Discussion of the Weinberg Salam Model

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Theoretische Teilchenphysik II

- Spontaneous symmetry breaking in the SM
- W and Z mass generation
- Charged and neutral current couplings of fermions
- Fermion mass generation
- Higgs boson couplings

Electroweak sector

From experimental facts (charged currents couple only to left-handed fermions, existence of a massless photon and a neutral Z), the gauge group is chosen as $SU(2)_L \times U(1)_Y$.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad \nu_{eR} \equiv \frac{1}{2}(1+\gamma_5)\nu_e \qquad e_R \equiv \frac{1}{2}(1+\gamma_5)e$$

- $SU(2)_L$: weak isospin group. Three generators $\Longrightarrow$ three gauge bosons: W^1, W^2 and W^3.
  Generators for doublets are $T^a = \sigma^a/2$, where σ^a are the 3 Pauli matrices
  For gauge singlets (e_R, ν_R) $T^a \equiv 0$. All satisfy $\left[T^a, T^b\right] = i\epsilon^{abc}T^c$.
  The gauge coupling will be indicated with g.
- $U(1)_Y$: weak hypercharge Y. One gauge boson B with gauge coupling g'.
  One generator (charge) $Y(\psi)$, whose value depends on the fermion field

W^3 and B carry identical quantum numbers ($T_3 = 0$, $Y = 0$) $\Longrightarrow$ they will combine to produce two neutral gauge bosons: Z and γ.

EW gauge-boson sector of the SM

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, \epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z, i.e. forbidden are terms like

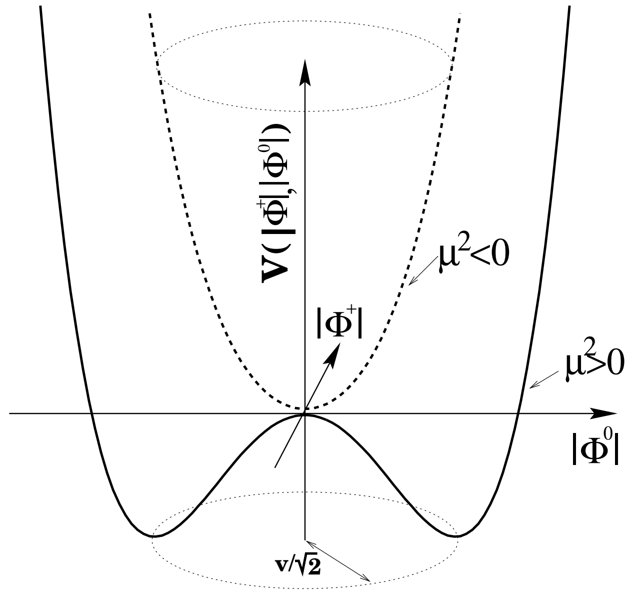
$$\mathcal{L}_{Mass} = \frac{1}{2} m_W^2 W^a_\mu W_a^\mu$$

Spontaneous symmetry breaking

Experimentally, the weak bosons are massive. We give mass to the gauge bosons through the Higgs mechanism: generate mass terms from the kinetic energy term of a scalar doublet field Φ that undergoes spontaneous symmetry breaking.

Introduce a complex scalar doublet

$$
\begin{aligned}
\Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad Y_\Phi = \frac{1}{2} \\
\mathcal{L}_{\text{Higgs}} &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) \\
D^\mu &= \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' Y_\Phi B^\mu \\
V\left(\Phi^\dagger \Phi\right) &= V_0 - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad \mu^2, \lambda > 0
\end{aligned}
$$



Notice the "wrong" mass sign.

$V\left(\Phi^\dagger \Phi\right)$ is $SU(2)_L \times U(1)_Y$ symmetric.

Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i\theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an SU(2)$_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i\theta^i(x)}{v}\right]$.

This gauge choice, called unitary gauge, is equivalent to absorbing the Goldstone modes $\theta^i(x)$.

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Notice that only a scalar field can have a vacuum expectation value.The VEV of a fermion or vector field would break Lorentz invariance.

Consequences for the scalar field H

The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x)\end{pmatrix}$$

becomes

$$V = \frac{\lambda}{4}\left(2vH + H^2\right)^2 = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

Consequences:

- the scalar field H gets a mass which is given by the quartic coupling λ

$$m_H^2 = 2\lambda v^2 \qquad \Longrightarrow \qquad \lambda \approx 0.13 \qquad \text{since} \quad m_H \approx 125\,\text{GeV} \quad \text{and} \quad v = 246.22\,\text{GeV}$$

- there is a term of cubic and quartic self-coupling.
- The coupling $\lambda \approx 0.13$ is small, i.e. perturbation theory is warranted.

