Euler Beta-function

$$\begin{aligned}
B(x,y) &= \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)} \\
&= \int\limits_0^\infty dt\; t^{x-1}\,(1+t)^{-x-y} \\
&= 2\int\limits_0^{\pi/2} (\sin\theta)^{2x-1}(\cos\theta)^{2y-1}\,d\theta \\
&= \int\limits_0^1 dt\; t^{x-1}(1-t)^{y-1} \\
&= r^y(r+1)^x \int\limits_0^1 dt\,\frac{t^{x-1}\,(1-t)^{y-1}}{(r+t)^{x+y}} \qquad \forall r
\end{aligned}$$

Expansion of the Γ-function

$$\begin{aligned}
\Gamma(\epsilon) &= \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon}\Big[\Gamma(1) + \epsilon\Gamma'(1) + \cdots\Big] \\
&= \frac{1}{\epsilon} + \Gamma'(1) + \mathcal{O}(\epsilon) = \frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)
\end{aligned}$$

with the Euler constant

$$\gamma = 0.57721\,56649\ldots$$

$$\Gamma(-n+\epsilon) = \frac{(-1)^n}{n!}\left[\frac{1}{\epsilon} + \left(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \gamma\right) + \mathcal{O}(\epsilon)\right]$$