Karlsruhe Institute of Technology

WiSe 2017

Einführung in Theoretische Teilchenphysik

Lecture: PD. Dr. S Gieseke – Exercises: Dr. D. López-Val, Dr. S. Patel, Dr. M. Rauch

Exercise Sheet 3

Submission: Mo, 13.11.17, 12:00

Discussion:			
	Mon, 13.11.17	14:00	Room 11/12
	Wed, 15.11.17	09:45	Room 10/1

WHEN HANDING IN, PLEASE MAKE SURE:

- your **NAME** and **GROUP** **(Mon/Wed)** are clearly indicated
- all sheets are **stapled** together
- you include the **question sheet** as **cover**

Exercise 1: Hadron decays & isospin invariance

Recall that the quark flavors u, d are members of an isospin $SU(2)$ doublet $I = \frac{1}{2}$,

$$|\mathrm{I\,I_3}\rangle_u = \left|u,\, \frac{1}{2}\, +\frac{1}{2}\right\rangle \qquad |\mathrm{I\,I_3}\rangle_d = \left|d,\, \frac{1}{2}\, -\frac{1}{2}\right\rangle \tag{1}$$

Accordingly, also the nucleons constitute an isospin doublet

$$|\mathrm{I\,I_3}\rangle_p = \left|uud,\, \frac{1}{2}\, +\frac{1}{2}\right\rangle \qquad |\mathrm{I\,I_3}\rangle_n = \left|d,\frac{1}{2}\, -\frac{1}{2}\right\rangle\,,$$

as well as the nucleon resonances

$$|\mathrm{I\,I_3}\rangle_{N^+} = \left|uud,\frac{1}{2}\, +\frac{1}{2}\right\rangle \qquad |\mathrm{I\,I_3}\rangle_{N^0} = \left|udd,\frac{1}{2}\, -\frac{1}{2}\right\rangle\,,$$

In turn, the pions form an isospin triplet $I = 1$,

$$|\mathrm{I\,I_3}\rangle_{\pi^+} = \left|u\bar{d},\, 1\, +1\right\rangle \qquad |\mathrm{I\,I_3}\rangle_{\pi^0} = \left|\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),\, 1\, -1\right\rangle \qquad |\mathrm{I\,I_3}\rangle_{\pi^-} = \left|d\bar{u},\, 1\, -1\right\rangle \quad .$$

Finally, we consider as well the isospin quadruplet $I = \frac{3}{2}$ states,

$$|\mathrm{I\,I_3}\rangle_{\Delta^{++}} = \left|uuu,\frac{3}{2}\, +\frac{3}{2}\right\rangle \qquad |\mathrm{I\,I_3}\rangle_{\Delta^{+}} = \left|uud,\frac{3}{2}\, +\frac{1}{2}\right\rangle$$
$$|\mathrm{I\,I_3}\rangle_{\Delta^{0}} = \left|udd,\frac{3}{2}\, -\frac{1}{2}\right\rangle \qquad |\mathrm{I\,I_3}\rangle_{\Delta^{-}} = \left|ddd,\frac{3}{2}\, -\frac{3}{2}\right\rangle$$

Notice that the states $\{N^+, \Delta^+\}$ have the same quark content, and thence the same isospin quantum numbers as the proton. In this sense, they can be considered as excited states (resonances) of the proton. Both resonances decay primarily through

$$\Delta^+ \to p + \pi^0\,,\, n + \pi^+ \qquad N^+ \to p + \pi^0\,, n + \pi^+$$

(a) Represent each decay mode using a quark diagram.

(b) Verify that in all these processes the quark flavor, electric charge, and total isospin are conserved. Which of the fundamental interactions (strong, weak, electromagnetic) is therefore governing these decays?

(c) Evaluate the decay width ratios $\frac{\Gamma(N^+ \to n + \pi^+)}{\Gamma(N^+ \to p + \pi^0)}$ and $\frac{\Gamma(\Delta^+ \to n + \pi^+)}{\Gamma(\Delta^+ \to p + \pi^0)}$.

Hint: Recall the fact that the total isospin is conserved, and make use of Clebsch-Gordan coefficients to rewrite the isospin of the invididual final states into the coupled basis.

