Karlsruher Institut für Technologie

# Emission kernels of parton shower

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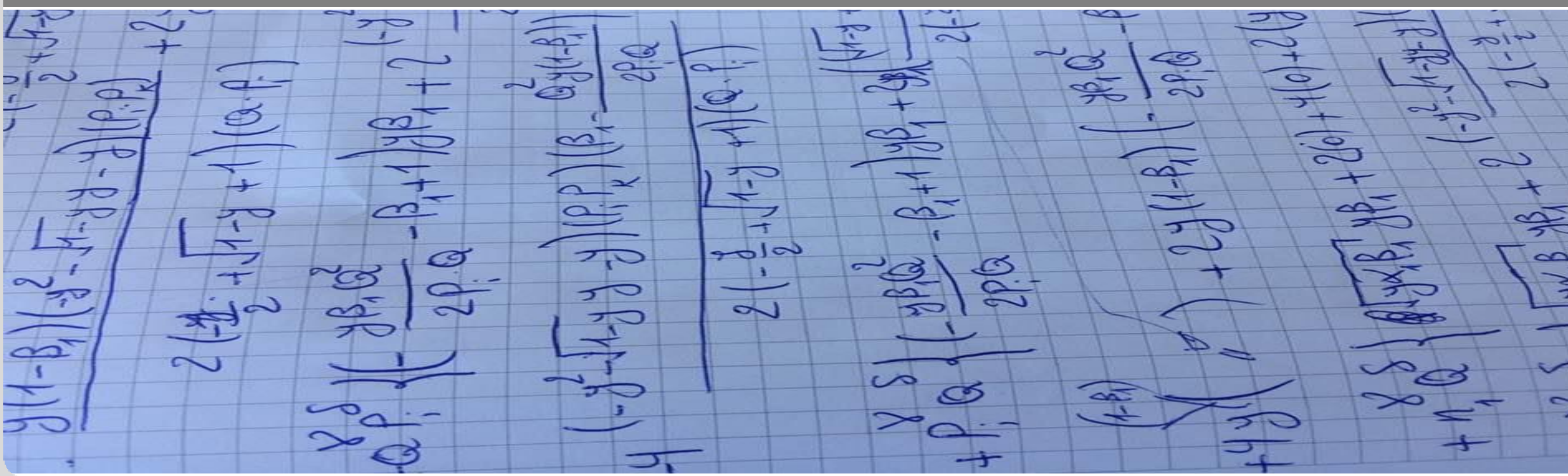
**ITP Seminar | 04.07.2019**

Institut for theoretical physics (ITP)



# Motivation

- To compare experimental data with theoretical predictions ➡ (**GPMC**)
- (**GPMC**) ➡ different energy scales
- Higher simulation accuracy ➡ compare NNLO and higher order calculations with parton showers
- Increasing the accuracy of the shower models

➡ higher-order splitting functions.

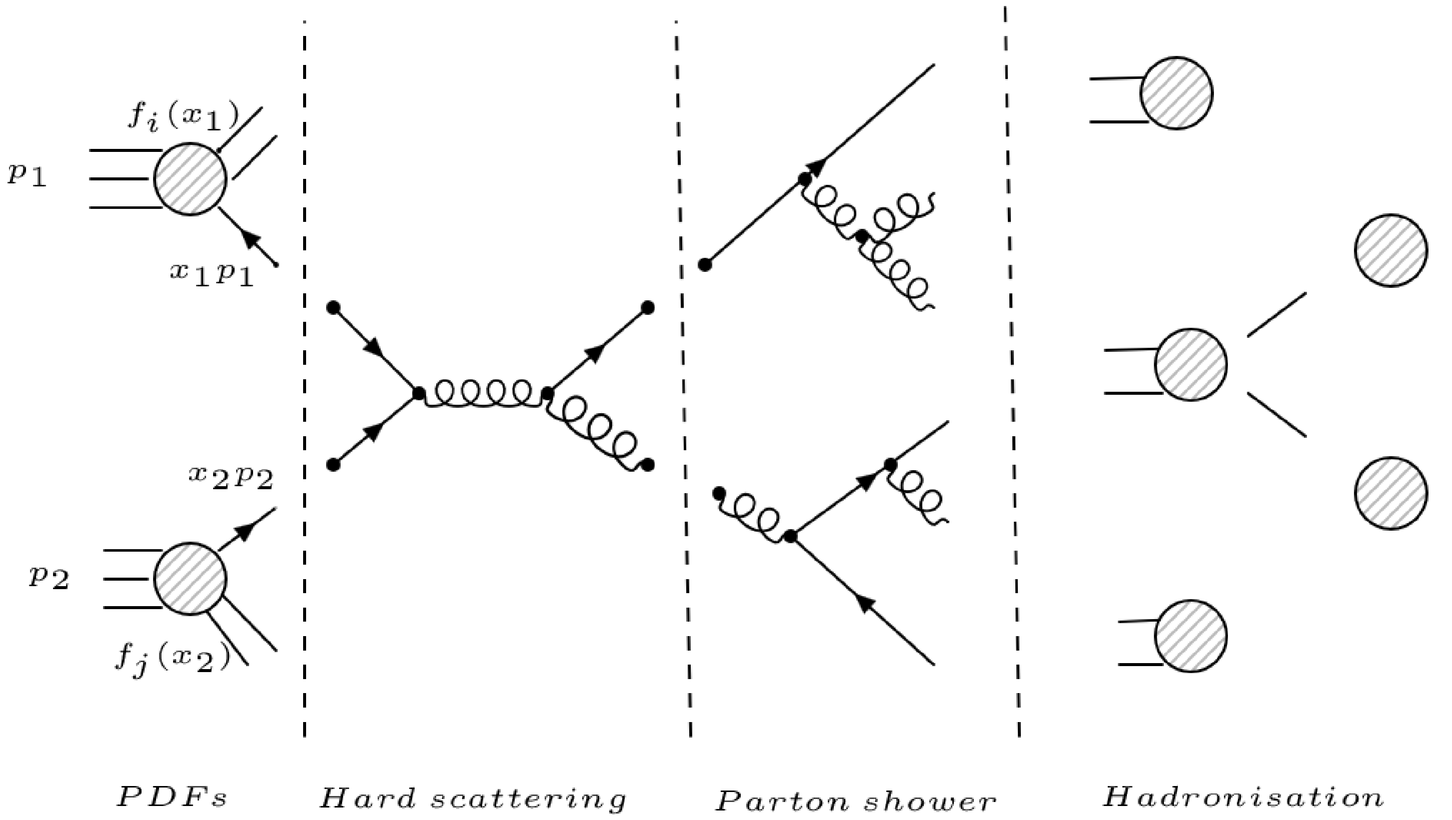
- The three major Monte Carlo programs **Pythia**, **Sherpa** and **Herwig**

