WiSe 2017

Einführung in Theoretische Teilchenphysik

Lecture: PD Dr. S Gieseke – Exercises: Dr. D. López-Val, Dr. S. Patel, PD Dr. M. Rauch

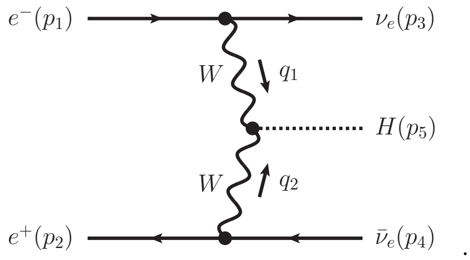
Exercise Sheet 12

Submission: Mo, 29.01.18, 12:00

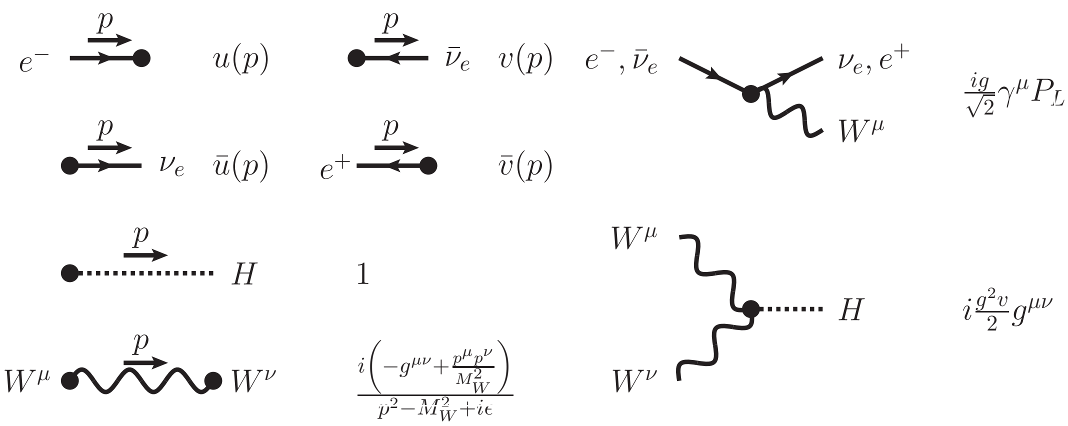
Discussion:	Mon, 29.01.18	14:00	Room 11/12
	Wed, 31.01.18	09:45	Room 10/1

Exercise 1: Higgs Production via W Fusion

Higgs production via W fusion is one of the main Higgs production processes at electron-positron colliders. At leading order, this process is given by a single Feynman diagram:



The corresponding Feynman rules are

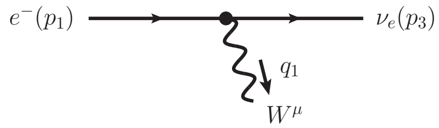


with the weak coupling constant $g \simeq 0.65$ and the vacuum expectation value of the Higgs field $v \simeq 246$ GeV.

(a) Show the following properties of the chirality projection operators $P_{\substack{R\\L}} = \frac{1\pm\gamma^5}{2}$, which we will need in the following:

$$\left(P_{\substack{L\\R}}\right)^2 = P_{\substack{L\\R}}\,, \qquad P_R + P_L = 1\,, \qquad P_R - P_L = \gamma^5\,,$$
$$\left(P_{\substack{L\\R}}\right)^\dagger = P_{\substack{L\\R}}\,, \qquad P_{\substack{L\\R}}\gamma^\mu = \gamma^\mu P_{\substack{R\\L}}\,, \qquad \gamma^0\left(P_{\substack{L\\R}}\right)\gamma^0 = P_{\substack{R\\L}}\,.$$

Consider first the following sub-diagram:



where the W boson is "amputated", i.e. the contraction with its polarization vector is omitted and instead the matrix element contains an open Lorentz index μ.

(b) Write down the corresponding matrix element $\mathcal{M}_1^\mu$. Calculate $q_{1\mu}\mathcal{M}_1^\mu$ and eliminate the explicit momentum dependence of the expression, assuming that also the neutrino has a mass m_ν. What happens in the limit $m_e = m_\nu$? Show that for $m_e = m_\nu = 0$ $q_{1\mu}\mathcal{M}_1^\mu$ vanishes.

In the following we consider only massless fermions, $m_e = m_\nu = 0$.

(c) Show that for the squared matrix element of the sub-diagram one obtains after spin summation

$$\sum_{s_1,s_3} |\mathcal{M}_1|^{2,\mu\nu} \equiv \sum_{s_1,s_3} \mathcal{M}_1^\mu \mathcal{M}_1^{\dagger,\nu} = g^2 \left(p_1^\mu p_3^\nu + p_3^\mu p_1^\nu - p_1 \cdot p_3 g^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma} p_{1,\rho} p_{3,\sigma}\right) .$$

Then consider how the expression changes for the equivalent part of the lower fermion line.

(d) Use this to calculate the spin-averaged squared matrix element of the whole Feynman diagram. Consider first the result of the "middle part" (W propagators, HWW vertex) separately. The limit $\epsilon \to 0$ of the $i\epsilon$ terms in the propagators can be taken immediately. Take care to contract the correct Lorentz indices.
Result:

$$\overline{\sum} |\mathcal{M}|^2 = \frac{g^8 v^2}{4} \frac{1}{(q_1^2 - M_W^2)^2 (q_2^2 - M_W^2)^2} \; p_1 \cdot p_4 \; p_2 \cdot p_3 \, .$$

(e) no hand-in!
Write a program which calculates the corresponding cross section

$$\sigma = \frac{1}{2s} \int \mathrm{dPS(3\text{-particle})} \overline{\sum} |\mathcal{M}|^2$$

for a given centre-of-mass energy $\sqrt{s}$ of the electron-positron pair via numerical integration. The 3-particle phase space with one massive final-state particle can be written as

$$\begin{aligned}
\int \mathrm{d}\Phi_3 &\equiv \int \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4} \frac{\mathrm{d}^3 p_5}{(2\pi)^3 2E_5} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4 - p_5) \\
&= \frac{s}{512\pi^4} \int_0^1 \mathrm{d}x_3 \int_{1-x_3}^1 \mathrm{d}x_4 \int_{-1}^1 \mathrm{d}\cos\vartheta_3 \int_0^{2\pi} \mathrm{d}\varphi_4
\end{aligned}$$

where $s = (p_1 + p_2)^2$, $x_i = \frac{2E_i}{\sqrt{s}}$ and φ_4 is taken relative to the plane spanned by the vectors $\vec{p}_3$ and $\vec{e}_z$.