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Exercise 1: Higgs boson decays at tree level

In this exercise we want to discuss all $1 \to 2$ Higgs boson decays in the Standard Model of particle physics, which emerge at tree-level. The general formula for the decays is

$$d\Gamma(H \to XY) = \frac{1}{2m_H} d\Phi_2 \overline{\sum}_{\text{spins, colors}} |\mathcal{M}_{H\to XY}|^2 \,,$$

where $d\Phi_2$ is again the two-particle phase space as printed on the first exercise sheet and for $X = Y$ an additional symmetry factor $\frac{1}{2}$ has to be added. In TTP1 you already determined the decay width into fermions, which was found to be

$$\Gamma(H \to f\bar{f}) = N_c \frac{G_F m_f^2 m_H}{4\sqrt{2}\pi} \left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2}$$

with $G_F = \frac{1}{\sqrt{2}v^2} = 1.16638 \cdot 10^{-5}\,\text{GeV}$ and a color factor N_c, which is 3(1) for quarks(leptons).

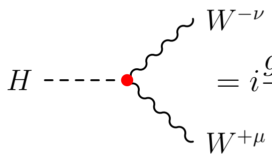
(a) Calculate the partial decay widths into a pair of gauge bosons, i.e. $\Gamma(H \to W^+W^-)$ and $\Gamma(H \to ZZ)$. Show that the result can be written in the form

$$\Gamma(H \to W^+W^-) = \frac{G_F m_H^3}{8\sqrt{2}\pi} \left(1 - \frac{4m_W^2}{m_H^2}\right)^{1/2} \left(1 - \frac{4m_W^2}{m_H^2} + \frac{12m_W^4}{m_H^4}\right) .$$

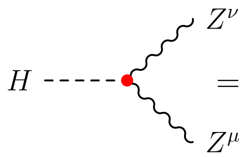
For the decays into a pair of Z bosons an additional factor of $\frac{1}{2}$ has to be added.
Hint: Make use of the polarisation sum for massive gauge bosons with mass m_V, being

$$\sum_{\lambda=1}^{3} \epsilon_\mu(p,\lambda)\epsilon_\nu^*(p,\lambda) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_V^2} .$$

The relevant Feynman rules are given by



$$= i\frac{g^2 v}{2} g^{\mu\nu} \,,$$



$$= i\frac{g^2 v}{2c_W^2} g^{\mu\nu} \,,$$

where we can identify $m_W = \frac{1}{2}gv$ and $m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v = \frac{m_W}{c_W}$ with the cosine of the weak mixing angle $c_W = \frac{m_W}{m_Z}$.

(b) For each final state $H \to XY$ we define its branching ratio through the ratio of the partial width through the total width of the Higgs boson, i.e.

$$\text{BR}(H \to XY) = \frac{\Gamma(H \to XY)}{\sum_{AB} \Gamma(H \to AB)} .$$

Consider the decays into the two massive gauge bosons as well as into the four heaviest fermions of the Standard Model, which have masses

$$\begin{aligned}
\text{Gauge bosons :}\quad & m_W = 80.4\,\text{GeV}\,, \quad m_Z = 91.2\,\text{GeV} \\
\text{Quarks with } N_c = 3 :\quad & m_t = 173.2\,\text{GeV}\,, \quad m_b = 4.8\,\text{GeV}\,, \quad m_c = 1.3\,\text{GeV} \\
\text{Leptons with } N_c = 1 :\quad & m_\tau = 1.8\,\text{GeV}\,.
\end{aligned}$$

Make a graphic, that shows the branching ratios of the Higgs boson as a function of the Higgs boson mass m_H between 10 and 1000 GeV.

Solution of exercise 1

After the introduction of the Standard Model of particle physics, we are able to discuss all $1 \to 2$ Higgs boson decay modes, that emerge at tree-level. As written in the exercise the result for decays into fermions was already derived in TTP1, which is why we don't repeat it here. The decay into gluons (and photons) is loop mediated and will be discussed later in the lecture. Before we proceed we remind the reader, that the two particle phase space for the decay into a pair of particles with mass m_F is given by

$$d\Phi_2 = d\Omega \frac{\lambda(m_H^2, m_F^2, m_F^2)}{32\pi^2 m_H^2}\,,$$

which integrated over $d\Omega$ as the matrix element does not develop an angular dependence results in

$$\begin{aligned}
\int d\Phi_2 &= \frac{1}{8\pi m_H^2}\sqrt{m_H^4 + m_F^4 + m_F^4 - 2m_F^4 - 4m_H^2 m_F^2} \\
&= \frac{1}{8\pi m_H^2} m_H^2 \sqrt{1 - \frac{4m_F^2}{m_H^2}} = \frac{1}{8\pi}\sqrt{1 - \frac{4m_F^2}{m_H^2}}\,.
\end{aligned}$$