Problem: ✗ the known structure of singularities present in a NNLO calculation

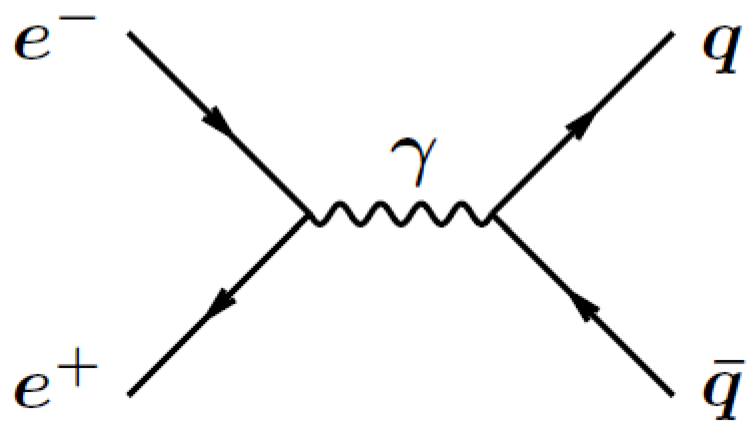
# Factorisation theorem

$$\sigma = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \sigma_{ij}(x_1, x_2, Q^2/\mu^2 \dots)$$

