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Theoretische Teilchenphysik II

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Exercise 1: One-loop calculation in ϕ^4 theory - Part 3

In Part 2 we deduced the renormalization constants δZ_λ and δZ_m in the $\overline{\text{MS}}$ renormalization scheme, which is e.g. often applied in QCD calculations. However, e.g. in electroweak theory the masses of the particles are measurable quantities, such that we want them to be constants. In a sloppy formulation we thus demand that one-loop corrections should vanish at physical masses for the self-energy corrections (and similarly at a certain momentum for the scattering amplitude). This leads to the concept of on-shell renormalization.

(a) We obtained the correction $-iM(p^2)$ in Part 2 of this exercise. Combine this correction to the infinite series of propagator corrections

and use the geometric series to get the all-order loop-corrected propagator

$$\frac{i}{p^2 - m^2 - M(p^2)} \approx \frac{i}{p^2 - m^2}\left(1 + \frac{M(p^2)}{p^2 - m^2}\right),$$

where the latter expression corresponds to the pure one-loop result. Demand that the location of the pole and the residue of the propagator do not change between one-loop and tree-level through

$$\lim_{p^2 \to m^2} \frac{i}{p^2 - m^2}\left(1 + \frac{M(p^2)}{p^2 - m^2}\right) \stackrel{!}{=} \lim_{p^2 \to m^2} \frac{i}{p^2 - m^2}.$$

Perform a Taylor expansion of $M(p^2)$ around $p^2 = m^2$ and thus show $M(p^2)|_{p^2=m^2} = 0$ and $\frac{d}{dp^2}M(p^2)|_{p^2=m^2} = 0$. Show that the first equation leads to

$$\delta Z_m^{\text{OS}} = \frac{\lambda}{32\pi^2}\left(\frac{1}{\epsilon} - \gamma_E + \log(4\pi) + 1 + \log\left(\frac{\mu^2}{m^2}\right)\right).$$

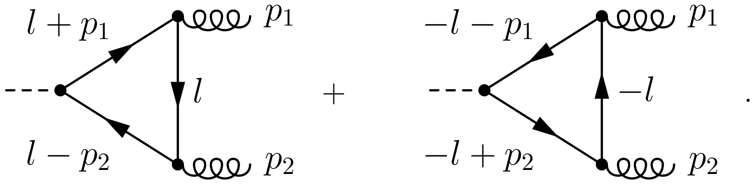
(b) We know that the bare mass parameter m_0^2 is independent of the chosen scheme. We can thus obtain a relation between the $\overline{\text{MS}}$ mass and the on-shell mass, which yields $m^2_{\text{OS}} = m^2_{\overline{\text{MS}}}(\mu) Z_m^{\overline{\text{MS}}} (Z_m^{\text{OS}})^{-1}$. Insert the expressions obtained for the renormalization constants and expand in λ to first order. You should obtain a finite expression, which for $\mu = m_{\overline{\text{MS}}}$ simplifies to

$$m^2_{\text{OS}} = m^2_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})\left(1 - \frac{\lambda}{32\pi^2} + \mathcal{O}(\lambda^2)\right).$$

Thus, for the choice $\mu = m_{\overline{\text{MS}}}$ potentially large logarithms cancel out of the relation.

Exercise 2: Higgs boson decay into gluons - Part 1

The aim of this exercise is to calculate the partial decay width of the Standard Model Higgs boson into a pair of gluons, $h^0 \to gg$, in the first non-vanishing order. The decay is loop-mediated, i.e. the Higgs boson couples to two gluons through quark loops. The quark running in the loop with mass m couples to the Higgs boson with the Yukawa coupling $y_q = \frac{m}{v}$ with the vacuum expectation value $v = 1/\sqrt{\sqrt{2}G_F}$. The relevant two Feynman diagrams for each quark, depicting the four-momenta, are given by



The two final-state gluons have outgoing momenta p_1 and p_2 as well as Lorentz indices μ and ν and colors a and b, respectively. Accordingly, the initial-state Higgs boson has momentum $p_1 + p_2$. We want all particles to be on-shell, i.e. $(p_1 + p_2)^2 = m_{h^0}^2$, $p_1^2 = p_2^2 = 0$. Since there are no tree-level diagrams and thus no counterterms, the final result of the loop diagrams cannot develop an ultraviolet divergence.