Higgs kinetic terms and coupling to W, Z

$$
\begin{aligned}
D^\mu \Phi &= \left(\partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig'\frac{1}{2}B^\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2\sqrt{2}}\left[g \begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix} + g' B^\mu \right] \begin{pmatrix} 0 \\ v + H \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}(v+H)\begin{pmatrix} g\left(W_1^\mu - iW_2^\mu\right) \\ -gW_3^\mu + g'B^\mu \end{pmatrix}\right] \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}\left(1 + \frac{H}{v}\right)\begin{pmatrix} vg\, W^{\mu +} \\ -v\sqrt{(g^2+g'^2)/2}\; Z^\mu \end{pmatrix}
\end{aligned}
$$

$$
(D^\mu \Phi)^\dagger D_\mu \Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu +} W_\mu^- + \frac{1}{2}\frac{\left(g^2+g'^2\right) v^2}{4} Z^\mu Z_\mu\right]\left(1 + \frac{H}{v}\right)^2
$$

Consequences

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad\qquad m_Z^2 = \frac{\left(g^2 + g'^2\right) v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_W W_\mu^+ W^{-\mu} H + \frac{1}{2} \frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Higgs coupling proportional to mass

- tree-level HVV (V = vector boson) coupling requires VEV! e.g. $g m_W = g^2 v/2$
Normal scalar couplings give $\Phi^\dagger \Phi V$ or $\Phi^\dagger \Phi V V$ couplings only.

Gauging the symmetry: fermion Lagrangian

Following the gauge recipe (for one generation of leptons, quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L\,\not{D}\,L_L + i\,\bar{\nu}_{eR}\,\not{D}\,\nu_{eR} + i\,\bar{e}_R\,\not{D}\,e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu\,T^i - ig'Y_\psi\,B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0, \qquad i = 1,2,3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L\,\not{\partial}\,L_L + i\,\bar{\nu}_{eR}\,\not{\partial}\,\nu_{eR} + i\,\bar{e}_R\,\not{\partial}\,e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\left[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L\right] + g'\,B_\mu\Big[Y_L\left(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L\right) \\
&\quad + Y_{\nu_{eR}}\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y_{e_R}\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

Fermion couplings fixed by renormalizability and gauge quantum numbers

				$SU(3)$	$SU(2)$	$U(1)_Y$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0

Weak mixing angle

W_μ^3 and B_μ mix to produce two orthogonal mass eigenstates

$$\begin{aligned}\text{massive partner}: \quad g\,W_\mu^3 - g'\,B_\mu &= \sqrt{g^2+g'^2}\,Z_\mu = \sqrt{g^2+g'^2}\left(W_\mu^3\cos\theta_W - B_\mu\sin\theta_W\right)\\ \text{orthogonal, massless}: \quad g'\,W_\mu^3 + g\,B_\mu &= \sqrt{g^2+g'^2}\,A_\mu = \sqrt{g^2+g'^2}\left(W_\mu^3\sin\theta_W + B_\mu\cos\theta_W\right)\end{aligned}$$

with mixing angle fixed by $\quad \cos\theta_W = \dfrac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \dfrac{g'}{\sqrt{g^2+g'^2}}$

Write the NC Lagrangian in terms of these mass eigenstates

$$\mathcal{L}_{NC} = \bar\psi\gamma_\mu\left(gT_3W_3^\mu + g'YB^\mu\right)\psi = \bar\psi\gamma_\mu\left(\frac{1}{\sqrt{g^2+g'^2}}(g^2T_3 - g'^2Y)Z^\mu + \frac{gg'}{\sqrt{g^2+g'^2}}(T_3+Y)A^\mu\right)\psi$$

Must identify electron charge, e, as

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

and the charge of a particle, as a multiple of the positron charge, is given by the Gell-Mann–Nishijima formula: $Q = T_3 + Y$

