Einführung in Theoretische Teilchenphysik

Lecture: PD Dr. S. Gieseke – Exercises: Dr. Christoph Borschensky, Dr. Cody B. Duncan

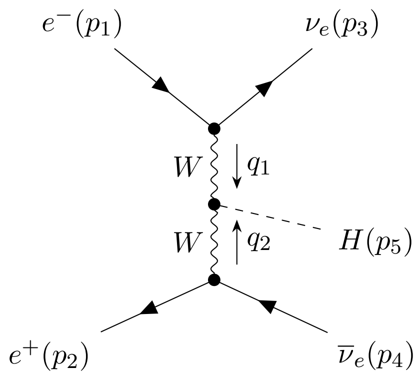
---

Exercise Sheet 12

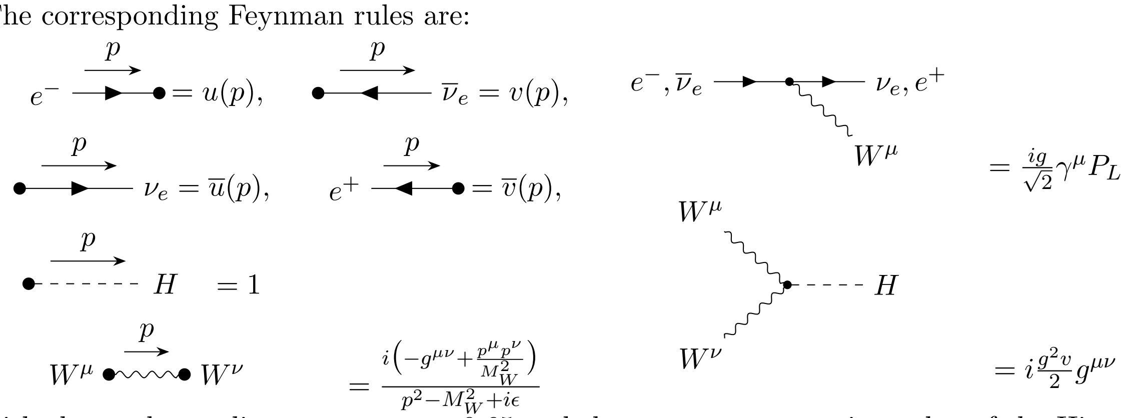
Hand-in Deadline: Mo 31.01.22, 12:00.
Discussion: Di 01.02.22, Mi 02.02.22.

[20 points] Higgs Production via W-Boson Fusion

Higgs production via W-boson fusion is one of the main Higgs production processes at electron-positron colliders. At leading order, this process is given by a single Feynman diagram:



The corresponding Feynman rules are:



with the weak coupling constant $g \approx 0.65$ and the vacuum expectation value of the Higgs field $v \approx 246$ GeV.

1. **[4 points]** Show that the following properties of the chirality projection operators $P_{R,L} = (1 \pm \gamma^5)/2$, which we will need in the following:

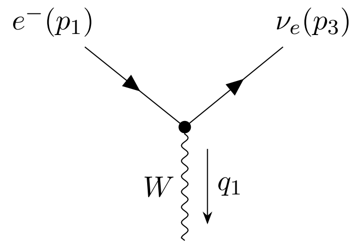
   (a) **[1 point]** $(P_{R,L})^2 = P_{R,L}$ and $P_{R,L}^\dagger = P_{R,L}$

   (b) **[1 point]** $P_R + P_L = 1$ and $P_R - P_L = \gamma^5$

(c) **[1 point]** $P_{R,L}\gamma^\mu = \gamma^\mu P_{L,R}$

(d) **[1 point]** $\gamma^0 P_{R,L}\gamma^0 = P_{L,R}$

Consider the following sub-diagram:



where the W-boson is "amputated", i.e. the contraction with its polarization vector is omitted and instead the matrix element contains an open Lorentz index μ.

2. **[6 points]** Write down the corresponding matrix element $\mathcal{M}_1^\mu$. Calculate $q_{1,\mu}\mathcal{M}_1^\mu$ and eliminate the explicit momentum dependence of the expression, assuming that the neutrino also has a mass m_v (Note that this is not the index "nu", but just the "v" of the neutrino). What happens in the limit of $m_e = m_v$? Show that the product $q_{1,\mu}\mathcal{M}_1^\mu$ vanishes in the massless limit $m_e = m_v = 0$.

3. **[6 points]** For the rest of the question (i.e. all the subsequent parts), we consider only massless fermions, i.e. $m_e = m_v = 0$.

   Show that for the squared matrix element of the sub-diagram, one obtains (after spin summation):

$$\sum_{s_1,s_3} |\mathcal{M}_1|^{2,\mu\nu} := \sum_{s_1,s_3} \mathcal{M}_1^\mu \mathcal{M}_1^{\dagger,\nu} = g^2\left(p_1^\mu p_3^\nu + p_3^\mu p_1^\nu - p_1\cdot p_3 g^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma} p_{1,\rho} p_{3,\sigma}\right)$$

   Then consider how the expression changes for the analogous part of the lower anti-fermion line.

4. **[4 points]** Use this to calculate the spin-averaged squared matrix element of the whole Feynman diagram. Consider first the result of the "middle part" (W propagators, HWW interaction vertex) separately. The limit $\epsilon \to 0$ of the $i\epsilon$ terms in the propagators can be taken immediately. Take care to contract the correct Lorentz indices.

   The result you should aim towards is:

$$\overline{\sum}|\mathcal{M}|^2 = \frac{g^8 v^2}{4} \frac{1}{\left(q_1^2 - M_W^2\right)^2} \frac{1}{\left(q_2^2 - M_W^2\right)^2} (p_1\cdot p_4)(p_2\cdot p_3)$$

5. **[0 points]**, no hand-in!

   Write a program which calculates the corresponding cross section:

$$\sigma = \frac{1}{2s}\int \mathrm{dPS}\ \text{(3-particle)} \overline{\sum}|\mathcal{M}|^2$$

   for a given centre-of-mass energy $\sqrt{s}$ of the electron-positron pair via numerical integration. The 3-particle phase space with one massive final-state particle can be written as:

$$\begin{aligned}\int \mathrm{d}\Phi_3 &:= \int \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4} \frac{\mathrm{d}^3 p_5}{(2\pi)^3 2E_5} (2\pi)^4 \delta^{(4)}\left(p_1 + p_2 - p_3 - p_4 - p_5\right) \\ &= \frac{s}{512\pi^4} \int_0^1 \mathrm{d}x_3 \int_{1-x_3}^1 \mathrm{d}x_4 \int_{-1}^1 \mathrm{d}\cos\theta_3 \int_0^{2\pi} \mathrm{d}\phi_4,\end{aligned}$$

   where $s = (p_1+p_2)^2$, $x_i = 2E_i/\sqrt{s}$ and ϕ_4 is taken relative to the plane spanned by the vectors $\vec{p}_3$ and $\vec{e}_z$.