Theoretische Teilchenphysik 1

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Exercise sheet 10

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Lecture website: `https://www.itp.kit.edu/courses/ss2018/ttp1`

Exercise 24: Compton scattering (P) (*1+1+4+5+3+4+2 = 20 points*)

In this exercise, we will consider the so-called Compton scattering, i.e. the scattering of an electron and a photon: $e^-\gamma \to e^-\gamma$.

(a) Draw all possible tree-level Feynman diagrams contributing to the QED process

$$e^-(p_1) + \gamma(k_1) \to e^-(p_2) + \gamma(k_2) \ ,$$

where all external particles shall be on-shell, i.e. $p_1^2 = p_2^2 = m^2$ with the electron mass m and $k_1^2 = k_2^2 = 0$.

(b) By using the Feynman rules of QED as presented in the lecture, derive the amplitudes of all tree-level diagrams.

(c) Show that the full amplitude, after simplification, reads

$$i\mathcal{M} = e^2 \epsilon_\mu^\lambda(k_1)\epsilon_\nu^{\lambda'*}(k_2)\bar{u}(p_2,s_2)\left[\frac{\gamma^\nu \not{k}_1\gamma^\mu + 2\gamma^\nu p_1^\mu}{2p_1k_1} + \frac{\gamma^\mu \not{k}_2\gamma^\nu - 2\gamma^\mu p_1^\nu}{2p_1k_2}\right]u(p_1,s_1)\,.$$

(d) We now want to calculate the squared amplitude which is averaged over all initial and summed over all final electron spins and photon polarizations. Show that the averaged and summed squared amplitude is given by

$$\frac{1}{4}\sum_{polarizations}|\mathcal{M}|^2 = 2e^4\left[\frac{p_1k_2}{p_1k_1} + \frac{p_1k_1}{p_1k_2} + 2m^2\left(\frac{1}{p_1k_1} - \frac{1}{p_1k_2}\right) + m^4\left(\frac{1}{p_1k_1} - \frac{1}{p_1k_2}\right)^2\right].$$

Hint You can use the spin sums from exercise 15(d) as well as the completeness relation from exercise 19(c), where you drop the term that goes with $\frac{1}{M}$.

(e) As the previous result is Lorentz-invariant, we can choose our reference frame as it fits us best. For the next part of the calculation, we choose the so-called "lab frame" where the initial electron is at rest and the initial photon moves along the positive z direction. We therefore have

$$p_1 = (m, 0)\ , \quad k_1 = (\omega_1, 0, 0, \omega_1)$$

with the initial photon energy ω_1. We can now further choose our coordinate system such that the scattering is happening in the y-z Plane with a scattering angle ϑ such that the four-momentum of the outgoing photon is given by

$$k_2 = (\omega_2, 0, \omega_2 \sin\vartheta, \omega_2 \cos\vartheta)\ .$$

Show that the energy of the outgoing photon can be written as

$$\omega_2 = \frac{\omega_1}{1 + \frac{\omega_1}{m}(1 - \cos\vartheta)}\ .$$

(f) We now want to compute the differential cross-section $\mathrm{d}\sigma$ in the lab frame. Show that

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\vartheta} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{\omega_2}{\omega_1} + \frac{\omega_1}{\omega_2} - \sin^2\vartheta\right)\ .$$

Hint: in natural units, the electric charge can be expressed as $e^2 = 4\pi\alpha$.

(g) Finally, we want to study the low-energy limit, i.e. $\omega_1 \to 0$. Show that in this so-called *Thomson limit*, you find the classical result

$$\sigma_{tot} = \int \mathrm{d}\cos\vartheta \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\vartheta} = \frac{8\pi\alpha^2}{3m^2}\ .$$