(a) We denote the momenta of the outgoing W bosons as p_1 and p_2 for W^+ and W^- and accordingly their helicities λ_1 and λ_2. The matrix element is then given by

$$\begin{aligned}
\mathcal{M} &= \epsilon_\mu^*(p_1,\lambda_1)\epsilon_\nu^*(p_2,\lambda_2)\frac{ig^2 v}{2} g^{\mu\nu} \\
\to \mathcal{M}^\dagger &= \epsilon_\mu(p_1,\lambda_1)\epsilon_\nu(p_2,\lambda_2)\frac{-ig^2 v}{2} g^{\mu\nu}
\end{aligned}$$

We square the matrix element and sum over the helicities using the provided helicity sum, that was derived in TTP1. We obtain

$$\begin{aligned}
\overline{\sum}|\mathcal{M}|^2 &= \sum_{\lambda_1,\lambda_2} \epsilon_\mu^*(p_1,\lambda_1)\epsilon^*(p_2,\lambda_2)\epsilon_\rho(p_1,\lambda_1)\epsilon_\sigma(p_2,\lambda_2)\frac{g^4 v^2}{4} g^{\mu\nu} g^{\rho\sigma} \\
&= \frac{g^4 v^2}{4}\left(-g_{\mu\rho} + \frac{p_{1\mu}p_{1\rho}}{p_1^2}\right)\left(-g_{\nu\sigma} + \frac{p_{2\nu}p_{2\sigma}}{p_2^2}\right) g^{\mu\nu} g^{\rho\sigma} \\
&= \frac{g^4 v^4 4}{16 v^2}\left(g_\mu^\mu - \frac{p_1^2}{p_1^2} - \frac{p_2^2}{p_2^2} + \frac{(p_1\cdot p_2)^2}{p_1^2 p_2^2}\right)\,.
\end{aligned}$$

Moreover it yields $m_H^2 = (p_1+p_2)^2 = p_1^2+p_2^2+2p_1\cdot p_2 = 2m_W^2 + 2p_1\cdot p_2$ and $m_W^2 = \frac{1}{4}g^2v^2$, such that we get

$$\begin{aligned}\overline{\sum}|\mathcal{M}|^2 &= \frac{4m_W^2}{v^2}\left(2+\frac{1}{m_W^4}\left(\frac{m_H^2}{2}-m_W^2\right)^2\right)\\ &= \frac{4m_W^4}{v^2}\left(2+\frac{m_H^4}{4m_W^4}-\frac{m_H^2}{m_W^2}+1\right)\\ &= \frac{m_H^4}{v^2}\left(1-\frac{4m_W^2}{m_H^2}+\frac{12m_W^4}{m_H^4}\right).\end{aligned}$$

Combined with the above phase space and using $G_F = \frac{1}{\sqrt{2}v^2}$ we thus get

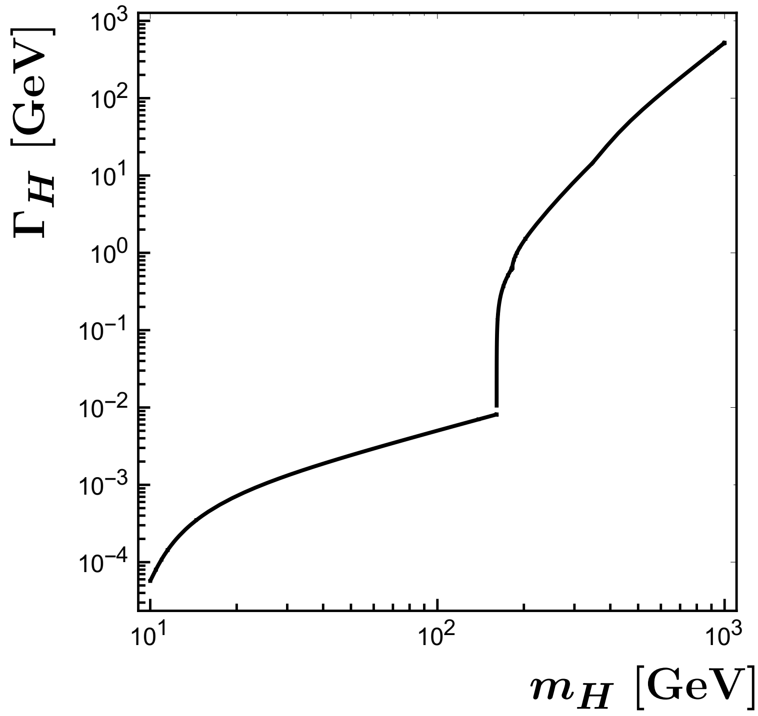
$$\begin{aligned}\Gamma(H\to W^+W^-) &= \frac{1}{2m_H}\frac{1}{8\pi}\sqrt{1-\frac{4m_W^2}{m_H^2}}\frac{m_H^4}{v^2}\left(1-\frac{4m_W^2}{m_H^2}+\frac{12m_W^4}{m_H^4}\right)\\ &= \frac{G_F m_H^3}{8\sqrt{2}\pi}\sqrt{1-\frac{4m_W^2}{m_H^2}}\left(1-\frac{4m_W^2}{m_H^2}+\frac{12m_W^4}{m_H^4}\right)\end{aligned}$$

