Karlsruhe Institute of Technology

Wintersemester 2019/20
Theoretische Teilchenphysik II
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Sheet 1
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Information regarding the exercise course:
The exercise sheets are published each Tuesday on the webpage of the course, see below.
Scheinkriterium: In order to successfully pass "Theoretische Teilchenphysik II" we request you to

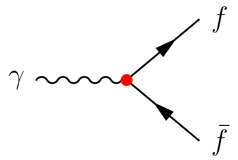
- visit at least 10 exercise classes (We collect signatures!).
- present parts of one/two exercises at the blackboard (in total $\sim 45 - 60$ minutes).
- register by writing an email to "stefan.liebler@kit.edu" specifying your full name, your "Matrikelnummer" and your class preference (see webpage) before 20.10.2019.

Exercise 1: Deep inelastic scattering

We consider the scattering of an electron and a quark, i.e. the process

$$e^-(p_1) + q(p_2) \to e^-(p_3) + q(p_4)\,.$$

This process is mediated through the exchange of a virtual photon. The relevant Feynman rule of two fermions interacting with a photon is given by



$$= -ieq_f\gamma^\mu\,.$$

Therein q_f is the electromagnetic charge of the fermion in units of the elementary charge e.

(a) Draw the relevant Feynman diagrams for this process.

(b) Deduce the matrix element $\mathcal{M}$ for an arbitrary quark with charge q_q. *Hints:* Neglect the exchange of a virtual Z boson and color factors, as the average/sum over the color charge will not yield more than a factor of 1. Use Dirac spinors for the external fermions.

(c) Calculate the squared matrix element $|\mathcal{M}|^2$ for massless electrons and quarks. Calculate $\overline{\sum}_{\text{spins}}|\mathcal{M}|^2$, i.e. average/sum over the spins of incoming/outgoing fermions, and use techniques from TTP1 to calculate the traces over the γ matrices.
Hint: Useful identities are:

$$\begin{aligned}
&\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}\\
&\mathrm{Tr}(\gamma^{\mu_1}\dots\gamma^{\mu_n}) = 0 \qquad \text{for odd } n\\
&\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4\cdot(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})\,.
\end{aligned}$$

(d) We introduce the Mandelstam variables $s = (p_1 + p_2)^2$ and $t = (p_1 - p_3)^2$. The cross section formula for the scattering of two particles with momenta p_1 and p_2 and masses m_1 and m_2 into two particles with momenta p_3 and p_4 and masses m_3 and m_4 reads

$$d\sigma = d\Phi_2 \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \overline{\sum}_{\text{spins}} |\mathcal{M}|^2$$

with the two-particle phase space

$$d\Phi_2 = \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4)\,.$$

Therein E_3 and E_4 are the energies of the two outgoing particles. In TTP1 you already showed that in the center-of-mass frame

$$d\Phi_2 = d\Omega\, \frac{\lambda(s, m_3^2, m_4^2)}{32\pi^2 s}\,,$$

where $d\Omega = d\cos\theta_3 d\phi_3$ includes the angles of particle 3 with respect to particle 1 and we neglect Θ functions and introduced the *Källén function*

$$\lambda(a, b, c) = \sqrt{a^2 + b^2 + c^2 - 2ab - 2ac - 2bc}\,.$$

Show that in the center-of-mass frame this leads to

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\lambda^2(s, m_1^2, m_2^2)} \overline{\sum}_{\text{spins}} |\mathcal{M}|^2\,,$$

when the remaining integration over the angle $\cos\theta_3$ is expressed in terms of the Mandelstam variable t. This latter expression is independent of the reference frame.

(e) Calculate the differential cross section $d\sigma/dt$ as a function of the Mandelstam variables s and t for deep inelastic scattering, again in the limit of massless fermions.
Note: Deep inelastic scattering is actually the scattering of an electron with the quarks and gluons in the proton, as free quarks do not exist.

Exercise 2: Massive quantum electrodynamics

Consider quantum electrodynamics with a single fermion ψ and a photon field A^μ, that enter the Lagrangian density as follows

$$\mathcal{L}_{\text{QED}} = \overline{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

with

$$D_\mu = \partial_\mu + ieA_\mu\,, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu\,.$$

We consider the U(1) gauge transformation

$$\psi(x) \to e^{i\alpha(x)}\psi(x)\,, \qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha(x)\,.$$

(a) Show that $\mathcal{L}_{\text{QED}}$ is invariant under the specified transformation.

(b) Show that on the other hand a mass term $\mathcal{L}_{\text{mass}} = \frac{m_A^2}{2} A^\mu A_\mu$ is not invariant under the specified transformation.