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Exercise 1: Propagator of the gluon field in axial gauge

Rather than choosing the gauge fixing term in the covariant gauge we want to discuss an alternative gauge choice in this exercise, namely the axial gauge (Arnowitt-Fickler gauge), which in covariant form can be written in the form $F^a[A] = n_\mu A^{\mu,a} = 0$ with an arbitrary constant vector n_μ. The for our purposes relevant part of the action therefore takes the form

$$S = i \int d^4x \left[-\frac{1}{4} F^{\mu\nu,a} F^a_{\mu\nu} - \frac{1}{2\xi} (n_\mu A^{\mu,a})^2 - \overline{\eta}^a \, (n^\mu) \, (D_\mu)_{ab} \eta^b \right] . \tag{1}$$

We now work in a non-Abelian theory, which explains the additional Roman color indices. The last term of Eq. 1, which due to the Abelian structure was omitted on the previous exercise sheet, includes ghost fields η and $\overline{\eta}$ and the covariant derivative $(D_\mu)_{bc} = \partial_\mu \delta_{bc} - igT^a_{bc} A^a_\mu$ in the adjoint representation.

(a) Derive the propagator of the gluon field from the first two terms in Eq. 1 in the same way it was done on the previous sheet. Use the ansatz

$$\Delta_{\mu\nu} = A g_{\mu\nu} + B k_\mu k_\nu + C(k_\mu n_\nu + k_\nu n_\mu) + D n_\mu n_\nu$$

though. Take the limit $\xi \to 0$. The result should remind you of the polarisation sum for massless gauge fields. Show that $n^\mu \Delta_{\mu\nu} = 0$ for $\xi \to 0$.
Hint: The term $+g f^{abc} A^b_\mu A^c_\nu$ in $F^{\mu\nu,a}$ is not of relevance for the propagator, since it leads to terms with three gauge fields.

(b) Show that the Fadeev-Popov determinant as it was defined in the lecture is independent of A^a_μ. This implies that the ghost field decouples and need not be considered in practical calculations in this gauge.

Exercise 2: R_ξ gauge in broken QED

We consider quantum electrodynamics (QED) broken through electroweak symmetry breaking, which is described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi)$$

with $D_\mu = \partial_\mu + ieA_\mu$, where ϕ is a complex scalar field, which obtains a vacuum expectation value according to

$$\phi = \frac{1}{\sqrt{2}} (v + h + i\varphi) \; .$$

We choose the gauge fixing $G = \frac{1}{\sqrt{\xi}}(\partial_\mu A^\mu - \xi e v \varphi)$, which results in the contribution $\mathcal{L}_{\text{fix}} = -\frac{1}{2}G^2$ to the Lagrangian density. The fields transform as follows under the infinitesimal gauge transformation

$$\delta h = -\alpha(x)\varphi, \qquad \delta\varphi = \alpha(x)(v+h), \qquad \delta A_\mu = -\frac{1}{e}\partial_\mu \alpha(x)\,.$$

(a) Derive the photon and Goldstone boson masses, m_A and m_φ, respectively.

(b) Derive the ghost Lagrangian $\mathcal{L}_{\text{ghost}}$. Rescale the fields, such that the kinetic term is of canonical form. Deduce the mass of the ghost field. Why can this term not be neglected in comparison to ordinary QED?

(c) We add another chiral interaction with a fermion field f by

$$\mathcal{L}_f = \overline{f}_L(i\not{D})f_L + \overline{f}_R(i\not{\partial})f_R - \lambda_f(\overline{f}_L\phi f_R + \overline{f}_R\phi^* f_L)$$

and consider fermion-fermion scattering $\overline{f}f \to \overline{f}f$. Which Feynman diagrams contribute at lowest order in perturbation theory?

(d) Write down the matrix element of the process $\overline{f}f \to \overline{f}f$ and show that it is independent of the gauge parameter ξ.
Hint: Make use of the following Feynman propagators

$$\text{propagator scalar/ghost with mass } m_j : \frac{i}{k^2 - m_j^2}$$

$$\text{propagator massive photon} : \frac{i}{k^2 - m_A^2}\left(-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi m_A^2}\right)$$

and the subsequent vertices:

$$\overline{f}_L f_R h : \quad \frac{\lambda_f}{\sqrt{2}} \qquad\qquad \overline{f}_R f_L h : \quad \frac{\lambda_f}{\sqrt{2}}$$

$$\overline{f}_L f_R \varphi : \quad \frac{\lambda_f}{\sqrt{2}} \qquad\qquad \overline{f}_R f_L \varphi : \quad -\frac{\lambda_f}{\sqrt{2}}$$

$$\overline{f}_L f_L A^\mu : \quad -ie\gamma^\mu \qquad\qquad \overline{\eta}\eta h : \quad \xi\frac{m_A^2}{v}$$

(e) Can you think of a process at tree-level, which contains ghost contributions?