORMAL GAUGE (USING A TERMINOLOGY BORROWED FROM GENERAL RELATIVITY AND GAUGE THEORIES). THIS OPERATION CAN ONLY BE PERFORMED FOR 1-BRANES.

· WE INTRODUCE NOW THE LIGHT-CONE COORDINATES

$$\xi^+ = \xi^0 + \xi^1, \qquad \xi^- = \xi^0 - \xi^1$$

$$g_{\alpha\beta} = \begin{pmatrix} & 1/2 \\ 1/2 & \end{pmatrix}$$

$$\partial_\pm = \frac{1}{2}\left(\partial_0 \pm \partial_1\right)$$

- THE COORDINATES DESCRIBE TWO MASSLESS PARTICLES MOVING IN OPPOSITE DIRECTIONS ALONG THE LINE ξ^1.

- 1-BRANES ARE PARTICULARLY SIMPLE, SINCE WE CAN COMPLETELY REMOVE ANY GAUGE FREEDOM AND HENCE "FIX" THE WHOLE DYNAMICS ON THE BRANE:

$g_{\alpha\beta}$	dim (dim+1)/2	3
REPARAMETRIZATION	− dim	−2
WEYL ($p=1$)	−1	−1
		0

- RESIDUAL REPARAMETRIZATION FREEDOM FROM THE INTEGRAL: $\xi^+ = f(\xi^+)$, $\xi^- = f(\xi^-)$

- THE POLYAKOV ACTION RETURN THE SAME TYPE OF EOM AS THE NAMBU−GOTO ACTION, WHICH IN LIGHT-CONE GAUGE READ (VON NEUMANN FOR OPEN STRING)

$$\underline{\underline{\partial_+ \partial_- \vec{Y} = 0}}$$

- REWRITING THE EOM FOR $h_{\alpha\beta}$ OF THE POLYAKOV ACTION:

$$\left.\begin{aligned} &\partial_0 Y^\mu \partial_1 Y_\mu = 0 \\ &\partial_0 Y^\mu \partial_0 Y_\mu + \partial_1 Y^\mu \partial_1 Y_\mu = 0 \end{aligned}\right\} \Longleftrightarrow \left\{\begin{aligned} &\partial_+ Y^\mu \partial_+ Y_\mu = 0 \\ &\partial_- Y^\mu \partial_- Y_\mu = 0 \end{aligned}\right.$$

<u>VIRASORO CONSTRAINTS</u>

- WE SEE THAT THE POLYAKOV ACTION RETURNS THE FOUR EQUATIONS DESCRIBING THE MOTION OF 1-BRANES WE SAW EARLIER.

OSCILLATIONS [SK 2.3]

· THE WAVE EQUATION GIVES

$$\partial_+ \partial_- Y^\mu = 0 \quad \Longrightarrow \quad Y^\mu = Y_+^\mu(\xi_+) + Y_-^\mu(\xi_-)$$

$$\cdot\; Y_+^\mu(\xi_+) = \frac{\bar{Y}^\mu}{2} + \frac{\ell^2 p^\mu}{2}\xi_+ + \frac{i\ell}{\sqrt{2}} \sum_{k \in \mathbb{R}_0} \frac{\alpha_k^\mu}{k} e^{-ik\xi_+}$$

$$\cdot\; Y_-^\mu(\xi_-) = \frac{\bar{Y}^\mu}{2} + \frac{\ell^2 \bar{p}^\mu}{2}\xi_- + \frac{i\ell}{\sqrt{2}} \sum_{k \in \mathbb{R}_0} \frac{\bar{\alpha}_k^\mu}{k} e^{-ik\xi_-}$$

