Extended Higgs Sectors Beyond the Standard Model

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Exercise sheet 1

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Exercise 3: Higgs decays into fermions

In this exercise we want to calculate the tree-level decay width of a Higgs boson into two fermions

$$h(\vec{p}) \to f(q_1, s_1)\overline{f}(q_2, s_2)$$

with incoming momentum p of the Higgs boson, $q_{1,2}$ being the outgoing momenta and $s_{1,2}$ being the spins of the fermions.

(a) Draw the Feynman graph for the process.

(b) Calculate the matrix element squared and sum over all spins and show that it can be cast into the form

$$\sum_s |\mathcal{M}_{h\to f\overline{f}}|^2 = \frac{2m_f^2}{v^2} m_h^2 \left(1 - \frac{m_f^2}{m_h^2}\right) .$$

Hint : The coupling between two fermions and a Higgs boson is given by $y_{hff} = \frac{m_f}{v}$ where m_f is the mass of the fermion. It will also be useful to remember the trace properties of the γ matrices.

(c) Calculate the decay width Γ defined by

$$\Gamma = \int \mathrm{d}\Gamma = \int \frac{1}{2m_h} N_c \sum_s |\mathcal{M}_{h\to f\overline{f}}|^2 \mathrm{d}\Phi_2 \,.$$

What is the factor N_c and which value does it have for quarks and which for leptons? Think about which degree of freedom was still missing in your calculation so far.

Exercise 4: Higgs-strahlung

In this exercise we want to calculate one of the possible production channels of the Higgs boson. For this we look at the Higgs-strahlung

$$\overline{f}_d(p_1, s_1)f(p_2, s_2)_u \to W^{+*}(q = p_3 + p_4) \to W^+(p_3, \lambda) + H(p_4)$$

where W^{+*} denotes an virtual W-boson.

(a) Draw the Feynman graph for the process.

(b) Show that the matrix element can be cast into the form

$$\mathcal{M}_{\overline{f}f\to hW^+} = \frac{2m_W^2}{v}\frac{g}{2\sqrt{2}}\epsilon_\lambda^\nu \overline{v}(p_1, s_1)\gamma^\mu (1-\gamma_5) u(p_2, s_2)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}\right)\frac{1}{q^2 - m_W^2}\,.$$

Hint : For this calculation we assume that the CKM Matrix is the identity matrix.

(c) Show that the averaged squared matrix element can be written as

$$\overline{\sum}|\mathcal{M}|^2 = \frac{1}{N_c}\frac{g^2 m_W^2}{v^2}\frac{s m_W^2 + \frac{s}{2}p_f^2 \sin^2\vartheta}{(s - m_W^2)^2}\,,$$

where s is the Mandelstamm variable, $p_f = |\vec{p_f}|$ and $\vec{p}_f$ the momentum of one of the final state particles in the center-of-mass frame of the decaying W-boson.

(d) The cross section is defined as

$$\sigma(\overline{f}_d f_u \to hW^+) = \frac{1}{2s}\int \overline{\sum}|\mathcal{M}|^2\, \mathrm{d}\Phi_2\,.$$

Show that it can be cast into the form

$$\sigma(\overline{f}_d f_u \to hW^+) = \frac{m_W^4 \lambda(s, m_W^2, m_h^2)}{144\pi s^2 v^4}\frac{\lambda^2(s, m_W^2, m_h^2) + 12 s m_W^2}{(s - mW^2)^2}\,.$$

Hint : Use the relation of 1a) between the *Källén function* and the momentum $\vec{p}_f$. You can also use that $g = \frac{2m_W}{v}$.