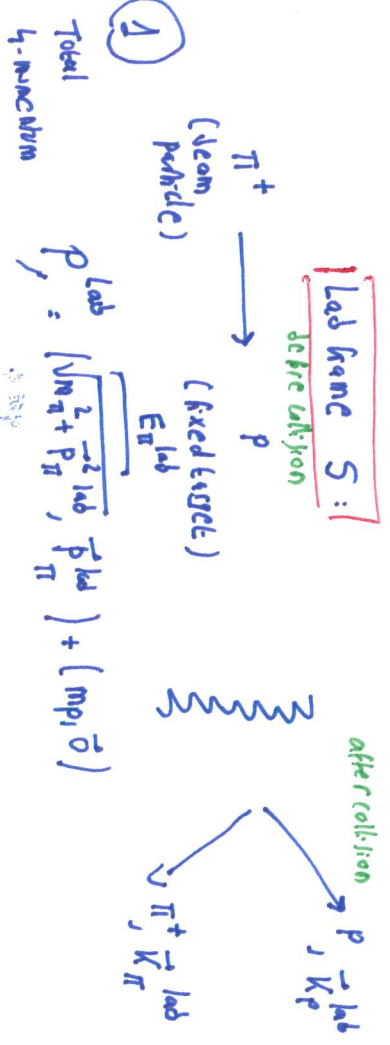


[Hints to solve Ex 2, part (c) of sheet 3
 (useful tricks for relativistic kinematics)]

⇒ Key idea: use Lorentz transformations to switch between lab frame and center-of-mass frame



p, p^μ | lab = S (Lorentz-invariant, valid in all frames)

$$S = (\mathbf{E}_\pi^{lab})^2 + m_p^2 - \mathbf{p}_\pi^{lab} \cdot \mathbf{p}_p = m_\pi^2 + m_p^2 + 2m_p E_\pi^{lab}$$

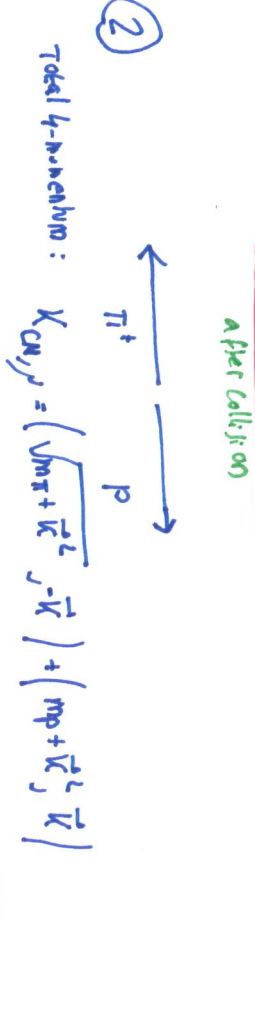
$$\Rightarrow \frac{+ 2E_\pi^{lab} m_p}{E_\pi^{lab}} = \frac{S - m_p^2 - m_\pi^2}{2m_p} \quad [1]$$

5) back to S, we simply recall that

$$K_\pi^{lab} = \gamma(\sqrt{u}^{lab}) \sqrt{u}^{lab} m_\pi$$

for the pion after the collision
 → we get \sqrt{u}^{lab}

Center of mass frame S'



$$K_{CM}^{\mu\nu} = S = m_\pi^2 + K^2 + m_p^2 + K^2 + 2\sqrt{(K^2 + m_p^2)(K^2 + m_\pi^2)} \quad [2]$$

From [2], we get $\vec{K}_\pi = -\vec{K}_p$ = momenta of π in the final state, CM frame

3) we find the relative velocity $|u|$ between S and S':

$$\vec{p}_\pi^{lab} \rightarrow \vec{p}_\pi^{CM} = \gamma(u) [\vec{p}_\pi^{lab} - u E_\pi^{lab}] \quad \text{(known from [1])}$$

$$\vec{p}_p^{lab} = 0 \rightarrow \vec{p}_p^{CM} = -m_p \vec{u} \gamma(u)$$

$$\text{By construction, } \vec{p}_\pi^{CM} + \vec{p}_p^{CM} = 0 \Leftrightarrow u = \frac{\sqrt{(E_\pi^{lab})^2 - m_\pi^2}}{E_\pi^{lab} + m_p} \quad [3]$$

4) we use [3] to transform [2] back to the lab frame:

$$K_\pi^{lab} = \gamma(u) [\vec{K}_\pi^{CM} + u E_\pi^{CM}] \quad [4]$$

leaving that