Einführung in Theoretische Teilchenphysik

Lecture: PD Dr. S. Gieseke – Exercises: Dr. Christoph Borchensky, Dr. Cody B Duncan

Exercise Sheet 1

Hand-in Deadline: Mo. 25.10.21, 12:30.
Discussion: Di. 26.10.21, Mi. 27.10.21.

1. **[7 points] Warm Up: Natural Units.**

   Particle physicists commonly employ *natural units* ($\hbar = c = 1$) to express quantities in mass/energy units. In this system, velocities are in units of c, while foces are measured in units of Planck's constant $\hbar$.

   (a) [3 point] How do length (m), time (s), and energy (eV) relate to each other in this system?
   Hint: Use that $c = 3 \times 10^8 \text{ms}^{-1}$, $\hbar = 1.1 \times 10^{-34} \text{kgm}^2\text{s}^{-1}$ and $\text{eV} = 1.6 \times 10^{-19}\text{kg m}^2\text{s}^{-2}$.

   (b) [1 points] Ultraviolet radiation, such as that emitted by the sun, has a wavelength between $10 - 400\,\text{nm}$. Express a ray of 200 nm in eV.

   (c) [3 points] The dark energy density of the universe is roughly $(2\text{meV})^4$. Express this in $\text{kg}\,\text{m}^{-3}$.

2. **[5 points] Energy and distance scales**

   (a) [2 points] Estimate the typical length scale that is probed by experiments: at the
      i. Large Electron-Positron Collider (LEP) ($\sqrt{s} = 45 - 209$ GeV);
      ii. at the Run II Large Hadron Collider (LHC) ($\sqrt{s} = 13$ TeV)

   (b) [1 point] In light of these results, discuss why the notions of Particle Physics and High Energy Physics are so often used as synonyms.

   Hint: Recall the notion of wave-particle duality and how it is implemented by the de Broglie relation

   (c) [2 point] Estimate the energy necessary to test the characteristic length scales of a quantum theory of gravitation.

3. **[5 points] Conservation of probability.** Using the non-relativistic Schrödinger equation,

$$i\partial_t \psi(\mathbf{x}, t) = -\frac{\nabla^2}{2m}\psi(\mathbf{x}, t) + V\psi(\mathbf{x}, t),$$

   and its complex conjugate, derive the continuity equation for the probability density

$$\frac{\partial \rho(\mathbf{x}, t)}{\partial t} + \nabla \cdot \mathbf{j}(\mathbf{x}, t) = 0\,,$$

with

$$\rho(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2, \qquad \mathbf{j}(\mathbf{x},t) = -\frac{i}{2m}\left(\psi^*(\mathbf{x},t)\nabla\psi(\mathbf{x},t) - \psi(\mathbf{x},t)\nabla\psi^*(\mathbf{x},t)\right) .$$