Karlsruhe Institute of Technology

Einführung in Theoretische Teilchenphysik

Lecture: PD. Dr. S Gieseke – Exercises: Dr. D. López-Val, Dr. S. Patel, PD. Dr. M. Rauch

Exercise Sheet 5

Submission: Mo, 27.11.17, 12:00

Discussion:	Mon, 27.11.17	14:00	Room 11/12
	Wed, 29.11.17	09:45	Room 10/1

Exercise 1: Relativistic Wave Equation for a scalar particle

(a) Explain why the Schrödinger Equation is not suitable for a relativistic formulation of Quantum Mechanics

(b) Identify, and briefly discuss, TWO reasons why the Klein-Gordon Equation does not yet provide a fully satisfactory alternative

(c) Starting from the Klein-Gordon Equation for a free scalar field,

$$\left(-\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2 - m^2\right) \varphi(\vec{x},t) = 0 \ : \qquad (1)$$

(i) Apply the *minimal coupling* prescription

$$-i\vec{\nabla} \to -i\vec{\nabla} + q\,\vec{A}. \qquad (2)$$

to derive the equation that describes a relativistic scalar particle coupled to a classical magnetic field.

(ii) Making (and justifying) suitable approximations, study the equation obtained in i) in the non-relativistic limit.

Hint: It is useful to rewrite the Klein-Gordon field as $\varphi(x) = \psi(\vec{x},t)\exp\left(-imt\right)$, where $\psi(\vec{x},t)$ encodes the non-relativistic part. Why?

(iii) Show that the result is equivalent to imposing the ansatz (2) on the Schrödinger Equation.

Exercise 2: Scalar field Lagrangians: invariance properties

Consider the following transformation to the Lagrangian density: $\mathcal{L} = \mathcal{L}(\varphi_i(x), \partial_\mu \varphi_i(x))$:

$$\mathcal{L}' = \mathcal{L} + \partial_\mu \Lambda^\mu \,,$$

where Λ^μ denotes a generic function of the set of scalar fields $\{\varphi_i(x)\}$. Show that these transformations leave the equations of motion for φ_i unchanged.

Exercise 3: Scalar field Lagrangians: Noether Theorem

(a) Show that the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left((\partial_\mu\varphi_1)^2 + (\partial_\mu\varphi_2)^2\right) - \frac{m^2}{2}\left(\varphi_1^2+\varphi_2^2\right) - \frac{\lambda}{4}\left(\varphi_1^2+\varphi_2^2\right)^2$$

remains invariant under the transformation ($\vartheta \in \mathrm{R}$)

$$\begin{aligned} \varphi_1 &\to \varphi_1' = \varphi_1 \cos\vartheta - \varphi_2 \sin\vartheta & \qquad x_\mu &\to x'_\mu = x_\mu \\ \varphi_2 &\to \varphi_2' = \varphi_1 \sin\vartheta + \varphi_2 \cos\vartheta & & .. \end{aligned}$$

(b) Compute the Noether current associated to the above transformation,

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} \delta\varphi_i - T^{\mu\nu}\delta x_\nu$$

with

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} (\partial^\nu \varphi_i) - \mathcal{L} g^{\mu\nu}$$

as well as its corresponding Noether charge

$$Q = \int \mathrm{d}^3 x J^0 \, .$$