Exercises: Stefan Liebler (stefan.liebler@kit.edu) (Office 12/03 - Build. 30.23)

Information regarding sheet 6:
Due to the holiday on Monday, 10.06.19, you don't have to hand in solutions to this sheet. We still encourage you to have a look at the exercise, for which a solution will be provided on the webpage. On Wednesday, 12.06.19, the individual exercise classes are canceled, but only a **combined exercise class** will take place at **14:00 in room 6/1** in the physics highrise. If you wish to gain insight into a special topic within this class, please let us know beforehand.

Exercise 1: Momentum operator and charge of the Dirac field **0 points**

The solution of the Dirac equation can be expanded in plain waves as follows

$$\psi(x) = \int d\tilde{p} \sum_{\lambda=\pm} \left[c_\lambda(p) u(p,\lambda) e^{-ip\cdot x} + d_\lambda^\dagger(p) v(p,\lambda) e^{ip\cdot x} \right] .$$

Therein $u(p,\lambda)$ and $v(p,\lambda)$ are Dirac spinors associated with positive and negative energies, respectively. They obey relations as shown on sheet 2, i.e. $u^\dagger(p,\lambda)u(p,\lambda') = 2\omega_p \delta_{\lambda\lambda'}$, $v^\dagger(p,\lambda)v(p,\lambda') = 2\omega_p\delta_{\lambda\lambda'}$, $u^\dagger(\tilde{p},\lambda)v(p,\lambda') = v^\dagger(\tilde{p},\lambda)u(p,\lambda') = 0$ with $\tilde{p} = (\omega_p, -\vec{p})^T$. A priori $c_\lambda^{(\dagger)}$ and $d_\lambda^{(\dagger)}$ are plain coefficients, which we assume to be not-nessesarily anti-commuting.

(a) Show that the componenents $T^{0\mu}$ of the energy-momentum tensor are given by $T^{0\mu} = \psi^\dagger i\partial^\mu \psi$. Express the four-momentum of the Dirac field

$$P^\mu = \int d^3x T^{0\mu}$$

in terms of $c_\lambda(p), c_\lambda^\dagger(p), d_\lambda(p)$ and $d_\lambda^\dagger(p)$.

(b) The charge of the Dirac field is given by

$$Q = \int d^3x \overline{\psi}(x)\gamma^0\psi(x) .$$

Express the charge again through the coefficients $c_\lambda(p), c_\lambda^\dagger(p), d_\lambda(p)$ and $d_\lambda^\dagger(p)$.

(c) For both subexercises (a) and (b) argue why having anti-commutation relations for d and $d^\dagger$ leads to physically sensible results.