· ℓ IS THE STRING LENGTH, α_k^μ AND $\bar{\alpha}_k^\mu$ ARE FOURIER MODES, AND TO KEEP Y^μ REAL WE MUST HAVE

$$(\alpha_k^\mu)^* = \alpha_{-k}^\mu$$

$$(\bar{\alpha}_k^\mu)^* = \bar{\alpha}_{-k}^\mu$$

· CLOSED STRINGS

· BCS: $Y^\mu(t, \sigma + 2\pi) = Y^\mu(t, \sigma)$ PERIODICITY

$$\begin{cases} \cdot\; Y_+^\mu(\xi^+) = \dfrac{\bar{Y}^\mu}{2} + \dfrac{\ell}{\sqrt{2}} \Big[\underbrace{\alpha_0^\mu}_{\ell p^\mu/\sqrt{2}} + i \underbrace{\sum_{n \in \mathbb{Z}_0}}_{\text{DUE TO PERIODICITY OF } 2\pi} \dfrac{\alpha_n^\mu}{n} e^{-in\xi^+} \Big] \\[2ex] \cdot\; Y_-^\mu(\xi^-) = \dfrac{\bar{Y}^\mu}{2} + \dfrac{1}{2} \Big[\underbrace{\bar{\alpha}_0^\mu}_{\ell \bar{p}^\mu/\sqrt{2} = \ell p^\mu/\sqrt{2}} + i \sum_{n \in \mathbb{Z}_0} \dfrac{\bar{\alpha}_n^\mu}{n} e^{-in\xi^-} \Big] \end{cases}$$

DUE TO PERIODICITY

· CALCULATING THE C.O.M. USING Y^μ AS WRITTEN ABOVE

$$Y_{CM}^\mu = \frac{1}{2\pi} \int_0^{2\pi} Y^\mu(\xi)\, d\xi_1 = \underbrace{\bar{Y}^\mu}_{\text{C.O.M. AT } \xi_0 = 0} + \underbrace{\ell^2 p^\mu \xi_0}_{\text{FREE PARTICLE MOTION WITH MOMENTUM } p^\mu}$$

$$P_{CM}^\mu = T \int_0^{2\pi} \frac{\partial Y^\mu(\xi)}{\partial \xi^0}\, d\xi^1 = p^\mu$$

· A CLOSED STRING MOTION IS A COMPOSITION OF A FREE MOTION PLUS AN INFINITE TOWER OF VIBRATIONS.

· OPEN STRINGS

· WE CAN APPLY EITHER DIRICHLET OR VON NEUMANN BCS INDEPENDENTLY ON EACH OF THE TWO ENDPOINTS.

· VON NEUMANN – VON NEUMANN : $\left.\frac{\partial Y^\mu}{\partial \xi^1}\right|_{\xi^1=0,\pi} = 0$

$$\cdot\ \partial_1 Y^\mu\Big|_{\xi^1=0} = \frac{\ell^2}{2}\underbrace{\left(p^\mu - \bar{p}^\mu\right)}_{p^\mu = \bar{p}^\mu} + \frac{\ell}{\sqrt{2}}\sum_{k\in\mathbb{R}_0} e^{-ik\xi^0}\underbrace{\left(\alpha_k^\mu - \bar{\alpha}_k^\mu\right)}_{\alpha_k^\mu = \bar{\alpha}_k^\mu} = 0$$

THE TWO PROPAGATION MODES ARE NOT INDEPENDENT: THE STRING MUST ALWAYS END WITH $\partial Y^\mu/\partial\xi_1 = 0$.

$$\cdot\ \partial_1 Y^\mu\Big|_{\xi^1=\pi} = 0 \longrightarrow k\in\mathbb{Z}_0$$

THE SOLUTION READS

$$Y^\mu(\xi) = \bar{Y}^\mu + \sqrt{2}\,\ell\Big[\underbrace{\alpha_0^\mu}_{\sqrt{2}\,\ell\, p^\mu} + i\sum_{n\in\mathbb{Z}_0}\frac{\alpha_n^\mu}{n}e^{-in\xi_0}\cos(n\xi_1)\Big]$$

ALSO HERE, THE MOTION IS THE COMPOSITION OF A FREE MOVING C.O.M. WITH OSCILLATIONS:

$$Y^\mu_{CM} = \bar{Y}^\mu + 2\ell^2 p^\mu \xi^0$$

$$P^\mu_{CM} = p^\mu$$

· DIRICHLET – DIRICHLET : $\left.\frac{\partial \vec{Y}}{\partial \xi^0}\right|_{\xi^1=0,\pi} = 0$