R. Keith Ellis, W. James Stirling, and B. R. Webber



# $e^+e^- \to 2$ jets:



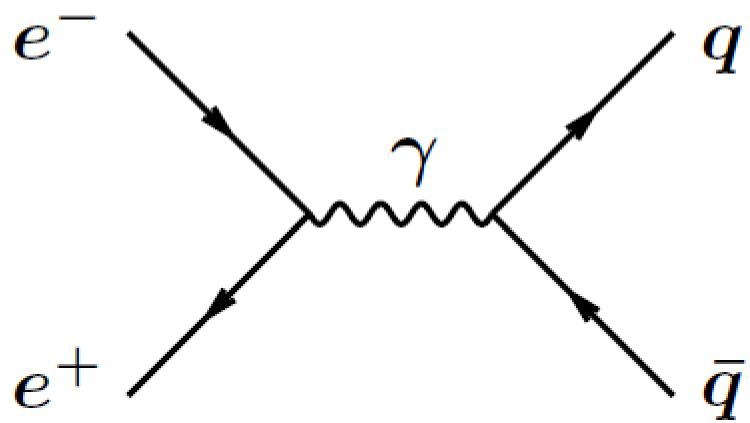

$$\sigma^{LO} = \int_m d\sigma^B$$

$m$-parton phase space integral

Born x-sec for $e^+e^- \to q\bar{q}$ $(m = 2)$

# $e^+e^- \to 2$ jets:

**Finite!**
Calculation in d=4, no regularisation needed

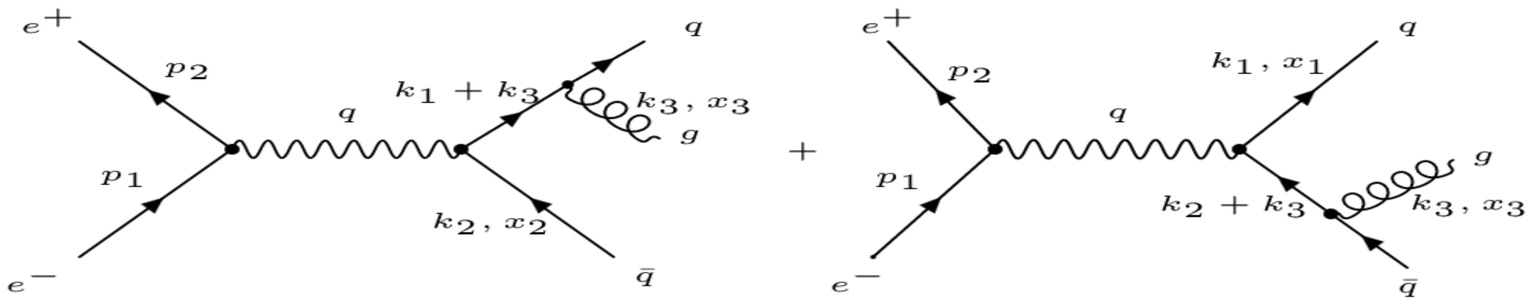



$$\sigma^{LO} = \int_m d\sigma^B$$

$m$-parton phase space integral

Born x-sec for $e^+e^- \to q\bar{q}$ $(m = 2)$

# NLO: IR and Collinear Divergences

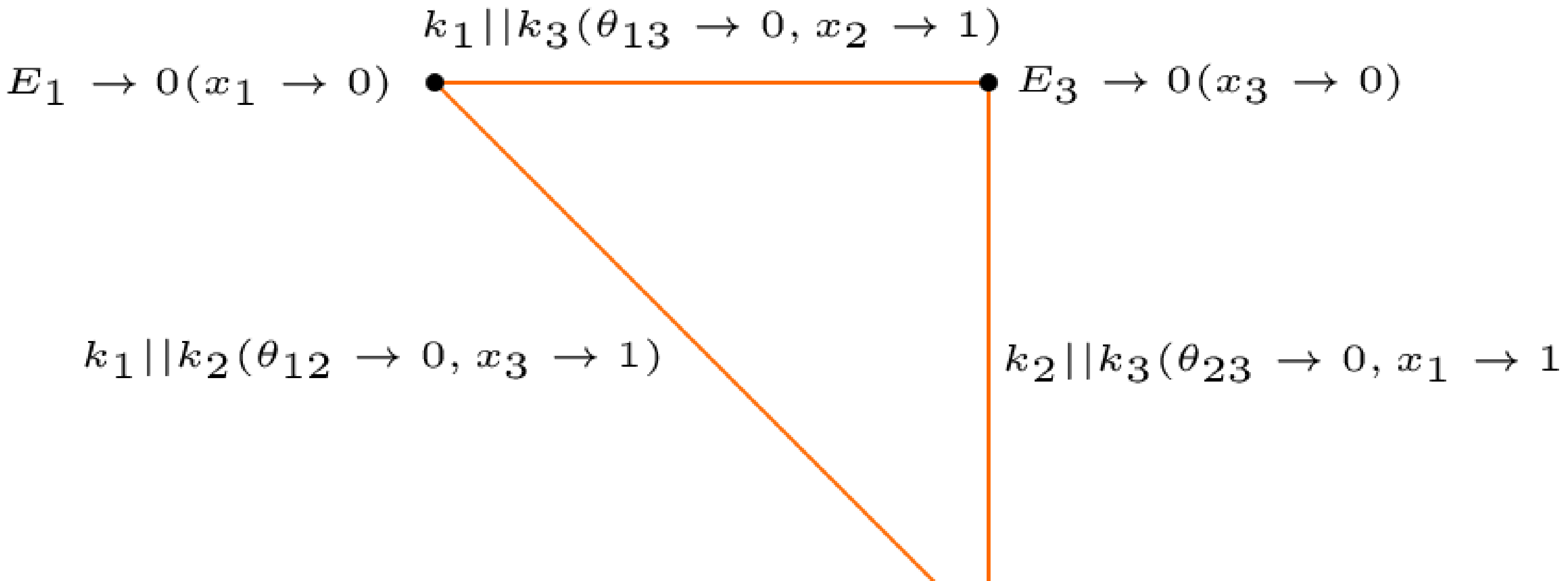


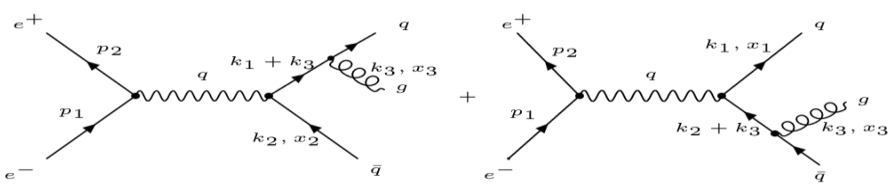

Michiel Botje, QCD lecture Nov. 2013

$$\frac{d^2\sigma}{dx_1 dx_2} = \left(\frac{4\pi\alpha}{s}\right) \sum {e_i}^2 \frac{2\alpha_s}{3\pi} \frac{{x_1}^2 + {x_2}^2}{(1-x_1)(1-x_2)}$$

* $x_1 \to 1$
* $x_2 \to 1$
* $x_1 \to 1 \wedge x_2 \to 1$

# Three-parton configurations



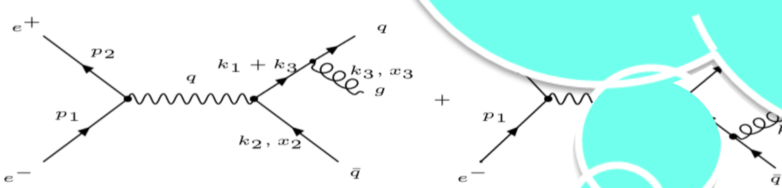
# Three-parton configurations

$E_1 \to 0(x_1$ … $0)$

T. Grandou. The Kinoshita-Lee-Nauenberg (KLN) theorem

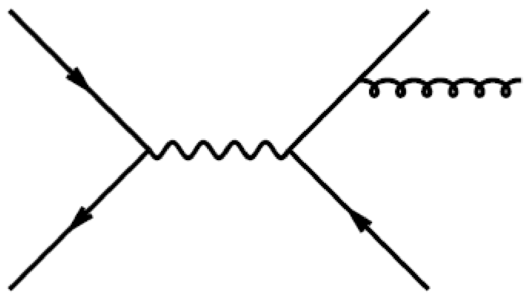
Soloution: KLN-Theorem
a summation over degenerate initial and final states removes all IR divergences. The sum of the integrals $\int_R$ and $\int_V$ over the phase space is finite.

$k_1||k$ … $, x_1 \to 1)$

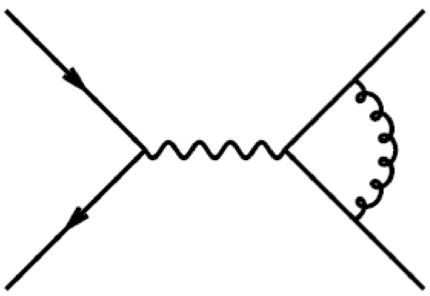



… $\to 0(x_2 \to 0)$

# Subtraction method



real emission contributions
$m+1$ parton kinematics



virtual corrections
$m$ parton kinematics

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

IR divergent
☞regularize in $d = 4 - 2\varepsilon$ dim

introduce local counterterm $d\boldsymbol{\sigma}^A$ with same singularity structure as $d\boldsymbol{\sigma}^R$:

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \underbrace{\left[d\boldsymbol{\sigma}^R - d\boldsymbol{\sigma}^A\right]}_{\text{finite}} + \int_{m+1} d\boldsymbol{\sigma}^A + \int_m d\boldsymbol{\sigma}^V$$

↓

can safely set $\varepsilon \longrightarrow 0$

perform integral numerically in four dimension

# Behaviour of the Counterterm:

- matches singular behaviour of $d\sigma_R$ exactly in $d$ dim
- convenient for Monte Carlo integration
- exactly integrable analytically over one-parton PS in $d$ dim
- extra feature: universal structure

**Solution: Dipol factorisation**

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

# Determination of emission kernels

color/spin projection of Born x-sec

universal dipole factors

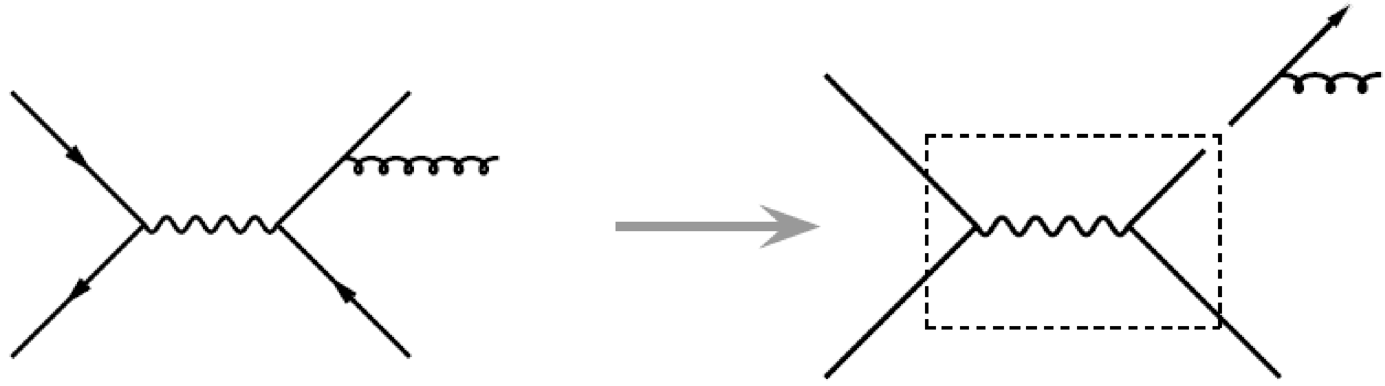
$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

dipoles for all $(m+1)$ configurations corresponding to given $m$-parton state

PS convolution & color/spin summation

# Singularity Structure

e. g.:



