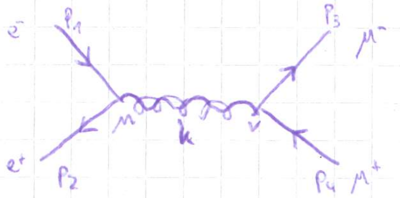
QED und Kolliderphysik

ExSh 1

A 1

(a)



$$i\mathcal{M} = \bar v_{s_2}(p_2)\, ie\gamma^\mu u_{s_1}(p_1) \frac{-ig_{\mu\nu}}{k^2} \bar u_{s_3}(p_3)\gamma^\nu v_{s_4}(p_4)$$

(b)
$$\overline{\sum}|\mathcal{M}|^2 = \frac{1}{2}\sum_{s_1}\frac{1}{2}\sum_{s_2}\sum_{s_3}\sum_{s_4}|\mathcal{M}|^2$$

$$= \frac{1}{4}\sum_{s_1 \dots s_4} \frac{e^4}{k^4} \underbrace{v_{s_4}^+ \gamma_\nu^+ \gamma^{0+}}_{\bar v_{s_4}\gamma_\mu} u_{s_3}(p_3)\, u_{s_1}^+(p_1)\gamma^{\mu +}\gamma^{0+} v_{s_2}(p_2)\bar v_{s_2}(p_2)\gamma^\nu u_{s_1}(p_1)\bar u_{s_3}(p_3)\gamma_\nu v_{s_4}(p_4)$$

$$= \frac{1}{4}\frac{e^4}{k^4}\sum_{s_1 \dots s_4} \bar v_a^{s_4}\gamma_{\mu,ab} u_b^{s_3} \bar u_c^{s_1}\gamma^\mu_{cd} \underbrace{v_d^{s_2}\bar v_f^{s_2}}_{(\not p_2 - m)_{df}} \gamma^\nu_{fg} u_g^{s_1}\bar u_k^{s_3}\gamma_{\nu,ke} v_e^{s_4}$$

$$= \frac{1}{4}\frac{e^4}{k^4} \underbrace{(\not p_4 - m)_{ea}\gamma_{\mu,ab}(\not p_3 + m)_{bk}\gamma_{\nu,ke}}_{tr[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu p_4^\rho p_3^\sigma - m^2\gamma_\mu\gamma_\nu]} \underbrace{\not p_{1,gc}\gamma^\mu_{cd}\not p_{2,df}\gamma^\nu_{fg}}_{tr[\not p_1\gamma^\mu\not p_2\gamma^\nu]}$$

$m_e, m_\mu \to 0$

$$= -4m^2 g_{\mu\nu} + 4(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\sigma}g_{\mu\nu} + g_{\rho\nu}g_{\mu\sigma})p_4^\rho p_3^\sigma$$

$$\overset{m\to 0}{=} 4(p_{4,\mu}p_{3,\nu} - p_4 p_3 g_{\mu\nu} + p_{4,\nu}p_{3,\mu})$$

$$= \frac{e^4}{4k^4}\cdot 16\left((p_4 \cdot p_1)(p_3\cdot p_2)\cdot 2 + 2(p_4\cdot p_3)(p_2\cdot p_1)\right)$$

$$= \frac{8e^4}{k^4}\left((p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right)$$

(c)
$$k^4 = s^2 = (p_1+p_2)^4 = (2p_1p_2)^2 = (2p_3p_4)^2$$

$$t = (p_3-p_1)^2 = (p_4-p_2)^2 = -2p_3p_1 = -2p_4p_2$$

$$u = (p_4-p_1)^2 = (p_3-p_2)^2 = -2p_4p_1 = -2p_3p_2$$

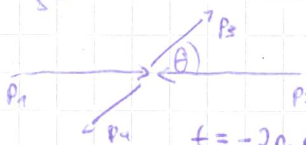
$$\Rightarrow \overline{\sum}|\mathcal{M}|^2 = \frac{8e^4}{s^2}\left(\frac{t^2}{4}+\frac{u^2}{4}\right) = \frac{32\pi^2\alpha^2}{s^2}(t^2+u^2)$$

(d)
$$d\Phi_2 = \frac{1}{32\pi^2}d\Omega$$

$$= \frac{1}{16\pi}\frac{2}{s}dt$$

$$= \frac{1}{8\pi s}dt$$

CMS:



$$t = -2p_3p_1 = -2E^2(1-\cos\theta) = \frac{s}{2}(\cos\theta - 1)$$

$$\Rightarrow dt = \frac{s}{2}d\cos\theta$$

(e)
$$d\sigma = \frac{1}{2s}d\Phi_2\overline{\sum}|\mathcal{M}|^2$$

$$\sigma = \frac{1}{2s}\frac{1}{8\pi s}\int_{-s}^0 dt\, \frac{32\pi^2\alpha^2}{s^2}(t^2+s^2+t^2+2st)$$

$$= \frac{1}{2s}\frac{4\pi\alpha^2}{s^3}\left[s^2t + \frac{2}{3}t^3 + st^2\right]_{-s}^0$$

$$= \frac{1}{2s}\frac{4\pi\alpha^2}{s^3}\frac{2}{3}s^3$$

$$= \frac{4\pi\alpha^2}{3s}$$

$$\frac{d\sigma}{dt} = \frac{|\mathcal{M}|^2}{16\pi\lambda(s,m_1^2,m_2^2)}$$

mit $\lambda(s,m_1^2,m_2^2) = [s-(m_1+m_2)^2]\times[s-(m_1-m_2)^2]$

A.2

(a) $C_2 = L^a L^a$

$$[C_2, L^a] = [L^b L^b, L^a] = L^b [L^b, L^a] + [L^b, L^a] L^b$$

$$= i f_{bac} L^b L^c + i \underbrace{f_{cab}}_{= -i f_{bac}} L^c L^b$$

$$\vdots$$

$$= 0$$

(b) $$[C_{2,F}]_{ik} = \sum_{a=1}^{N^2-1} \left(\frac{\lambda^a}{2}\right)_{ij} \left(\frac{\lambda^a}{2}\right)_{jk} \cancel{= \frac{1}{2}\left\{\frac{\lambda^a}{2}, \frac{\lambda^a}{2}\right\} = \sum_a d_{aab} \frac{\lambda^b}{4} + \frac{N^2-1}{N}}$$

$$= \frac{1}{2} \delta_{ik}\delta_{jj} - \frac{1}{2N} \delta_{ij} \delta_{jk}$$

$$= \frac{N}{2} \delta_{ik} - \frac{1}{2N} \delta_{ik}$$

$$= \frac{N^2-1}{2N} \delta_{ik}$$

(c) $$\left[\frac{\lambda^a}{2} \frac{\lambda^b}{2} \frac{\lambda^a}{2}\right]_{il} = \frac{1}{8} \lambda^a_{ij} \lambda^b_{jk} \lambda^a_{kl} = \frac{1}{2} \lambda^b_{jk} \left(\frac{1}{2} \delta_{il}\delta_{jk} - \frac{1}{2N} \delta_{ij}\delta_{kl}\right)$$

$$= \frac{1}{4}\left(\underbrace{\lambda^b_{kk}}_{=0} \delta_{il} - \frac{1}{N} \lambda^b_{il}\right)$$

$$= -\frac{1}{2N} \frac{\lambda^b}{2}$$