The neutral current

It is customary to write the Z coupling to fermions in terms of the electric charge Q and the third component of isospin ($T_3 = \pm 1/2$ for left-chiral fermions, 0 for right-chiral fermions)

$$\mathcal{L}_{NC} = \bar{\psi}\gamma_\mu \left(\frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2 Y) Z^\mu + \frac{g g'}{\sqrt{g^2+g'^2}}(T_3 + Y) A^\mu \right) \psi = e\bar{\psi}\gamma_\mu Q \psi A^\mu + \bar{\psi}\gamma_\mu Q_Z \psi Z^\mu$$

Q_Z is given by

$$Q_Z = \frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2(Q - T_3)) = \frac{e}{\cos\theta_W \sin\theta_W}\left(T_3 - Q\sin^2\theta_W\right)$$

This procedure works for leptons and also for the quarks (see more later)

$$Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \begin{array}{l} u_R^i = u_R, c_R, t_R \\ d_R^i = d_R, s_R, b_R \end{array}$$

Fermion mass generation

A direct mass term is not invariant under $\mathrm{SU}(2)_L$ or $\mathrm{U}(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

since left- and righthanded fields have different gauge quantum numbers

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d \bar{Q}_L \Phi d_R - \Gamma_d^* \bar{d}_R \Phi^\dagger Q_L \\
&\quad -\Gamma_u \bar{Q}_L \Phi_c u_R + \text{h.c.} \\
&\quad -\Gamma_e \bar{L}_L \Phi e_R + \text{h.c.} \\
&\quad -\Gamma_\nu \bar{L}_L \Phi_c \nu_R + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q, L are left-handed doublet fields and d_R, u_R, e_R, ν_R are right-handed $SU(2)$ -singlet fields.

Notice: neutrino masses can be implemented via Γ_ν term. Since $m_\nu \approx 0$ we neglect it in the following.

Fermion masses for three generations

Generate fermion masses for three generations of quarks and leptons by generalizing

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij}\bar{Q}_L'^{\,i}\Phi d_R'^{\,j} - \Gamma_d^{ij*}\bar{d}_R'^{\,j}\Phi^\dagger Q_L'^{\,i} \\
&\quad -\Gamma_u^{ij}\bar{Q}_L'^{\,i}\Phi_c u_R'^{\,j} + \text{h.c.} \\
&\quad -\Gamma_e^{ij}\bar{L}_L^i\Phi e_R^j + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2\Phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$\mathcal{L}_{\text{Yukawa}}$ is Lorentz invariant, gauge invariant and renormalizable, and therefore it can (actually it must) be included in the Lagrangian.

Expanding around the vacuum state

In the unitary gauge we have

$$
\begin{aligned}
\bar{Q}'^{\,i}_L\, \Phi\, d'^{\,j}_R &= \begin{pmatrix} \bar{u}'^{\,i}_L & \bar{d}'^{\,i}_L \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} d'^{\,j}_R = \frac{v+H}{\sqrt{2}}\, \bar{d}'^{\,i}_L\, d'^{\,j}_R \\
\bar{Q}'^{\,i}_L\, \Phi_c\, u'^{\,j}_R &= \begin{pmatrix} \bar{u}'^{\,i}_L & \bar{d}'^{\,i}_L \end{pmatrix} \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} u'^{\,j}_R = \frac{v+H}{\sqrt{2}}\, \bar{u}'^{\,i}_L\, u'^{\,j}_R
\end{aligned}
$$

and we obtain

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Yukawa}} &= -\Gamma^{ij}_d \frac{v+H}{\sqrt{2}}\, \bar{d}'^{\,i}_L\, d'^{\,j}_R - \Gamma^{ij}_u \frac{v+H}{\sqrt{2}}\, \bar{u}'^{\,i}_L\, u'^{\,j}_R - \Gamma^{ij}_e \frac{v+H}{\sqrt{2}}\, \bar{e}^{i}_L\, e^{j}_R + \mathrm{h.c.} \\
&= -\left[M^{ij}_u\, \bar{u}'^{\,i}_L\, u'^{\,j}_R + M^{ij}_d\, \bar{d}'^{\,i}_L\, d'^{\,j}_R + M^{ij}_e\, \bar{e}^{i}_L\, e^{j}_R + \mathrm{h.c.} \right] \left(1 + \frac{H}{v} \right)
\end{aligned}
$$

with mass matrices $M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$

Diagonalizing M_f

It is always possible to diagonalize M_f^{ij} ($f = u, d, e$) with a bi-unitary transformation ($U^f_{L/R}$ must be unitary in order to preserve the form of the kinetic terms in the Lagrangian)

$$
\begin{aligned}
f'_{Li} &= \left(U_L^f\right)_{ij} f_{Lj} \\
f'_{Ri} &= \left(U_R^f\right)_{ij} f_{Rj}
\end{aligned}
$$

with U_L^f and U_R^f chosen such that

$$\left(U_L^f\right)^\dagger M_f U_R^f = \text{diagonal}$$

For example:

$$(U_L^u)^\dagger M_u U_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad \left(U_L^d\right)^\dagger M_d U_R^d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Mass terms