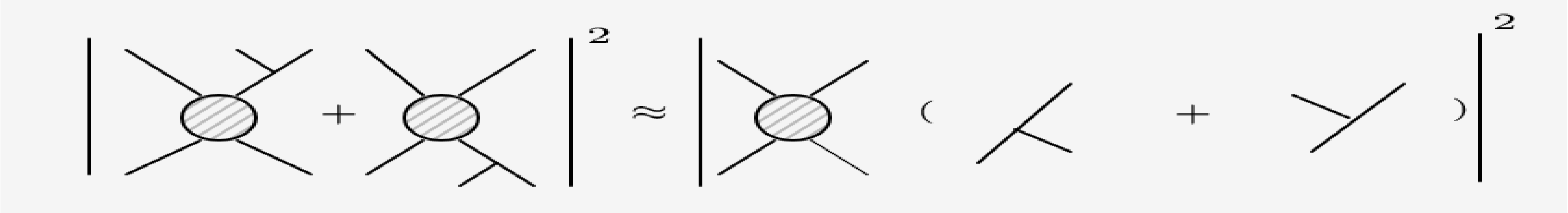
universal structure: for each singular configuration

$$|\mathcal{M}_{m+1}|^2 \to |\mathcal{M}_m|^2 \otimes \mathrm{V}_{ij,k}$$

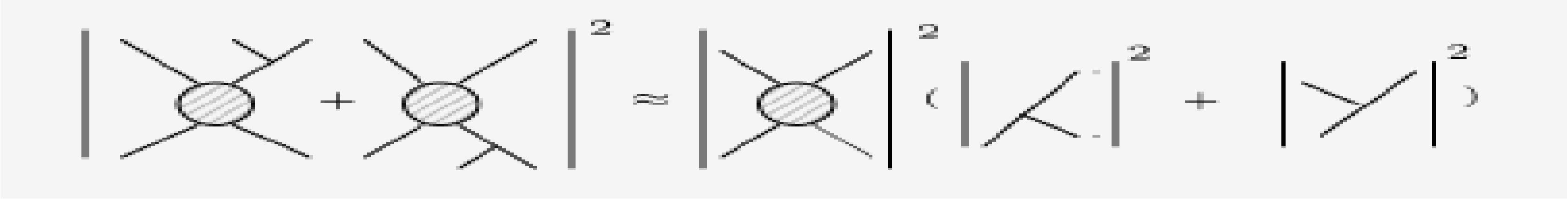
$\mathrm{V}_{ij,k}$ ... contains singularities, depends on momenta & quantum numbers of partons $i, j, k$

$ij$ and $k$ ... emitter and spectator

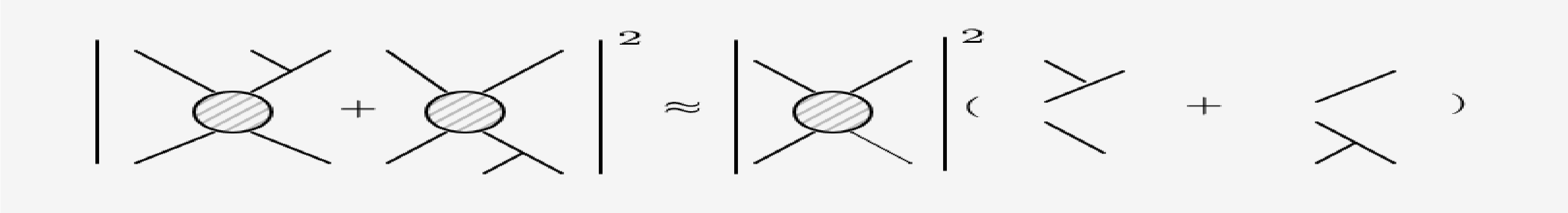
# Schematic depiction of the different types of factorizations




Factorisation in soft region



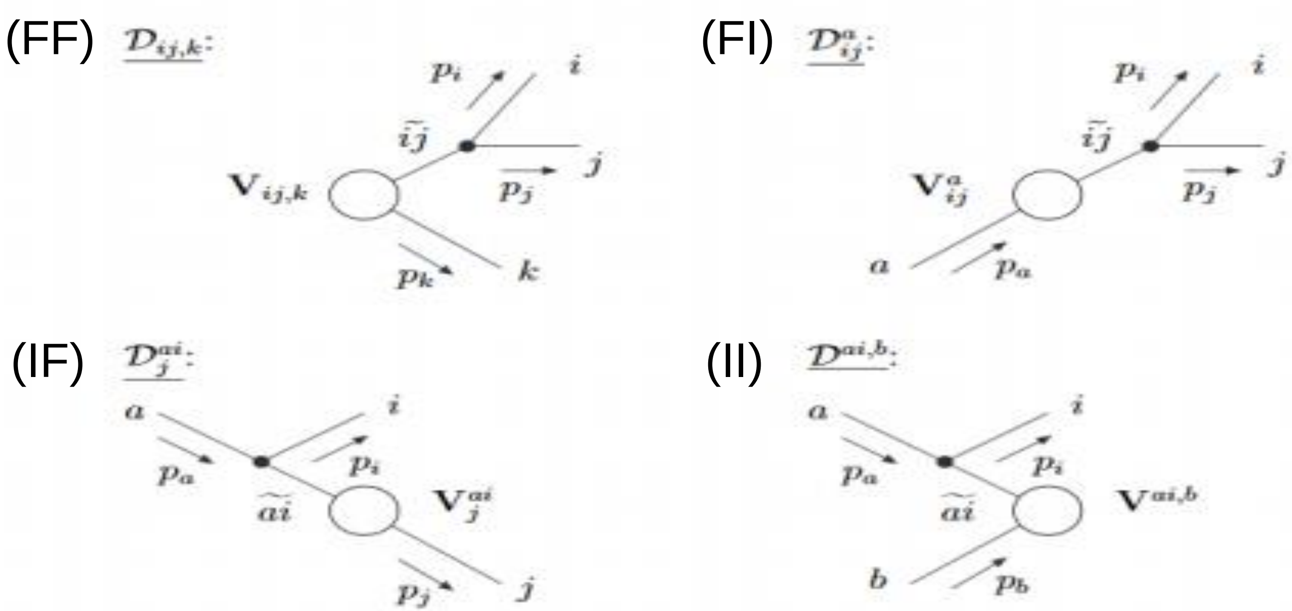

Factorisation in collinear region




Factorisation in dipole formalism

# Factorisation in dipole formalism

$$|\mathcal{M}_m + 1|^2 = \sum_{i,j} \sum_{k \neq i,j} \mathcal{D}_{ij,k} + \sum_{i,j} \sum_{a} \mathcal{D}^{a}{}_{ij} + \sum_{a,i} \sum_{k \neq i} \mathcal{D}^{ai}{}_{k} + \sum_{a,i} \sum_{b \neq a} \mathcal{D}^{aj,b} + \ldots$$




Catani, Dittmaier, Seymour, and Trocsanyi

# Kinematics

$$|\mathcal{M}_{m+1}|^2 =< 1,\ldots,m+1||1,\ldots,m+1 >= \sum_{k\neq i,j} \mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1}) + \text{finite terms}$$

$$\mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1}) = \frac{-1}{2p_i\cdot p_j}\ {}_m< 1,\ldots,\tilde{ij},\ldots,k,\ldots,m+1|\frac{T_k\cdot T_{ij}}{T_{ij}^2}V_{ij,k}|1,\ldots,\tilde{ij},\ldots,k,\ldots,m+1>_m$$

$\mathbf{V}_{\mathbf{ij,k}}$ splitting kernel in helicity space of emitter explicit form depends on parton type become proportional to Altarelli-Parisi splitting functions and Eikonal factors in collinear and soft limits, resp.

