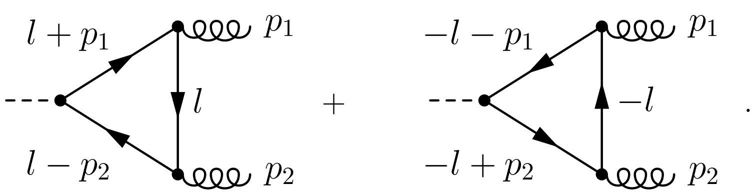
Exercises: Stefan Liebler (stefan.liebler@kit.edu) (Office 12/03 - Build. 30.23)
Shruti Patel (shruti.patel@kit.edu) (Office 12/14 - Build. 30.23)

Even if you finished your TTP2 duties, it's worth to come by and be amazed about the beauty of these exercises.

Exercise 1: Higgs boson decay into gluons - Part 1

The aim of this exercise is to calculate the partial decay width of the Standard Model Higgs boson into a pair of gluons, $h^0 \to gg$, in the first non-vanishing order. The decay is loop-mediated, i.e. the Higgs boson couples to two gluons through quark loops. The quark running in the loop with mass m couples to the Higgs boson with the Yukawa coupling $y_q = \frac{m}{v}$ with the vacuum expectation value $v = 1/\sqrt{\sqrt{2}G_F}$. The relevant two Feynman diagrams for each quark, depicting the four-momenta, are given by



The two final-state gluons have outgoing momenta p_1 and p_2 as well as Lorentz indices μ and ν and colors a and b, respectively. Accordingly, the initial-state Higgs boson has momentum $p_1 + p_2$. We want all particles to be on-shell, i.e. $(p_1 + p_2)^2 = m_{h^0}^2$, $p_1^2 = p_2^2 = 0$. Since there are no tree-level diagrams and thus no counterterms, the final result of the loop diagrams cannot develop an ultraviolet divergence.

(a) Show that the amplitude in dimensional regularisation ($d = 4 - 2\epsilon$) involving one quark q with mass m is of the form

$$\mathcal{M}_q = \epsilon_{1,\mu}^* \epsilon_{2,\nu}^* (ig_s)^2 (-iy_q)(-1) i^3 \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{T_{ij}^a T_{ji}^b \mathrm{Tr}[S^{\mu\nu}]}{(l^2 - m^2)((l + p_1)^2 - m^2)((l - p_2)^2 - m^2)}$$

with $S^{\mu\nu} = \gamma^\mu (\not{l} + \not{p}_1 + m)(\not{l} - \not{p}_2 + m)\gamma^\nu (\not{l} + m) + (-\not{l} + m)\gamma^\nu (-\not{l} + \not{p}_2 + m)(-\not{l} - \not{p}_1 + m)\gamma^\mu$.

(b) Show that $\mathrm{Tr}[S^{\mu\nu}] = 8m(g^{\mu\nu}(m^2 - l^2 - p_1 \cdot p_2) + 4 l^\mu l^\nu + p_2^\mu p_1^\nu)$. Argue why the second term in $S^{\mu\nu}$ yields the same contribution as the first term.

(c) Introduce Feynman parameters in the form

$$\frac{1}{abc} = 2 \int dx dy dz \frac{\delta(1 - x - y - z)}{(xa + yb + zc)^3} = 2 \int_0^1 dy \int_0^{1-y} dz \frac{1}{((1 - y - z)a + yb + zc)^3}$$

and shift the loop momentum l such that the denominator takes the form $(l^2 - (zp_2 - yp_1)^2 - m^2)^3 = (l^2 + yzm_{h^0}^2 - m^2)^3 =: (l^2 - M^2)^3$. Transform the numerator accordingly.

(d) Use the tensor integrals from sheet 7 to show that

$$\mathcal{M}_q = -\epsilon^*_{1,\mu}\epsilon^*_{2,\nu} g_s^2 y_q \frac{\delta^{ab}}{24\pi^2}\frac{m_{h^0}^2}{m}\epsilon^*_{1,\mu}\epsilon^*_{2,\nu}\left(g^{\mu\nu} - \frac{2}{m_{h^0}^2}p_1^\nu p_2^\mu\right) f\left(\frac{m_{h^0}^2}{m^2}\right)$$

with

$$f(x) = 3\int_0^1 dy \int_0^{1-y} dz \frac{1-4yz}{1-xyz}\,.$$

Hint: $g^{\mu\nu} I_d(0,2,M^2) + 4 I_d^{\mu\nu}(0,3,M^2)$ might be a useful relation.

(e) Check gauge invariance explicitly by replacing the polarisation vector of each gluon through the corresponding momentum. Introduce a sum over different quarks $\mathcal{M} = \sum_q \mathcal{M}_q$ with masses m_q. Square the amplitude $\mathcal{M}$ and perform the polarisation sum over the gluon polarisation. You should obtain

$$|\mathcal{M}|^2 = \alpha_s^2 \sqrt{2} G_F \frac{4 m_{h^0}^4}{9\pi^2}\left|\sum_q f\left(\frac{m_{h^0}^2}{m_q^2}\right)\right|^2 .$$

(f) Finally calculate the partial decay width, which is given by

$$\Gamma(h^0 \to gg) = \frac{\alpha_s^2 G_F m_{h^0}^3}{36\pi^3\sqrt{2}}\left|\sum_q f\left(\frac{m_{h^0}^2}{m_q^2}\right)\right|^2 .$$

Exercise 2: Higgs boson decay into gluons - Part 2

We continue with the previous exercise. *Hint:* This exercise can be performed independently. All relevant results are given in the previous exercise.

(a) Perform the integrations in the definition of $f(x)$ by using the results obtained for the dilogarithm on sheet 11. *Hint:* Perform the integration over z and determine the roots of the argument of the remaining logarithm named $y_\pm$, such that $1 - xy + xy^2 = (y_+ - y)(y_- - y)x$. Split the logarithm, use sheet 11 and use

$$\arcsin(z) = -i\ln(iz \pm \sqrt{1-z^2}) = -i\ln\left[\left(\frac{\sqrt{z^2-1}+z}{\sqrt{z^2-1}-z}\right)^2\right].$$

If you succeed, you should get for $x < 4$

$$f(x) = \frac{6}{x} - \frac{6(4-x)}{x^2}\arcsin^2\left(\frac{\sqrt{x}}{2}\right).$$

(b) What value does $f(x)$ take for heavy quarks (heavy-top limit), i.e. $m_q \to \infty$ and $m_{h^0}^2/m_q^2 \to 0$? Why does the measurement of the decay of a Higgs boson to gluons or the production of a Higgs boson from gluons allow to make a statement on the number of heavy quark generations?

(c) If you still don't have enough, try to obtain the squared amplitude using a computer, e.g. with `FeynArts` and `FormCalc`. You might encounter the Passarino-Veltman representation of the loop integrals, for which computer codes exist that allow their numerical evaluation.