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Exercise 1: Emission of a collinear photon

The focus of this exercise is on the emission of a collinear photon from an initial-state electron and its "absorbtion" into a distribution function associated with the electron. This collinear splitting is also the basis of the parton distribution functions employed at hadron colliders, which map from an initial-state proton to the proton ingredients, the quarks and the gluon. For simplicity we consider a massive gauge boson B with mass M, which couples to the left- and right-handed component of the electron equally, i.e. the Feynman rule of the e^+e^-B coupling is $-ig\gamma^\mu$.[4] Since we couple B to a conserved current, we can also use the simplified polarisation sum $\sum_i \epsilon_\mu^{(i)}\epsilon_\nu^{(i)*} = -g_{\mu\nu}$. In the first subexercise we consider the electron to be massless.

(a) Compute the cross section for $e^+e^- \to B$. You should obtain

$$\sigma(e^+e^- \to B) = \frac{\pi g^2}{2M}\delta(\sqrt{s} - M) = \pi g^2 \delta(s - M^2)\,.$$

(b) Compute the differential cross section for $e^+e^- \to \gamma B$, for which you can draw a t- and a u-channel diagram. The Feynman rule for the photon coupling to the electron is $-ie\gamma^\mu$, as discussed on sheet 13. The result for the squared amplitude should be

$$\frac{1}{4}\sum |\mathcal{M}|^2 = 2e^2 g^2 \left[\frac{u}{t} + \frac{t}{u} + \frac{2sM^2}{tu}\right]\,.$$

Go to the center-of-mass frame and introduce the scattering angle θ between the initial-state electron and the photon. The result is of the form

$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha g^2(1 - M^2/s)}{2s\sin^2\theta}\left[1 + \cos\theta^2 + \frac{4sM^2}{(s - M^2)^2}\right]\,.$$

(c) The result is divergent in the limit $\theta \to 0$ and $\theta \to \pi$, which are the collinear and anticollinear limit, respectively. These are the regions where the massless photon is close to the incoming massless electron or positron, respectively. We made a similar observation for $q\bar{q} \to gg$ on sheet 6. We consider the limit $\theta \to 0$ in more detail. We want to rearrange the formula to understand the physical origin of this (infrared) divergence. The divergence can be cut off by an electron mass m: Let the electron momentum be $p^\mu = (E, 0, 0, \sqrt{E^2 - m^2})$, and let the photon momentum carry away a fraction of it, $k^\mu = (xE, xE\sin\theta, 0, xE\cos\theta)$. Determine the Mandelstam variables for this case. The denominator of the propagator then never becomes smaller than $\mathcal{O}(m^2/s)$. Finally integrate the cross section over forward angles only, cutting off the θ integral at

[4] Considering instead a Z boson does not introduce new phenomena.

$\theta^2 \sim (m^2/s)$ keeping only the logarithmic term proportional to $\log(s/m^2)$. Show that in this approximation the cross section for forward photon emission can be written as

$$\sigma(e^+e^- \to \gamma B) \approx \int dx f(x) \sigma(e^+e^- \to B \quad \text{at} \quad E_{\text{cm}}^2 = (1-x)s) \,, \qquad (1)$$

where the annihilation cross section is evaluated for the collision of a positron of energy E and an electron of energy $(1-x)E$. The function $f(x)$ is the Weizsäcker-Williams distribution function and given by

$$f(x) = \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \log\left(\frac{s}{m^2}\right) \,.$$

It describes the collinear splitting of a photon from the initial-state electron, independent of the subsequent dynamics. *Hint:* Integrate over x in Eq. 1 by just employing $\delta(M^2 - (1-x)s)$ and show that the result equals the one with a limited integration range in θ.

Exercise 2: Higgs boson to leptons and quarks at one-loop

Read the article:

Higgs-boson decay and the running mass
E. Braaten and J.P. Leveille
Phys.Rev. D22 (1980) 715
`https://journals.aps.org/prd/abstract/10.1103/PhysRevD.22.715`[5]

(a) Prepare a $\sim$ 15–20 minutes blackboard presentation presenting the content of sections I., II. and III. with a focus on the cancellation of ultraviolet and infrared divergences, their regularisation and the renormalisation procedure.
Hint: The Spence function $\mathrm{Sp}(x)$ is exactly the dilogarithm we discussed on sheet 11.

(b) Prepare a $\sim$ 10–15 minutes blackboard presentation, which focuses on sections V. and VI.. Compare section VI. with the calculation we performed on sheet 10, exercise 2.

[5] Free access is granted from the KIT libraries and most KIT internal networks, not eduroam.