Einführung in Theoretische Teilchenphysik

Lecture: PD Dr. S Gieseke – Exercises: Dr. D. López-Val, Dr. S. Patel, PD Dr. M. Rauch

Exercise Sheet 11

Submission: Mo, 22.01.18, 12:00

Discussion:	Mon, 22.01.18	14:00	Room 11/12
	Wed, 24.01.18	09:45	Room 10/1

Exercise 1: Gamma-Algebra – part 2

In this continuation of exercise 1 on the previous sheet 10 we look once more at properties of the gamma matrices. Again, only the Clifford algebra,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \cdot \mathbb{1}_4 \,,$$

is necessary, not an explicit form of the γ matrices.

(a) Besides the four standard γ matrices, a particular combination called γ^5 is usually defined as $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.
Show that this is equivalent to

$$\gamma^5 = -\frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \,,$$

where $\epsilon_{0123} = -1$.

(b) Show the following relations for γ^5:
- (i) $\left(\gamma^5\right)^\dagger = \gamma^5$,
- (ii) $\left(\gamma^5\right)^2 = \mathbb{1}_4$,
- (iii) $\left\{\gamma^\mu, \gamma^5\right\} = 0$.

(c) Show the following identities for traces over γ matrices:
- (i) $\mathrm{Tr}[\gamma^\mu] = 0$,
- (ii) $\mathrm{Tr}[\gamma^{\mu_1}\cdots\gamma^{\mu_n}] = 0$, if n is odd,
- (iii) $\mathrm{Tr}[\gamma^{\mu_1}\cdots\gamma^{\mu_n}] = \mathrm{Tr}[\gamma^{\mu_n}\cdots\gamma^{\mu_1}]$,
- (iv) $\mathrm{Tr}[\gamma^\alpha\gamma^\beta] = 4g^{\alpha\beta}$,
- (v) $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right)$,
- (vi) $\mathrm{Tr}[\gamma^5] = 0$,
- (vii) $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^5] = 0$,
- (viii) $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5] = -4i\epsilon^{\mu\nu\rho\sigma}$.

Exercise 2: Electron-Positron Annihilation

Consider the process of electron-positron annihilation into two photons,

$$e^-e^+ \to \gamma\gamma \,.$$

(a) Draw all Feynman diagrams contributing at leading order. Label them with the momenta of the internal propagators and the wave function factors for the external particles.

(b) Use the Feynman rules of QED to write down the contribution of each diagram to the scattering amplitude. (Your result should be in the form $\bar{v}(p)\frac{1}{\not{p}+\not{k}-m}\ldots$, no further simplifications are necessary.)
Scetch, without doing any explicit calculations, what needs to be done further to obtain a cross section for this process.

(c) A correction to this process is given by the process with an additional photon in the final state, $e^-e^+ \to \gamma\gamma\gamma$. Draw the corresponding Feynman diagrams. Can you guess how many diagrams there are for n photons in the final state?