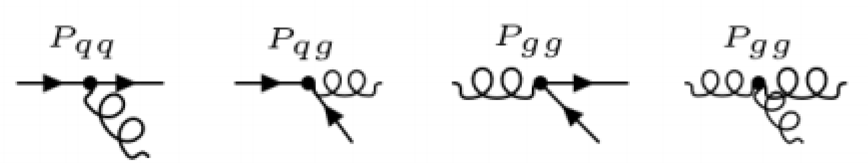
$\mathbf{T}_{\mathbf{k}}, \mathbf{T}_{\mathbf{ij}}$ colour charges of spectator and emitter

Barbara Jäger, The Dipole Subtraction Method, October 2006

# Altarelli-Parisi splitting functions

The spin-averaged unregularized Altarelli-Parisi splitting functions in *d*-dimensions the probability that a daughter parton i with momentum fraction z splits from a parent parton j

$$\left.\begin{aligned}
\langle \hat{P}_{qq} \rangle &= C_F\left[\frac{1+z^2}{1-z} - \varepsilon(1-z)\right] \\
\langle \hat{P}_{gq} \rangle &= T_R\left[1 - \frac{2z(1-z)}{1-\varepsilon}\right] \\
\langle \hat{P}_{qg} \rangle &= C_F\left[\frac{1+(1-z)^2}{z} - \varepsilon z\right] \\
\langle \hat{P}_{gg} \rangle &= 2C_A\left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right]
\end{aligned}\right\} \quad \text{Alterali-Parisi}$$




No S. No S.

# Mapping 3 partons to 2 for single emission

$$\left.\begin{aligned} q_i{}^\mu &= z p_i{}^\mu + y(1-z) p_j{}^\mu + \sqrt{zy(1-z)}\, m^\mu{}_\perp \\ q^\mu &= (1-z) p_i{}^\mu + yz p_j{}^\mu - \sqrt{zy(1-z)}\, m^\mu{}_\perp \\ q_j{}^\mu &= (1-y) p_j{}^\mu \end{aligned}\right\} \quad \text{parametrisation}$$

$m_\perp$ represents the transverse component

Includes soft limit ($z \to 1$) and collinear limit ($y \to 0$)

Problem:

- Catani Seymour method only works for single emission
- No singularity structure for NNLO matching

Dasgupta et al. 2018

# New kinematic for m+1 to m partons:

$$k_l{}^\mu = \alpha_l \alpha \Lambda^\mu{}_\nu p_i{}^\nu + y\beta n^\mu + \sqrt{y\alpha_l\beta_l}\, n^\mu{}_{\perp,l} \qquad l = 1,\ldots,m$$

$$q_i{}^\mu = (1 - \sum_{l=1}^{m} \alpha_l)\alpha\Lambda^\mu{}_\nu p_i{}^\nu + y(1 - \sum_{l=1}^{m}\beta_l) n^\mu - \sqrt{y\alpha_l\beta_l}\, n^\mu{}_{\perp,l}$$

$$q_k{}^\mu = \alpha\Lambda^\mu{}_\nu p_k{}^\nu \qquad k = 1,\ldots,n \qquad k \neq i$$

Subs.:

$$q_i \to q_i$$
$$q \to k_1$$
$$q_j \to q_k$$

# Recipe for the usage of the new parametrisation

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Parametrization in terms of $(k_1 \cdot q_i)(k_1 \cdot q_k)$

$$\boxed{(k_1 \cdot q_i)(k_1 \cdot q_k) \approx y(1-\beta_1)(1-y)\,(p_i \cdot p_k)(p_i \cdot Q)}$$

$$k_1{}^{\eta} k_1{}^{\eta'} = [(1-\beta_1)^2 - y^2 {\beta_1}^2 (\frac{Q^2}{2p_i \cdot Q})^2] p_i{}^{\eta} p_i{}^{\eta'} - y^2 {\beta_1}^2 (\frac{Q^2}{2p_i \cdot Q}) p_i{}^{\eta} Q^{\eta'} - y^2 {\beta_1}^2 (\frac{Q^2}{2p_i \cdot Q}) Q^{\eta} p_i{}^{\eta'}$$

$$k_1{}^{\eta} q_i{}^{\eta'} = [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i \cdot Q})] p_i{}^{\eta} p_i{}^{\eta'} + y{\beta_1}^2 Q^{\eta} p_i{}^{\eta'}$$

$$q_i{}^{\eta} k_1{}^{\eta'} = [\beta_1(1-\beta_1) - y{\beta_1}^2(\frac{Q^2}{2p_i \cdot Q})] p_i{}^{\eta} p_i{}^{\eta'} + y{\beta_1}^2 p_i{}^{\eta} Q^{\eta'}$$

$$q_i{}^{\eta} q_i{}^{\eta'} = {\beta_1}^2 p_i{}^{\eta} p_i{}^{\eta'}$$

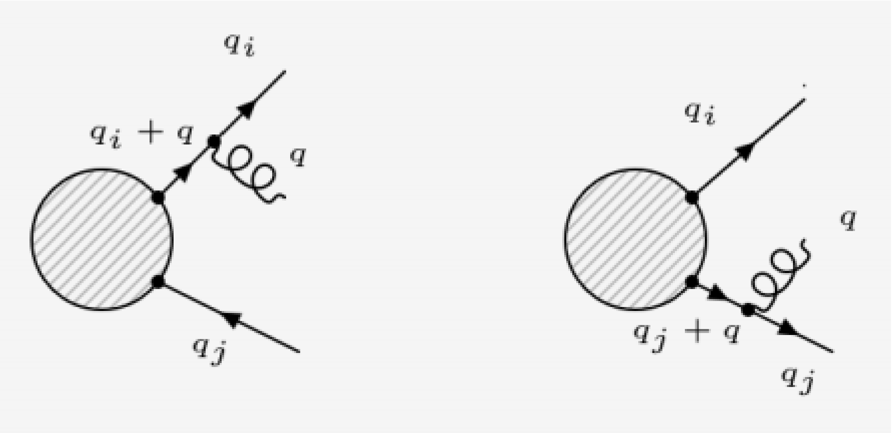
$$k_1{}^{\eta} q_k{}^{\eta'} = [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i \cdot Q})]\sqrt{1-y}\, p_i{}^{\eta} p_k{}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i \cdot Q}) A_1\, p_i{}^{\eta} p_i{}^{\eta'}$$
$$- y\beta_1(\frac{Q^2}{2p_i \cdot Q}) A_2\, p_i{}^{\eta} Q^{\eta'} + y\beta_1 A_1\, Q^{\eta} p_i{}^{\eta'} + y\beta_1 A_2\, Q^{\eta} Q^{\eta'} + y\beta_1\sqrt{1-y}\, Q^{\eta} p_k{}^{\eta'}$$

