

# Einführung in Theoretische Teilchenphysik

Lecture: PD. Dr. S Gieseke – Exercises: Dr. D. López-Val, Dr. S. Patel, PD. Dr. M. Rauch

## Exercise Sheet 5

Submission: Mo, 27.11.17, 12:00

Discussion: Mon, 27.11.17 14:00 Room 11/12  
Wed, 29.11.17 09:45 Room 10/1

### Exercise 1: Relativistic Wave Equation for a scalar particle

- Explain why the Schrödinger Equation is not suitable for a relativistic formulation of Quantum Mechanics
- Identify, and briefly discuss, TWO reasons why the Klein-Gordon Equation does not yet provide a fully satisfactory alternative
- Starting from the Klein-Gordon Equation for a free scalar field,

$$\left( \frac{\partial^2}{\partial t^2} + \vec{\nabla}^2 - m^2 \right) \varphi(\vec{x}, t) = 0 : \quad (1)$$

- Apply the *minimal coupling* prescription

$$-i\vec{\nabla} \rightarrow -i\vec{\nabla} + q\vec{A}. \quad (2)$$

to derive the equation that describes a relativistic scalar particle coupled to a classical magnetic field.

- Making (and justifying) suitable approximations, study the equation obtained in i) in the non-relativistic limit.

*Hint:* It is useful to rewrite the Klein-Gordon field as  $\varphi(x) = \psi(\vec{x}, t) \exp(imt)$ , where  $\psi(\vec{x}, t)$  encodes the non-relativistic part. Why?

- Show that the result is equivalent to imposing the ansatz (2) on the Schrödinger Equation.

### Exercise 2: Scalar field Lagrangians: invariance properties

Consider the following transformation to the Lagrangian density:  $\mathcal{L} = \mathcal{L}(\varphi_i(x), \partial_\mu \varphi_i(x))$ :

$$\mathcal{L}' = \mathcal{L} + \partial_\mu \Lambda^\mu,$$

where  $\Lambda^\mu$  denotes a generic function of the set of scalar fields  $\{\varphi_i(x)\}$ . Show that these transformations leave the equations of motion for  $\varphi_i$  unchanged.

### Exercise 3: Scalar field Lagrangians: Noether Theorem

- Show that the Lagrangian

$$\mathcal{L} = \frac{1}{2} ((\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2) - \frac{m^2}{2} (\varphi_1^2 + \varphi_2^2) - \frac{\lambda}{4} (\varphi_1^2 + \varphi_2^2)^2$$

remains invariant under the transformation ( $\vartheta \in \mathbb{R}$ )

$$\begin{aligned} \varphi_1 &\rightarrow \varphi'_1 = \varphi_1 \cos \vartheta - \varphi_2 \sin \vartheta & x_\mu &\rightarrow x'_\mu = x_\mu \\ \varphi_2 &\rightarrow \varphi'_2 = \varphi_1 \sin \vartheta + \varphi_2 \cos \vartheta & & \dots \end{aligned}$$

(b) Compute the Noether current associated to the above transformation,

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} \delta \varphi_i - T^{\mu\nu} \delta x_\nu$$

with

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} (\partial^\nu \varphi_i) - \mathcal{L} g^{\mu\nu}$$

as well as its corresponding Noether charge

$$Q = \int d^3x J^0.$$