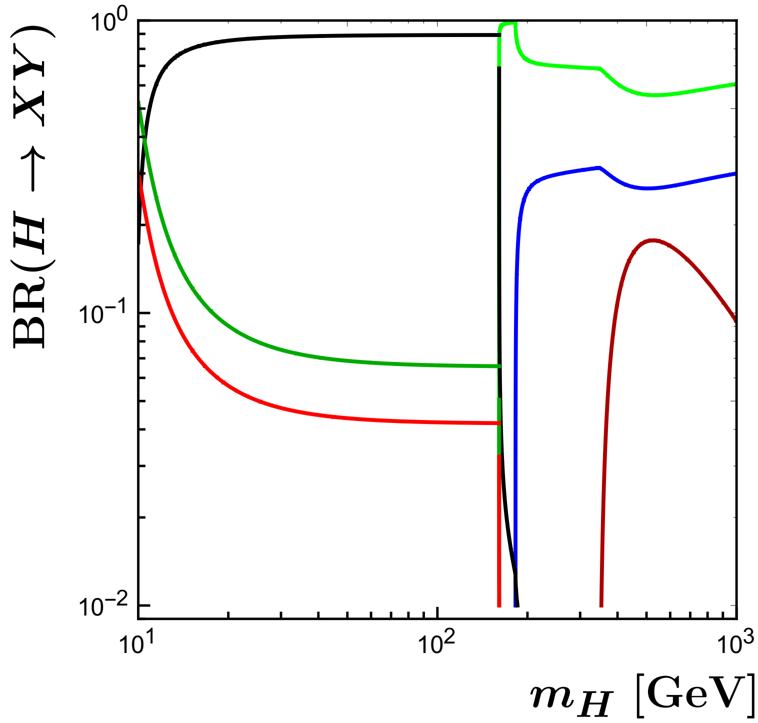
We remember that $m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2 = \frac{m_W^2}{\cos\theta_W^2}$ and thus identify the coupling of the Higgs boson to the Z boson as $\frac{2im_Z^2}{v}g^{\mu\nu}$ compared to $\frac{2im_W^2}{v}g^{\mu\nu}$ for the W boson. The calculation is thus completely analogous to the above case, with the exception that we produce two particles that are not distinguishable, for which we need to add another factor of $\frac{1}{2}$. This results in

$$\Gamma(H\to ZZ) = \frac{G_F m_H^3}{16\sqrt{2}\pi}\sqrt{1-\frac{4m_Z^2}{m_H^2}}\left(1-\frac{4m_Z^2}{m_H^2}+\frac{12m_Z^4}{m_H^4}\right).$$

For $m_H \gg \{m_Z, m_W\}$ this implies that the ratio of the branching ratios $\Gamma(H\to W^+W^-)/\Gamma(H\to ZZ)\to 2$. Note that in this limit the partial decay width are independent of the gauge boson masses!

(b) We follow the instructions of the subexercise and plot the branching ratios provided that the Higgs boson only decay in $1\to 2$ decays into the heavy gauge bosons and the four heaviest fermions. Please consider a Mathematica notebook for a graphical illustration of the branching ratios. It also shows the total decay width, which is only small below the ZZ and WW thresholds and increases rapidly for larger Higgs boson masses. Please have a look at the two subsequent plots. In the second plot the color coding is as follows: $H\to W^+W^-$ (green), $H\to ZZ$ (blue), $H\to t\bar{t}$ (dark red), $H\to b\bar{b}$ (black), $H\to c\bar{c}$ (dark green), $H\to\tau^+\tau^-$ (red).





At this point we emphasize that the Higgs boson at 125 GeV also decays into $H \to WW^* \to$ 4fermions and $H \to ZZ^* \to$ 4fermions via off-shell decays, which is why the subsequent graphic (provided by the mighty LHCHXSWG) shows rather smooth thresholds for the decays into WW and ZZ. Also it includes the loop-mediated decays $H \to gg$ and $H \to \gamma\gamma$, where the latter was crucial for the discovery of a scalar at 125 GeV. Lastly note that the decay into light quarks is actually smaller than suggested by the tree-level calculation, as the right mass to use is the running mass at the scale of the Higgs boson. We will return to this point when we calculate higher-order corrections.