$$q_i{}^{\eta} q_k{}^{\eta'} = A_1\beta_1 p_i{}^{\eta} p_i{}^{\eta'} + A_2\beta_1 p_i{}^{\eta} Q^{\eta'} + \beta_1\sqrt{1-y}\, p_i{}^{\eta} p_k{}^{\eta'}$$

$$q_k{}^{\eta} k_1{}^{\eta'} = [(1-\beta_1) - y\beta_1(\frac{Q^2}{2p_i \cdot Q})]\sqrt{1-y}\, p_k{}^{\eta} p_i{}^{\eta'} - y\beta_1(\frac{Q^2}{2p_i \cdot Q}) A_1\, p_i{}^{\eta} p_i{}^{\eta'}$$
$$- y\beta_1(\frac{Q^2}{2p_i \cdot Q}) A_2\, Q^{\eta} p_i{}^{\eta'} + y\beta_1 A_1\, p_i{}^{\eta} Q^{\eta'} + y\beta_1 A_2\, Q^{\eta} Q^{\eta'} + y\beta_1\sqrt{1-y}\, p_k{}^{\eta} Q^{\eta'}$$

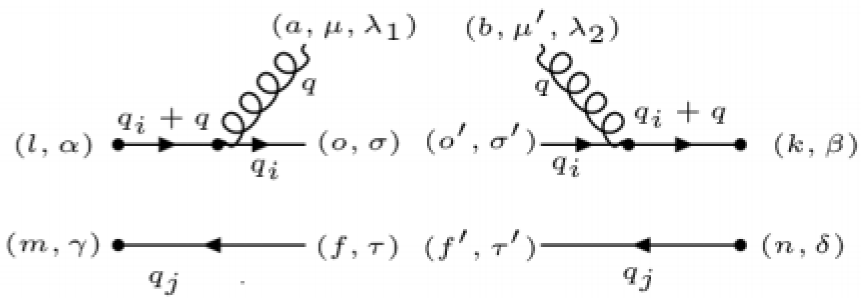
$$q_k{}^{\eta} q_i{}^{\eta'} = A_1\beta_1 p_i{}^{\eta} p_i{}^{\eta'} + A_2\beta_1 Q^{\eta} p_i{}^{\eta'} + \beta_1\sqrt{1-y}\, p_k{}^{\eta} p_i{}^{\eta'}$$

# Example: $q\overline{q}$ emission kernel




$$P_{\bar{q}_i\bar{q}_j} = P_{q_iq_j} \equiv P_{qq}\delta_{ij}$$
$$P_{\bar{q}_ig} = P_{q_ig} \equiv P_{qg}$$
$$P_{\bar{g}\bar{q}_i} = P_{gq_i} \equiv P_{gq}\delta_{ij}$$

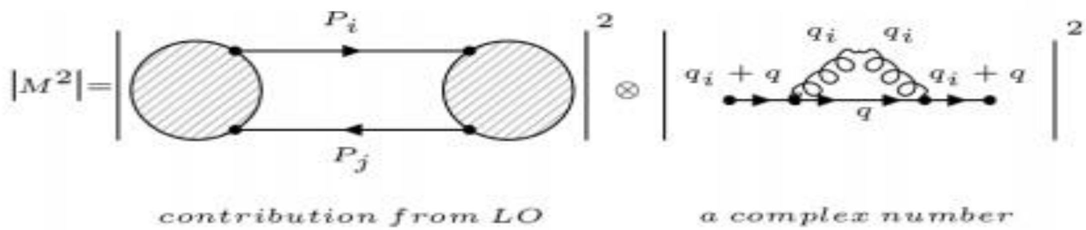
# Matrix element of a quark with a gluon radiation $|M_1|^2$




Can ignore finite terms $y^2$ and momenta are on-shell

Before mapping: $|M_1|^2 = \frac{-g_s^2[T^a]_o{}^k\,[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[(\not{q}_i + \not{q})\;\gamma^{\mu'}\;\;\not{q}_i\,\gamma_{\mu'}\,(\not{q}_i + q)][\not{q}_j]$

Expectation:

