Theoretische Teilchenphysik 1

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Exercise sheet 8

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Exercise 20: Gupta-Bleuler photon (P) *(3+4 = 7 points)*

In this exercise, we want to look at some of the properties and consequences of the Gupta-Bleuler formalism. We define the state of a single photon with polarization λ as

$$|\lambda\rangle = \int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3\sqrt{2\omega_p}} f_\lambda(p) a^{\dagger(\lambda)}(p) \left|\psi_0\right\rangle \,,$$

where $f_\lambda(p)$ is the probability distribution for the photon to have momentum p and polarization λ and $|\psi_0\rangle$ is the ground state of the photon system with $\langle\psi_0|\psi_0\rangle = 1$. The probability distribution f is normalized as

$$\int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3} f^*_\lambda(p) f_{\lambda'}(p) = \delta_{\lambda\lambda'} \,.$$

(a) Calculate $\langle\lambda|\lambda'\rangle$ and explain why $|\lambda\rangle$ cannot be an element of a Hilbert space.

(b) We now consider a photon state given by

$$|\gamma\rangle = |0\rangle + |3\rangle \,,$$

i.e. the photon has timelike and longitudinal polarization. Suppose that the photon state $|\gamma\rangle$ corresponds to a physical particle and show that $|\gamma\rangle$ can be written as

$$|\gamma\rangle = \int \frac{\mathrm{d}^3\mathrm{k}}{(2\pi)^3\sqrt{2\omega_k}} f_3(k) \left(a^{\dagger(3)}(k) - a^{\dagger(0)}(k)\right) \left|\psi_0\right\rangle \,.$$

Calculate $\langle\gamma|\gamma\rangle$ and, by using your result, explain why $|\gamma\rangle$ actually cannot describe a physical particle.

Exercise 21: Proving Wick's theorem *(4+4+5 = 13 points)*

In this exercise we want to prove Wick's theorem. The general form of the theorem for scalar particles φ_i can be written as

$$T[\varphi_1\varphi_2\dots\varphi_n] = \sum_{c=1}^{\lfloor n/2\rfloor}\sum_{\mathcal{P}_c}\left(\prod_{i=1}^{2c}\overline{\varphi_{a_{2i}}\varphi_{a_{2i-1}}}\right) : a_{2c+1}\dots a_n : \tag{1}$$

where $\mathcal{P}_c$ is the partition into c pairs of two indices (a_{2i}, a_{2i-1}) and the remaining $n-2c$ fields. The sum has to be performed over all possible partitions. In this exercise, we want to prove this relation by induction.

(a) A relation necessary for the proof is given by

$$[A_1, A_2\dots A_m] = \sum_{i=2}^{m} A_2\dots A_{i-1}[A_1, A_i]A_{i+1}\dots A_m \qquad m\geq 3\,.$$

Show that this relation is true through induction.

(b) By using the results from part (a), prove that

$$\varphi_1 : \varphi_2\dots\varphi_n :=: \varphi_1\varphi_2\dots\varphi_n : + \sum_{i=2}^{m} : \varphi_2\dots\varphi_{i-1}\overline{\varphi_1\varphi_i}\varphi_{i+1}\dots\varphi_n :$$

holds.

(c) Without loss of generality, we can assume $t_i < t_{i+1}$ for our n fields. With this, we get

$$T(\varphi_1\varphi_2\dots\varphi_n) = \varphi_1 T(\varphi_2\dots\varphi_n)\,.$$

Use this to prove Eq. (1).