(a) Show that the amplitude in dimensional regularisation ($d = 4 - 2\epsilon$) involving one quark q with mass m is of the form

$$\mathcal{M}_q = \epsilon_{1,\mu}^* \epsilon_{2,\nu}^* (ig_s)^2 (-iy_q)(-1) i^3 \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{T_{ij}^a T_{ji}^b \mathrm{Tr}[S^{\mu\nu}]}{(l^2 - m^2)((l + p_1)^2 - m^2)((l - p_2)^2 - m^2)}$$

with $S^{\mu\nu} = \gamma^\mu (\not{l} + \not{p}_1 + m)(\not{l} - \not{p}_2 + m)\gamma^\nu (\not{l} + m) + (-\not{l} + m)\gamma^\nu(-\not{l} + \not{p}_2 + m)(-\not{l} - \not{p}_1 + m)\gamma^\mu$.
Hint: Remember the quark-quark-gluon coupling $ig_s\gamma^\mu T^a$ and $T_{ij}^a T_{ji}^b = \frac{1}{2}\delta^{ab}$ (sheet 3).

(b) Show that $\mathrm{Tr}[S^{\mu\nu}] = 8m(g^{\mu\nu}(m^2 - l^2 - p_1 \cdot p_2) + 4l^\mu l^\nu + p_2^\mu p_1^\nu)$. Argue why the second term in $S^{\mu\nu}$ yields the same contribution as the first term.

(c) Introduce Feynman parameters in the form

$$\frac{1}{abc} = 2 \int dx dy dz \frac{\delta(1 - x - y - z)}{(xa + yb + zc)^3} = 2 \int_0^1 dy \int_0^{1-y} dz \frac{1}{((1 - y - z)a + yb + zc)^3}$$

and shift the loop momentum l such that the denominator takes the form $(l^2 - (zp_2 - yp_1)^2 - m^2)^3 = (l^2 + yzm_{h^0}^2 - m^2)^3 =: (l^2 - M^2)^3$. Transform the numerator accordingly.

(d) Use the tensor integrals from sheet 10 to show that

$$\mathcal{M}_q = -\epsilon_{1,\mu}^* \epsilon_{2,\nu}^* g_s^2 y_q \frac{\delta^{ab}}{24\pi^2} \frac{m_{h^0}^2}{m} \epsilon_{1,\mu}^* \epsilon_{2,\nu}^* \left(g^{\mu\nu} - \frac{2}{m_{h^0}^2} p_1^\nu p_2^\mu \right) f\left(\frac{m_{h^0}^2}{m^2} \right)$$

with

$$f(x) = 3 \int_0^1 dy \int_0^{1-y} dz \frac{1 - 4yz}{1 - xyz} .$$

Hint: $g^{\mu\nu} I_d(0, 2, M^2) + 4I_d^{\mu\nu}(0, 3, M^2)$ might be a useful relation.

(e) Check gauge invariance explicitly by replacing the polarisation vector of each gluon through the corresponding momentum. Introduce a sum over different quarks $\mathcal{M} = \sum_q \mathcal{M}_q$ with masses m_q. Square the amplitude $\mathcal{M}$ and perform the polarisation sum over the gluon polarisation. You should obtain

$$|\mathcal{M}|^2 = \alpha_s^2 \sqrt{2} G_F \frac{4m_{h^0}^4}{9\pi^2} \left| \sum_q f\left(\frac{m_{h^0}^2}{m_q^2} \right) \right|^2 .$$

(f) Finally calculate the partial decay width using $|\mathcal{M}|^2$ from the previous exercise.