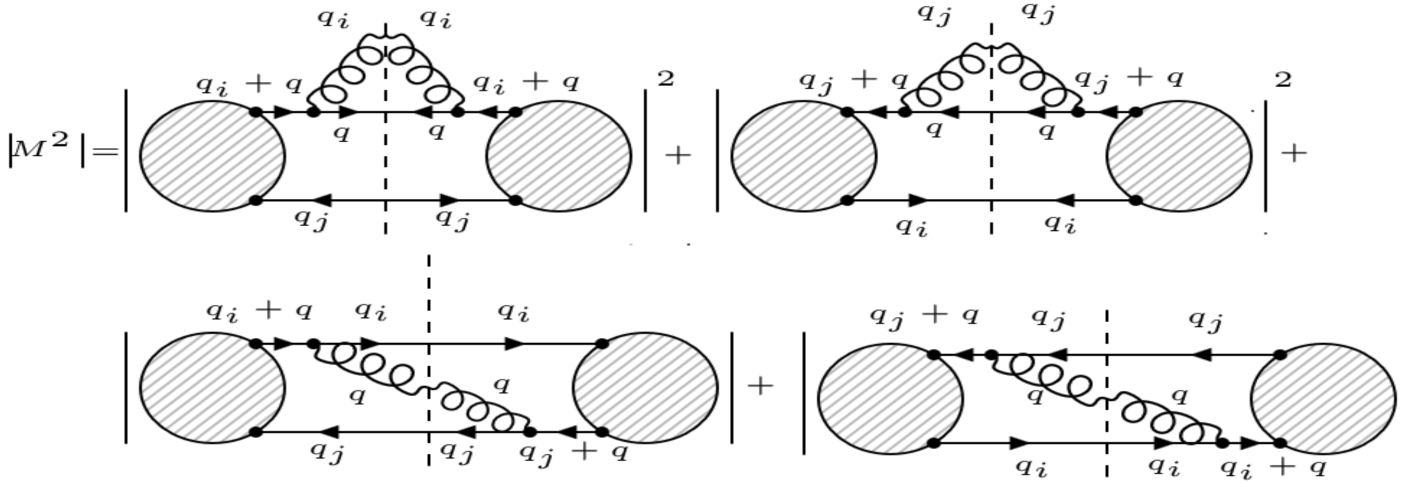


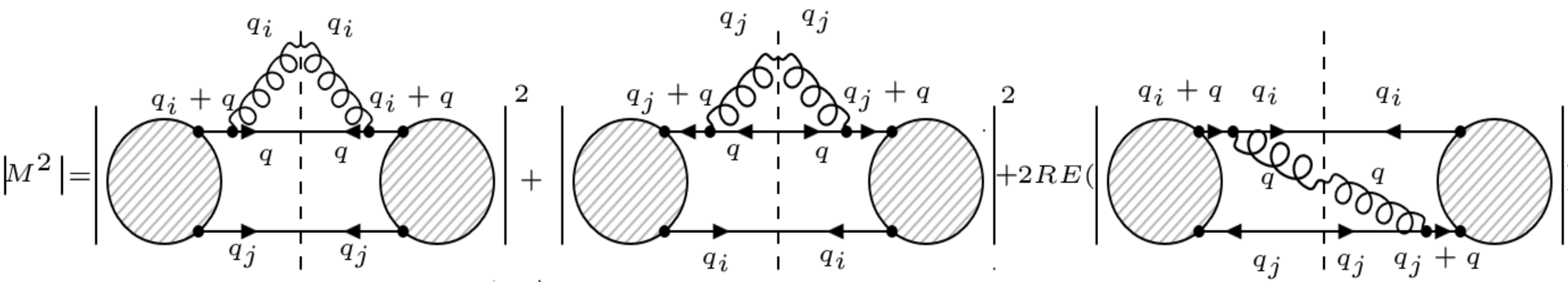

$$|M_1|^2 = \frac{-g_s^2[T^a]_o{}^k\,[T^a]_o{}^l}{(q_i+q)^2(q_i+q)^2}[\not{P}_i][\not{P}_j] \otimes \textcolor{red}{(a\ complex\ number)}$$

After mapping: $|M_1|^2 = \frac{g_s^2[T^a]_o{}^k\,[T^a]_o{}^l}{2y(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \otimes (d-2)(1-z)(1-y)$

# Final result:

$$| M|^2 = | M_1|^2 +| M_2|^2 + M_1 \, M_2^\dagger + M_1^\dagger M_2$$



$$| M|^2 = (d-2)(1-z)(1-y)\ \frac{g_s^2 [T^a]_o{}^k\ [T^a]_o{}^l}{2y(p_i\cdot p_j)}[\not{p}_i][\not{p}_j]$$

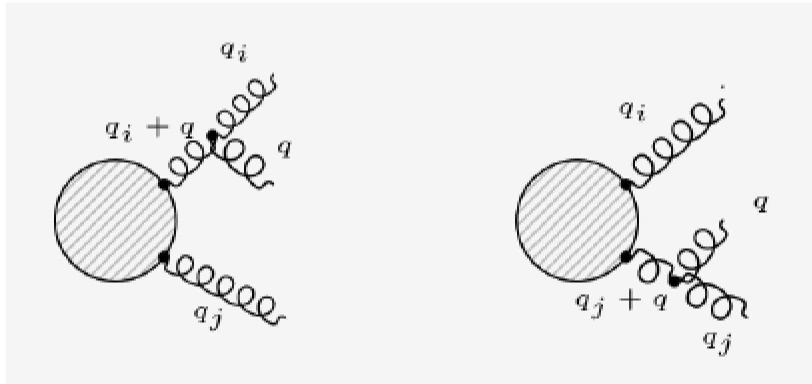
$$-\ (d-2)yz^2\ \frac{g_s^2\ [T^c]_f{}^m\ [T^c]_f{}^n}{2(1-z)(1-y)(p_i\cdot p_j)}[\not{p}_i][\not{p}_j]$$

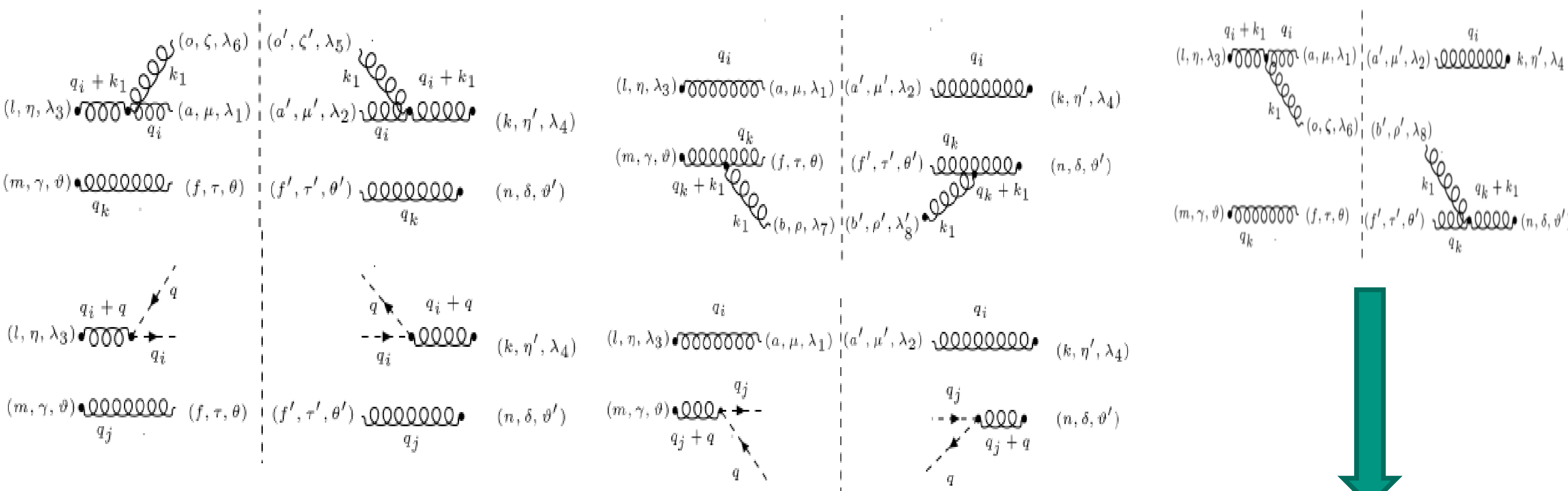
$$+\ 2RE((\frac{-2z}{z-1})\frac{g_s^2\ \ [T^a]_o{}^l\ [T^a]_f{}^n}{2y(p_i\cdot p_j)}[\not{p}_i][\not{p}_j])$$

$$| M|^2\ = \frac{g_s^2}{y(p_i\cdot p_j)}[\not{p}_i][\not{p}_j] \times C_F(\frac{(1+z^2)}{1-z} - \epsilon(1-z))$$

$$= \frac{g_s^2}{q_i\cdot q}[\not{p}_i][\not{p}_j] \times \langle\ \hat{P}_{qq}\rangle$$