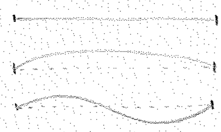
$$\partial_0 \vec{Y}\Big|_{\xi^1=0} = \frac{\ell^2}{2}\underbrace{\left(\vec{p}+\vec{\bar{p}}\right)}_{\vec{p}=-\vec{\bar{p}}} + \frac{\ell}{\sqrt{2}}\sum_{k\in\mathbb{Z}_0} e^{-ik\xi^0}\underbrace{\left(\vec{\alpha}_k + \vec{\bar{\alpha}}_k\right)}_{\vec{\alpha}_k = -\vec{\bar{\alpha}}_k} = 0$$

($k\in\mathbb{Z}_0$) FROM $\partial_0\vec{Y}\Big|_{\xi^1=\pi} = 0$

· WE SEE THAT $\vec{P}_{CM} = \vec{p} = 0$: BOTH ENDPOINTS ARE GLUED TO D_0-BRANES, WHICH COUNTERBALANCE THE MOMENTUM VARIATION DURING OSCILLATIONS.

· THE SOLUTION READS

$$\vec{Y}(\xi) = \vec{\bar{Y}} + \sqrt{2}\,\ell\sum_{n\in\mathbb{Z}_0}\frac{\vec{\alpha}_n}{n}e^{in\xi^0}\sin(n\xi^1)$$

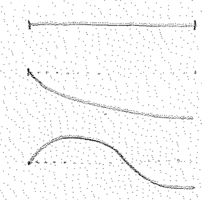


· DIRICHLET – VON NEUMANN : $\partial_0\vec{Y}\Big|_{\xi^1=0} = 0$, $\partial_1 Y^\mu\Big|_{\xi^1=\pi} = 0$

$$\cdot\ \partial_1\vec{Y}\Big|_{\xi^1=\pi} = \frac{\ell^2}{2}\left(\vec{p}-\vec{\bar{p}}\right) + \frac{\ell}{\sqrt{2}}\sum_{k\in\mathbb{R}_0}\underbrace{e^{-ik(\xi^0-\pi)}}_{2k\in\mathbb{Z}_0}\left[(-1)^{2k}\vec{\alpha}_k - \vec{\bar{\alpha}}_k\right] = 0$$

$$\cdot\ \partial_0\vec{Y}\Big|_{\xi^1=0} = 0 \rightleftharpoons \underbrace{\vec{p} = -\vec{\bar{p}}}_{\vec{p}=0},\ \vec{\bar{\alpha}}_k = -\vec{\alpha}_k$$

THE SOLUTION NOW READS



$$\vec{Y}(\xi) = \vec{\bar{\bar{Y}}} - \sqrt{2}\,\ell \sum_{k \in \mathbb{Z}+\frac{1}{2}} \frac{\vec{\alpha}_k}{k}\, e^{-ik\xi^0} \sin(\xi^1 n)$$

· HAMILTONIAN FRAMEWORK [SK 2.3]

· WE CAN MOVE FROM THE LAGRANGIAN DENSITY TO THE HAMILTONIAN DENSITY BY MEANS OF THE LEGENDRE TRANSFORMATION

$$\mathcal{H} = \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_0 Y^\mu)}}_{\Pi^0_\mu = \partial^0 Y_\mu \text{ ON EOM}} \partial_0 Y^\mu - \mathcal{L} = \mathcal{L} - \mathcal{L} = 0$$

THE DYNAMICS IS COMPLETELY DETERMINED BY THE CONSTRAINTS ON THE MOTION

THE HAMILTONIAN FOR A 1-BRANE (AKA ITS T^{00}) READS

$$H = \underbrace{\int \mathcal{H} \, d\xi^1}_{\text{SPACE-VOLUME INTEGRAL}} = \underbrace{\frac{T}{2} \int \partial_\alpha Y^\mu \partial^\alpha Y_\mu \, d\xi^1}_{\text{UNCONSTRAINED DYNAMICS}}$$

