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Exercise 1: One-loop calculation in ϕ^4 theory - Part 2

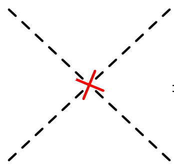
After the calculation of the one-loop corrections on sheet 7 and the proof of the optical theorem on sheet 8 we get to the renormalization of ϕ^4 theory. For this purpose we understand the parameters of the Lagrangian

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \phi_0)(\partial^\mu \phi_0) - \frac{1}{2} m_0^2 \phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

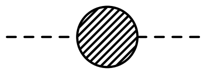
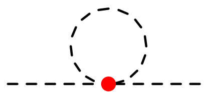
as bare parameters, which is indicated by the additional subscript 0. We introduce the subsequent renormalization constants that map to the physical parameters denoted without subscript

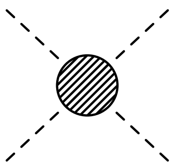
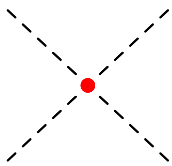
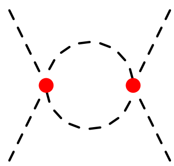
$$\begin{aligned} \phi_0 &= \sqrt{Z}\phi \quad &\text{with} \quad & Z = 1 + \delta Z\,, \\ m_0^2 &= Z_m m^2 \quad &\text{with} \quad & Z_m = 1 + \delta Z_m\,, \\ \lambda_0 &= Z_\lambda \lambda \quad &\text{with} \quad & Z_\lambda = 1 + \delta Z_\lambda\,. \end{aligned}$$

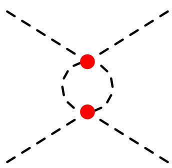
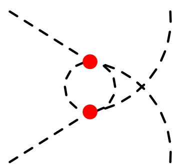
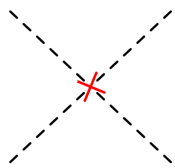
(a) Insert the above relations into the Lagrangian and split it into the physical Lagrangian and the counterterm Lagrangian that contains the renormalization constants linear in either δZ, δZ_m and δZ_λ. Higher-order terms in $\delta Z_{(\lambda,m)}$ are only needed for beyond one-loop calculations. Motivate that the Feynman rules for the counterterms at one-loop level are given by

 $= i((p^2 - m^2)\delta Z - m^2 \delta Z_m)\,,$  $= -i(\delta Z_\lambda + 2\delta Z)\lambda\,.$

(b) Combine the results from previous exercises (sheet 7 and subexercise (a)) to get the analytical form of the subsequent expressions:

$-iM(p^2) =$  $=$  $+$ 

$i\mathcal{M}(q_1 q_2 \to k_1 k_2) =$  $=$  $+$ 

$+$  $+$  $+$ 

Argue from $M(p^2)$ why $\delta Z = 0$ at the one-loop level by checking the momentum-dependence of the expression.

(c) In the previous formulas we can still shift finite parts between the master integrals I_d and the renormalization constants. A renormalization prescription resolves this ambiguity and defines which divergent and finite parts to absorb in the renormalization constants. We start with the simplest case of the minimal subtraction (MS) scheme, where the renormalization constants absorb only the divergent part $\frac{1}{\epsilon}$ (MS) or additionally associated constants $\frac{1}{\epsilon} - \gamma_E + \log(4\pi)$ ($\overline{\text{MS}}$). Expand the master integrals for $d = 4 - 2\epsilon$ in small ϵ and show that in the $\overline{\text{MS}}$ scheme the renormalization constants are given by

$$\delta Z_m^{\overline{\text{MS}}} = \frac{\lambda}{32\pi^2}\left(\frac{1}{\epsilon} - \gamma_E + \log(4\pi)\right) \quad \text{and} \quad \delta Z_\lambda^{\overline{\text{MS}}} = \frac{3\lambda}{32\pi^2}\left(\frac{1}{\epsilon} - \gamma_E + \log(4\pi)\right) .$$

Also provide the results for $-iM(p^2)$ and $i\mathcal{M}(q_1 q_2 \to k_1 k_2)$ in the $\overline{\text{MS}}$ scheme and check the dependence on the renormalization scale μ.

Exercise 2: Higgs boson decays into fermions at tree-level

Determine the partial width of a Higgs particle into a pair of fermions, more precisely the partial decay width $\Gamma(h^0 \to f\bar{f})$ in lowest order perturbation theory. The Feynman rule for the coupling yields

$$[f\bar{f}h^0] \to -ig\frac{m_f}{2m_W} ,$$

where g is the coupling constant of $\text{SU}(2)_L$, m_f is the mass of the fermion and m_W is the mass of the W boson. Express your result in terms of Fermi's constant

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} ,$$

which is given by $G_F = 1.166 \cdot 10^{-5}\,\text{GeV}^{-2}$. Provide values for the decay widths of the Standard Model Higgs boson with $m_{h^0} = 125\,\text{GeV}$ decaying into bottom quarks with $m_b = 4\,\text{GeV}$ and tau leptons with $m_\tau = 1.78\,\text{GeV}$.
Hint: Again you have the chance to use the tools presented on sheet 6!