# Gluon radiation from a parent gluon

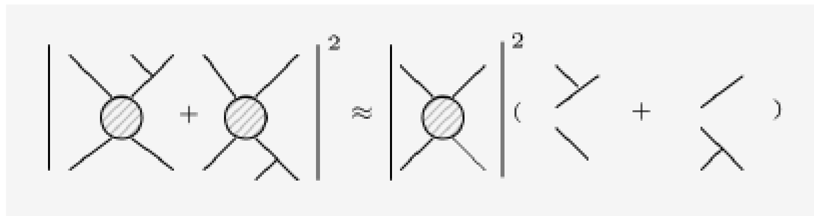



Swapping for indistinguishable partons

# Example applications

$$\frac{d^2\sigma}{dx_1 dx_2} = \hat{\sigma}_0 \frac{\alpha_s}{2\pi} C_F \frac{{x_1}^2 + {x_2}^2}{(1 - x_1)(1 - x_2)}$$



$$|\mathcal{M}_m + 1|^2 = \sum_{i,j} \sum_{k \neq i,j} \mathcal{D}_{ij,k}$$

$$\mathcal{D}_{13,2}(q_i, q_j, q) = \frac{1}{2 p_i \cdot p_j} [\frac{(d-2)(1-z)(1-y)}{y} + \frac{(d-2) y z^2}{(1-z)(1-y)} (\frac{-2z}{z-1}) \frac{1}{y}] |\mathcal{M}_2|^2$$

# Comparison of results in the collinear limit

$$p_i = Q/2\,(1, \vec{0}_\perp, 1)$$

$$p_i = Q/2\,(1, \vec{0}_\perp, 1)$$

$$x_1 = z + y(1-z)$$

$$x_2 = 1 - y$$

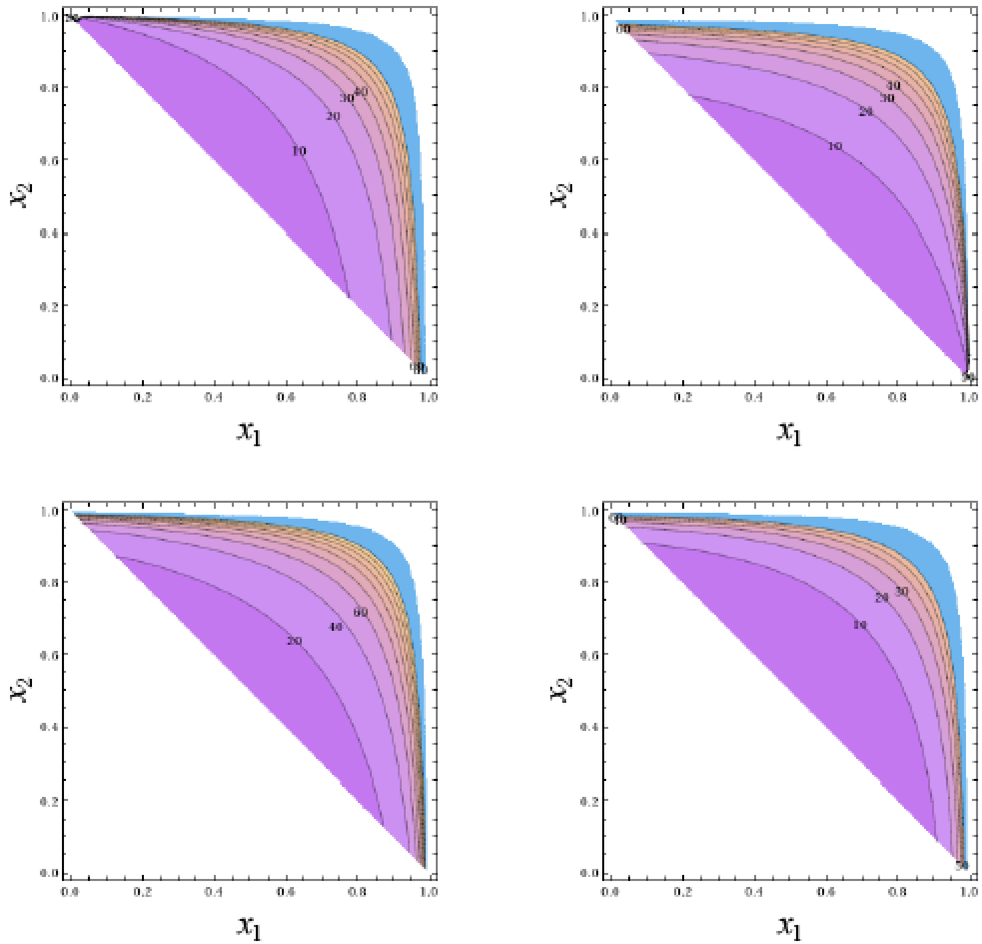
$$x_3 = (1-z) + yz$$

$$\frac{d^2\sigma}{dx_1 dx_2}_{PS_q} = \hat{\sigma_0}\frac{\alpha_s}{2\pi}C_F[\frac{1}{y_{13,2}}(\frac{2}{1-\tilde{z_1}} - (1+\tilde{z_1})]$$

$$\Rightarrow \frac{d^2\sigma}{dx_1 dx_2}_{PS_q} = \hat{\sigma_0}\frac{\alpha_s}{2\pi}C_F[\frac{1}{1-x_2}(\frac{2}{2-x_1-x_2} - (1+x_1)) + \frac{1-x_1}{x_2}]$$

$$\frac{d^2\sigma}{dx_1 dx_2}|_{PS} = \frac{d^2\sigma}{dx_1 dx_2}|_{PS_q} + \frac{d^2\sigma}{dx_1 dx_2}|_{PS_{\bar{q}}} = \hat{\sigma_0}\frac{\alpha_s}{2\pi}C_F[\frac{{x_1}^2 + {x_2}^2}{(1-x_1)(1-x_2)} + \frac{1-x_1}{x_2} + \frac{1-x_2}{x_1}]$$

# Graphical explanation of the results



# Summary and Future outlook

- a mapping 3 -> 2-partons in single emission
- one for $m$ + 1 -> $m$ *general emission* case
- Confirmation of LO Altarelli-Parisi splitting functions
- An approach for simplifying results
- Comparison of  quark-antiquark emission kernel with the known result from electron-positron annihilation in the collinear limit

Future outlook:

- Full matrix element
- Behaviour of the recoil
- Study of double emission

**Thanks for your attention**

**Any questions?**