· IN THE HAMILTONIAN FRAMEWORK WE CAN DEFINE POISSON BRACKETS AS

$$\{A,B\} = \frac{\partial A}{\partial Y^\mu}\frac{\partial B}{\partial \Pi_\mu} - \frac{\partial A}{\partial \Pi_\mu}\frac{\partial B}{\partial Y^\mu} \underset{\Pi_\mu = \partial_0 Y_\mu}{=} \frac{\partial A}{\partial Y^\mu}\frac{\partial B}{\partial(\partial_0 Y_\mu)} - \frac{\partial B}{\partial Y^\mu}\frac{\partial A}{\partial(\partial_0 Y_\mu)}$$

· $\{Y^\mu(\xi), \underbrace{\partial_0 Y^\nu(\xi')}_{\Pi^\nu_0}\}_{\underbrace{\xi^0 = \xi'^0}_{\text{EQUAL-TIME RELATIONS}}} = \frac{1}{T}\delta(\xi^1 - \xi'^1)\eta^{\mu\nu}$

· $\{Y^\mu(\xi), Y^\nu(\xi')\}_{\xi^0 = \xi'^0} = 0$

· $\{\partial_0 Y^\mu(\xi), \partial_0 Y^\nu(\xi')\}_{\xi^0 = \xi'^0} = 0$

$$\Longleftrightarrow \begin{cases} \{\alpha^\mu_m, \alpha^\nu_n\} = \{\bar{\alpha}^\mu_m, \bar{\alpha}^\nu_n\} = -im\,\delta_{m+n,0}\,\eta^{\mu\nu} \\ \{\alpha^\mu_m, \bar{\alpha}^\nu_n\} = 0 \\ \{Y^\mu, p^\nu\} = \eta^{\mu\nu} \end{cases}$$

· REWRITING THE HAMILTONIAN IN TERMS OF OSCILLATION MODES:

· CLOSED $\quad H = \frac{\ell^2 p^2}{2} + \sum_{n=1}^{+\infty} \alpha_{-n}\cdot\alpha_n \;\; \bar{\alpha}_{-n}\cdot\bar{\alpha}_n$

· OPEN, NN $\quad H = \ell^2 p^2 + \sum_{n=1}^{+\infty} \alpha_{-n}\cdot\alpha_n$ $\quad$ (NO $\bar{\alpha}$ IN THE OPEN CASE)

· OPEN, DD $\quad H = \frac{[Y^\mu(\xi^1=0) - Y^\mu(\xi^1=\pi)]^2}{(2\pi\ell)^2} + \sum_{n=1}^{+\infty} \alpha_{-n}\cdot\alpha_n$

· OPEN, DN $\quad H = \sum_{n=1}^{+\infty} \alpha_{\frac{1}{2}-n} \cdot \alpha_{n-\frac{1}{2}}$

· WE CAN NOW WRITE THE VIRASORO CONSTRAINTS IN TERMS OF OSCILLATION MODES. SINCE OSCILLATION MODES ARE DETERMINED BY THE BCS, WE NEED TO DISTINGUISH AMONG DIFFERENT CASES. FOR THE CLOSED STRING:

$$L_m = \frac{1}{2}\sum_{n\in\mathbb{Z}} \alpha_{m-n}\cdot\alpha_n = 0 \qquad \bar{L}_m = \frac{1}{2}\sum_{n\in\mathbb{Z}} \bar{\alpha}_{m-n}\cdot\bar{\alpha}_n = 0$$

WITH

$$L^*_m = L_{-m} \qquad \bar{L}^*_m = \bar{L}_{-m}$$

• THE HAMILTONIAN (WHICH IS EQUAL TO ZERO) READS, IN TERMS OF VIRASORO MODES,

$$H = L_0 + \bar{L}_0 = 0 \longrightarrow \text{THE SYSTEM IS TIME INVARIANT}$$

• ANOTHER INTERESTING QUANTITY IS ZERO:

$$\sigma = L_0 - \bar{L}_0 = 0 \longrightarrow \text{THE SYSTEM IS INVARIANT FOR TRANSLATIONS ALONG THE STRING}$$