(d) To cross-check your results: verify that the corresponding branching fractions yield

$$
\begin{aligned}
\mathrm{BR}_{N^+ \to n + \pi^+} &= \frac{\Gamma(N^+ \to n + \pi^+)}{\Gamma(N^+ \to n + \pi^+) + \Gamma(N^+ \to p + \pi^0)} \simeq 66\% \\
\mathrm{BR}_{\Delta^+ \to n + \pi^+} &= \frac{\Gamma(N^+ \to n + \pi^+)}{\Gamma(N^+ \to n + \pi^+) + \Gamma(N^+ \to p + \pi^0)} \simeq 33\%
\end{aligned}
\tag{2}
$$

Exercise 2: Pion-Nucleon scattering & isospin invariance

Consider all possible pion-nucleon scattering processes $\{\Pi + N\} \to \{\Pi' + N'\}$, with $\Pi = \{\pi^-, \pi^0, \pi^+\}$ and $N = \{p, n\}$, mediated by the strong force.

(a) List down all of them (there are 10!) and classify them according to the total isospin of the initial (final) states in the coupled basis.

(b) If we denote the 3rd component of the isospin of each initial (resp. final) state particle as

$$\mathrm{I}_3^{\Pi} = \mu \qquad \mathrm{I}_3^{N} = \nu \qquad\qquad \mathrm{I}_3^{\Pi'} = \mu' \qquad \mathrm{I}_3^{N'} = \nu' \, ,$$

show that the S-matrix elements $\langle \Pi' + N' | \, \hat{S} \, | \Pi + N \rangle$ in the isospin space may be written as:

$$
\begin{aligned}
\left\langle 1\,\mu' \, \frac{1}{2}\,\nu' \right| \hat{S} \left| 1\,\mu\,\frac{1}{2}\,\nu \right\rangle = \Big[& C_{\frac{1}{2}\mu\frac{1}{2}\nu,\frac{1}{2}\mu+\nu} \, C_{\frac{1}{2}\mu'\frac{1}{2}\nu',\frac{1}{2}\mu'+\nu'} \, \mathcal{A}^{\frac{1}{2}}_{\Pi+N\to\Pi'+N'} + \\
& + C_{\frac{1}{2}\mu\frac{1}{2}\nu,\frac{3}{2}\mu+\nu} \, C_{\frac{1}{2}\mu'\frac{1}{2}\nu',\frac{3}{2}\mu'+\nu'} \, \mathcal{A}^{\frac{3}{2}}_{\Pi+N\to\Pi'+N'} \Big] \, \delta_{\mu+\nu,\mu'+\nu'}
\end{aligned}
$$

in terms of the elements of the S matrix $\mathcal{A}^{\mathrm{I}}_{\Pi + N \to \Pi' + N'} \equiv \langle I\,\mu' + \nu' | \, \hat{S} \, | \mathrm{I}\,\mu + \nu \rangle$ in the coupled isospin basis, and the corresponding Clebsch-Gordan coefficients. How does this result reflect the Wigner-Eckart theorem?

(c) Apply the above result to the specific channels $\pi^- + p \to \pi^- + p$ and $\pi^- + p \to \pi^0 + n$ to demonstrate that

$$
\begin{aligned}
\langle \hat{S} \rangle_{\pi^- + p \to \pi^- + p} &= \frac{1}{3} \left[\mathcal{A}^{\frac{3}{2}}_{\pi^- + p \to \pi^- + p} + 2\mathcal{A}^{\frac{1}{2}}_{\pi^- + p \to \pi^- + p} \right] \\
\langle \hat{S} \rangle_{\pi^- + p \to \pi^0 + n} &= \frac{\sqrt{2}}{3} \left[\mathcal{A}^{\frac{3}{2}}_{\pi^- + p \to \pi^0 + n} - \mathcal{A}^{\frac{1}{2}}_{\pi^- + p \to \pi^0 + n} \right] .
\end{aligned}
$$

(d) Verify that the cross-section ratio between the scattering processes with $\pi^+ + p$ and $\pi^- + p$ initial states reads

$$\frac{\sigma(\pi^+ + p)}{\sigma(\pi^- + p)} = 3 \, .$$

(e) We consider the fixed-target experiment $\pi^+ + p \to \pi^+ + p$, where the proton is initially at rest. Compute the minimum kinetic energy of the incident pion $T^{\min}_{\pi^+}$ in the lab frame for the reaction to take place.

Evaluate the lab-frame velocity of the outgoing pion as a function of an arbitrary center-of-mass energy.

Data:: $m_{\pi^+} = 139.57$ MeV; $\qquad m_p = 938.27$ MeV


DAS PHYSIKERTHEATER PRÄSENTIERT
EINER FLOG ÜBER DAS KUCKUCKSNEST
VON DALE WASSERMAN
AM 17.+18. NOVEMBER 2017 IM GAEDE-HÖRSAAL
BEGINN 20:00 – EINLASS 19:30 – EINTRITT FREI