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{f',i,j} M_f^{ij}\, \bar{f}_L'^{\,i}\, f_R'^{\,j} \left(1+\frac{H}{v}\right) + \text{h.c.}$$

$$= -\sum_{f,i,j} \bar{f}_L^i \left[\left(U_L^f\right)^\dagger M_f U_R^f\right]_{ij} f_R^j \left(1+\frac{H}{v}\right) + \text{h.c.}$$

$$= -\sum_f m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right)\left(1+\frac{H}{v}\right)$$

We succeed in producing fermion masses and we got a fermion-antifermion-Higgs coupling proportional to the fermion mass.

The Higgs Yukawa couplings are flavor diagonal: no flavor changing Higgs interactions.

Mass diagonalization and charged current interaction

The charged current interaction is given by

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}_L'^{\,i}\, \not{W}^+\, d_L'^{\,i} + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}_L^i\left[(U_L^u)^\dagger\, U_L^d\right]_{ij}\not{W}^+ d_L^j + \text{h.c.}$$

and we define the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}

$$V_{CKM} = (U_L^u)^\dagger\, U_L^d$$

- V_{CKM} is not diagonal and then it mixes the flavors of the different quarks.

- It is a unitary matrix and the values of its entries must be determined from experiments.

Higgs boson couplings

We have identified the relevant terms in the SM Lagrangian for Higgs boson couplings
to gauge bosons:

$$\mathcal{L}_{\text{kin}}^{\Phi} = (D^\mu \Phi)^\dagger D_\mu \Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[m_W^2 W^{\mu +} W_\mu^- + \frac{1}{2} m_Z^2 Z^\mu Z_\mu \right] \left(1 + \frac{H}{v}\right)^2$$

which produces the HVV coupling term

$$\frac{2m_V^2}{v} V_\mu V^\mu H = \frac{2m_V^2}{v} g^{\mu\nu} V_\mu V_\nu H$$

to fermions:

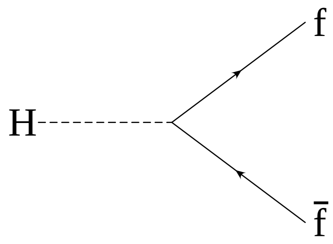
$$\mathcal{L}_{\text{Yukawa}} = -\sum_f m_f \bar{f} f \left(1 + \frac{H}{v}\right) = -\sum_f m_f \bar{f} f - \sum_f \frac{m_f}{v} H \bar{f} f$$

and the Higgs self-couplings

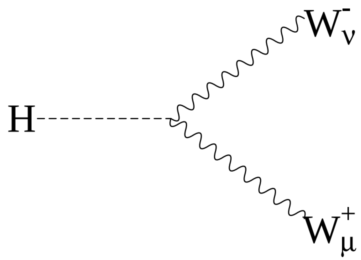
$$\mathcal{L}_V = -\frac{1}{2}(2\lambda v^2) H^2 \; - \; \lambda v H^3 \; - \; \frac{\lambda}{4} H^4 = -\frac{1}{2} m_H^2 H^2 \; - \; \frac{m_H^2}{2v} H^3 \; - \; \frac{m_H^2}{8v^2} H^4$$

Note that the Higgs couplings increase with the mass of particles the Higgs boson couples to.

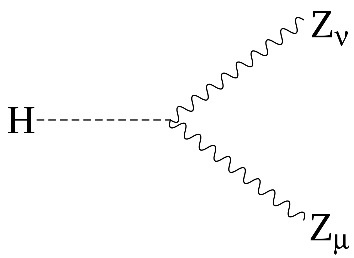
Feynman rules for Higgs couplings



$$-i\frac{m_f}{v}$$



$$ig\, m_W\, g_{\mu\nu}$$



$$i\, g\, \frac{1}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

Within the Standard Model, the Higgs couplings are completely constrained since the masses of all SM particles[a] have been measured.

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[a] except neutrinos