• WE CAN CHECK THAT σ IS GENERATOR OF INFINITESIMAL TRANSLATIONS ON THE STRING. IN FLAT SPACETIME:

$$L_0 = \frac{\ell^2}{4} p^2 + \sum_{n \in \mathbb{N}} \alpha_{-n} \cdot \alpha_n \qquad \bar{L}_0 = \frac{\ell^2}{4} p^2 + \sum_{n \in \mathbb{N}} \bar{\alpha}_{-n} \cdot \bar{\alpha}_n$$

SO WE HAVE

$$\{\sigma, \underbrace{Y^\mu}_{\text{FUNCTION OF } \xi}\} = \sum_{n \in \mathbb{N}} \Big\{ \alpha_{-n} \cdot \alpha_n - \bar{\alpha}_{-n} \cdot \bar{\alpha}_n \,,\, \underbrace{\bar{Y}^\mu}_{\text{CONSTANT TERM}} + \ell^2 p^\mu \xi^0 + \frac{i\ell}{\sqrt{2}} \sum_{m \in \mathbb{Z}_0} \frac{1}{m} \left(e^{-im\xi^+} \alpha_m^\mu + e^{-im\xi^-} \bar{\alpha}_m^\mu \right) \Big\}$$

$$= -i \frac{i}{\sqrt{2}} \left[\sum_{m \in \mathbb{N}} \left[e^{-im\xi^+} \left(\alpha_{-m}^\mu + \alpha_m^\mu \right) - e^{-im\xi^-} \left(\bar{\alpha}_{-m}^\mu + \bar{\alpha}_m^\mu \right) \right] \right]$$

$$= \frac{\partial Y^\mu}{\partial \xi^1}$$

GENERATOR OF INFINITESIMAL TRANSLATIONS ALONG THE STRING PARAMETRIZATION

• WE SEE THAT THE SYSTEM DOES NOT EVOLVE (I.E. CHANGE) UNDER PARAMETERS TRANSLATIONS (AS WE EXPECT, SINCE IT IS A CLOSED STRING).

• FOR AN <u>OPEN STRING</u> $\bar{\alpha}$ IS MAPPED INTO α, AND FOR ALL CASES THE VIRASORO OPERATORS READ

$$L_m = \frac{1}{2} \sum_{n \in \mathbb{Z}} a_{m-n} \cdot a_n \,, \qquad H = L_0$$

• BY MEANS OF POISSON BRACKETS WE GET THE RELATIONS OBEYED BY THE VIRASORO OPERATORS L: THE <u>VIRASORO ALGEBRA</u>.

$$\{L_m, L_n\} = -i(m-n) L_{m+n}$$
$$\{\bar{L}_m, \bar{L}_n\} = -i(m-n) \bar{L}_{m+n}$$
$$\{L_m, \bar{L}_n\} = 0$$

ALGEBRA OF 2-DIMENSIONAL CONFORMAL FIELD THEORY

• WE JUST DISCARD $\bar{L}$ FOR OPEN STRINGS.

· LIGHT-CONE QUANTIZATION

· WE APPLY ALL CONSTRAINTS TO THE MOTION AT THE CLASSICAL LEVEL, BEFORE QUANTIZATION. WE THEN QUANTIZE, OBTAINING A THEORY WHICH IS NOT MANIFESTLY LORENTZ INVARIANT.
· WE USE THE RESIDUAL RESCALING FROM INTEGRATION MEASURE TO FIX

$$\underbrace{Y^+}_{\text{NEW LIGHT-CONE COORDINATES}} = \underbrace{Y^+ + \ell^2 p^+ \tau}_{\text{OLD LIGHT-CONE COORDINATES}}$$

