Extended Higgs Sectors Beyond the Standard Model

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Exercise 1: Two-particle phase space (*0 points*)

For the calculation of decay rates and cross sections, we need to integrate over the phase space of the particles in the final state. For a generic process with two particles with momenta p_1 and p_2 and masses m_1 and m_2 in the final state, this phase space integral is given by

$$\int d\Phi_2 = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^{(4)}(q - p_1 - p_2) \ ,$$

where q is the total four-momentum of all incoming particles. This integral is performed over the absolute squared of the matrix element as well as over some additional Heaviside step functions Θ to implement the proper momentum cuts for the particles in the final state.

(a) Show that in the center-of-mass frame of the two final-state particles, the absolute value of the incoming three-momenta is given by

$$|\vec{p}_1| = |\vec{p}_2| = \frac{\lambda(q^2, m_1^2, m_2^2)}{2\sqrt{q^2}} \ ,$$

where λ is the *Källén function* given by

$$\lambda(a^2, b^2, c^2) \equiv \sqrt{a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2} \ .$$

(b) Show that in the center-of-mass frame of the two final-state particles, the phase space integral can be written as

$$\int d\Phi_2 = \int d\Omega \frac{1}{32\pi^2 q^2} \lambda(q^2, m_1^2, m_2^2) \Theta(q_0) \Theta(q^2 - (m_1 + m_2)^2) \ ,$$

where $d\Omega \equiv d(\cos\vartheta_1) d\varphi_1$ is the integration over the solid angle of particle 1 in the center-of-mass frame.

Hint: Prove and use the relation

$$\frac{d^3p}{2E} = d^4p \Theta(p_0) \delta(p^2 - m^2) \ .$$

Exercise 2: Tree-Level Higgs decay into massive gauge bosons (*0 points*)

One application for Exercise 1 is given by the decay of a Higgs boson h into two gauge bosons $V = (W^\pm, Z)$. The process is given by

$$h(p) \to V(q_1, \lambda_1) + V(q_2, \lambda_2)\,,$$

where p is the incoming momentum of the Higgs, $q_{1,2}$ are the outgoing momenta of the gauge bosons and $\lambda_{1,2}$ are the polarizations of the gauge bosons.
Assume now that the on-shell decay is possible ($m_h \geq 2m_V$).

(a) Draw the Feynman graph for the process.

(b) Calculate the squared matrix element and sum over all possible polarizations. What is the difference between $V = W^\pm$ and $V = Z$? Express the result as a polynomial in $x = m_V^2/m_h^2$. Show that it can be cast into the form

$$\sum_{\lambda_i} |\mathcal{M}_{h\to VV}|^2 = \frac{m_h^4}{v^2}\left[12x^2 + 1 - 4x\right]\,.$$

Hint: The coupling between the Higgs boson and two gauge bosons is given by $g_{hVV} = \frac{2m_V^2}{v}$ where v is the electroweak vacuum expectation value. You can replace the product $q_1 q_2$ by the masses by using energy-momentum conservation.

(c) The decay width Γ is now given as

$$\mathrm{d}\Gamma = \frac{1}{2m_h}\sum_{\lambda_i} |\mathcal{M}_{h\to VV}|^2 \mathrm{d}\Phi_2\,,$$

where $\mathrm{d}\Phi_2$ is the two-particle phase space calculated in exercise 1b). Simplify Γ as far as possible.