· WE USE THE VIRASORO CONSTRAINTS TO LINK Y^+ AND Y^- TO THE TRANSVERSE COORDINATES Y^i. WE ARE THEN LEFT WITH x, p, AND TRANSVERSE OSCILLATIONS.
· THE LIGHT-CONE OSCILLATORS NOW READ

$$\alpha_n^+ = \bar{\alpha}_n^+ = \frac{\ell}{\sqrt{2}} p^+ \delta_{0,n} \longrightarrow \text{ONLY ZERO MODES}$$

$$\alpha_n^- = \frac{\sqrt{2}}{\ell p^+}\left[\sum_{m\in\mathbb{Z}} :\alpha^i_{n-m}\alpha^i_m: - 2A\,\delta_{n,0}\right]$$

$$\bar{\alpha}_n^- = \frac{\sqrt{2}}{\ell p^+}\left[\sum_{m\in\mathbb{Z}} :\bar{\alpha}^i_{n-m}\bar{\alpha}^i_m: - 2A\,\delta_{n,0}\right]$$

LIGHT-CONE OSCILLATORS IN TERMS OF TRANVERSAL ONES

NORMAL ORDERING INDUCED CONSTANT IN $n=0$

NORMAL ORDERING: ALL POSITIVE-FREQUENCY MODES ON THE RIGHT OF THE NEGATIVE-FREQUENCY ONES

· WE CAN NOW QUANTIZE THE SETS

$$Y^\mu \text{ AND } p^\nu: \qquad [x^\mu, p^\nu] = i\eta^{\mu\nu}$$

$$\alpha^\mu_k \text{ AND } \alpha^\nu_h: \qquad [\alpha^\mu_k, \alpha^\nu_h] = k\,\delta_{k+h,0}\,\eta^{\mu\nu}$$

$$\bar{\alpha}^\mu_k \text{ AND } \bar{\alpha}^\nu_h: \qquad [\bar{\alpha}^\mu_k, \bar{\alpha}^\nu_h] = k\,\delta_{k+h,0}\,\eta^{\mu\nu}$$

SIMILAR TO FIELD THEORIES BUT WITH k, h DISCRETE

· SINCE WE REPLACED POISSON BRACKETS WITH COMMUTATOR, THIS IS A BOSONIC STRING.

· SINCE WE KNOW THAT

$$\bar{\alpha}^{\mu\dagger}_k = \bar{\alpha}^\mu_{-k}, \qquad \alpha^{\mu\dagger}_k = \alpha^\mu_{-k}$$

WE WILL USE NEGATIVE FREQUENCY MODES AS CREATION OPERATORS, POSITIVE FREQUENCY MODES AS CORRESPONDING ANNIHILATION OPERATORS.
· TO CONSTRUCT OSCILLATIONS WE START FROM THE GROUND STATE

$$|p^\mu\rangle$$

(C.O.M. MOVING WITH MOMENTUM p^μ, NO TRANSVERSE OSCILLATIONS).

· CLOSED STRING

· FIRST EXCITATION:

$$\alpha^i_{-1}\bar{\alpha}^j_{-1}|p^\mu\rangle = \underbrace{\alpha^{[i}_{-1}\bar{\alpha}^{j]}_{-1}|p^\mu\rangle}_{\text{ANTISYMMETRIC TENSOR } B^{ij}} + \underbrace{\frac{1}{d-2}\delta^{ij}\alpha^k_{-1}\bar{\alpha}^k_{-1}|p^\mu\rangle}_{\text{SCALAR } \Phi} + \underbrace{\left[\alpha^{\{i}_{-1}\bar{\alpha}^{j\}}_{-1} - \frac{1}{d-2}\delta^{ij}\alpha^k_{-1}\bar{\alpha}^k_{-1}\right]|p^\mu\rangle}_{\text{"GRAVITON" } G^{ij}}$$

· OPEN STRING

· FIRST EXITATION: $\alpha^{i}_{-1}|p^{\mu}\rangle$ VECTOR FIELD

· FOR CONSISTENCY REASONS, SUCH THAT THE FIRST EXITATION OF THE CLOSED STRING SHOULD PRODUCE MASSLESS FIELD, THE DIMENSION OF THE INDICES IS 26. THIS DIMENSION IS CALLED THE CRITICAL DIMENSION.