ZARM, University of Bremen July 28, 2010

Towards a derivation of G

Frans R. Klinkhamer

Institute for Theoretical Physics, University of Karlsruhe,

Karlsruhe Institute of Technology

Email: frans.klinkhamer@kit.edu

a somewhat unusual talk . . .

Physical constants

Fundamental physical constants (Roemer, Cavendish, Planck):

$$\boxed{c, \quad G, \quad \hbar}$$

Theorists often set $c = 1$, $G = 1$, and $\hbar = 1$,
by using appropriate units for length, time, and energy.

This practice considers SR (= special relativity), GR (= general relativity), and QM (= quantum mechanics), to be <u>closed chapters</u>.

But what does Nature say?

Physical constants

Table 1: Known constants of nature [1].

quantum matter (Planck & Bohr)	classical relativity (Einstein)	quantum spacetime (Wheeler)
$\hbar$	$c\,,\,G$	$l_P \equiv \sqrt{\hbar\,G/c^3}$

Possible argument for a single constant $\hbar$ controlling the quantum nature of both matter (e.g., photons & electrons) and spacetime:

- quantized electrons $\leftrightarrow$ quantized electromagnetic field $\Rightarrow$ QED [exps: Geiger and Bothe, 1925; Compton and Simon, 1925];
- similarly, quantized electrons $\leftrightarrow$ quantized metric field? $\Rightarrow$???

[1] P.J. Mohr, B.N. Taylor, and D.B. Newell, RMP 80, 633 (2008), arXiv:0801.0028.

Physical constants

Table 2: Alternative constants of nature [2].

quantum matter	classical relativity	quantum space
$\hbar$	$c\,,\, G \equiv f\, c^3\, l^2/\hbar$	l^2

Possible arguments for a new constant l^2 of quantized space:

- space (and gravity) may be emergent phenomena;
- natural to have a constant with dimension of length/area/volume.

Conceptual remark:

If Table 2 holds true, there may be physical situations where matter quantum effects are negligible ("$\hbar = 0$") but not spacetime quantum effects ("$l^2 \neq 0$"), which is impossible if Table 1 holds ($l_P = 0$ for $\hbar = 0$).

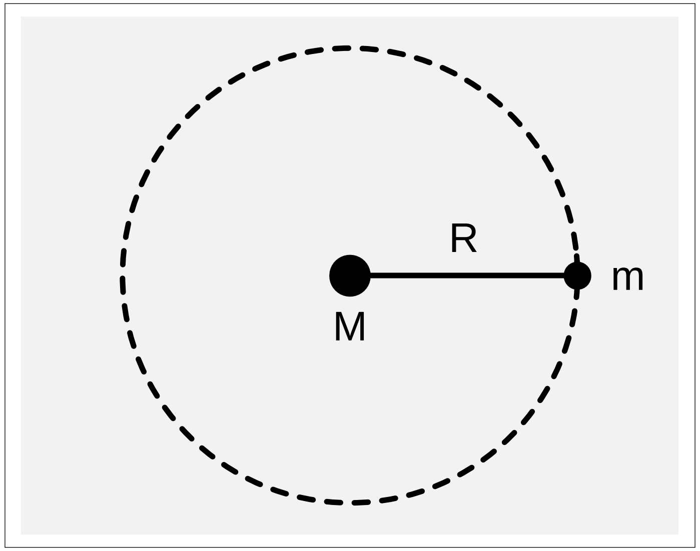
---

[2] F.R. Klinkhamer, JETPL 86, 73 (2007), arXiv:gr-qc/0703009.

Entropic gravity

Now, consider **Newtonian gravity**, specifically, the inward acceleration $\vec{A}_{\text{grav}}$ on a test mass m produced by a point mass M at a distance R:

$$\vec{A}_{\text{grav}} = -(G\,M/R^2)\,\widehat{e}_R\,. \tag{1}$$



Newton (1713): "*hypotheses non fingo*"

Entropic gravity

Using the G formula from Table 2,

$$G = f\, c^3\, l^2/\hbar\,, \tag{2}$$

with a factor $f > 0$, the magnitude of this acceleration reads [2]

$$A_{\text{grav}} = GM/R^2 = f\, c\left(M c^2/\hbar\right)\left(l^2/R^2\right), \tag{3}$$

where all microscopic quantities are indicated by lower-case symbols.

Possible interpretation of the two factors in brackets on the RHS of (3):

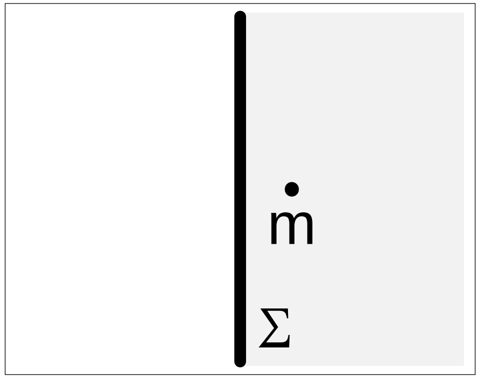
- first factor is a *decay rate of space* triggered by external mass M;
- second factor is a *geometric dilution factor*.

Interpretation perhaps suggestive but definitely vague.

Progress from a recent idea of Verlinde . . .

Entropic gravity

Verlinde's proposal [3] is that Newtonian gravity arises as an **entropic force** from a **holographic** [4] microscopic theory.



Main steps [3]:

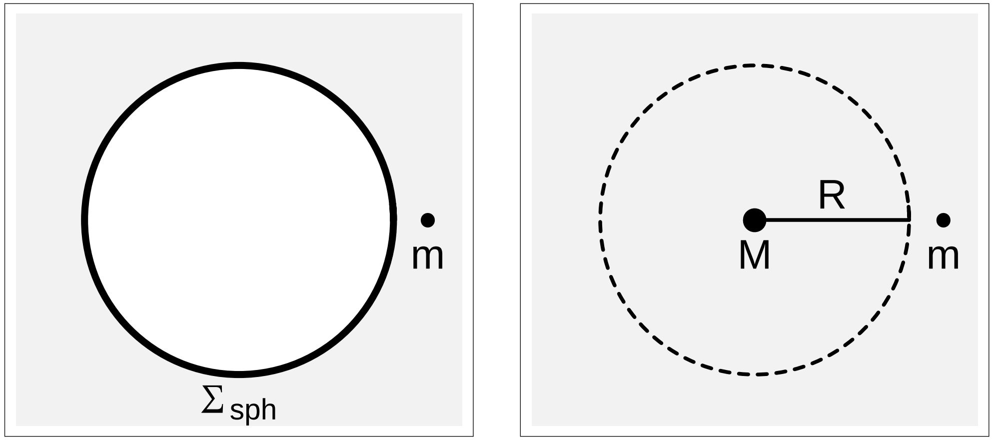
- holographic screen $\Sigma(x_1, x_2)$ with orthogonal dimension x_3 emerging from coarse-graining degrees of freedom (d.o.f.) on Σ;
- entropy change from nearby mass m at distance Δx_3 is given by $\Delta S_\Sigma \propto (m\, c/\hbar)\, \Delta x_3$;
- first law of thermodynamics: $T \Delta S_\Sigma = F_{\text{grav}} \Delta x_3 \Rightarrow F_{\text{grav}} \propto mM/R^2$, with mass equivalent M of spherical screen with area $4\pi R^2$.

---

[3] E. Verlinde, arXiv:1001.0785v1.

[4] G. 't Hooft, arXiv:gr-qc/9310026; L. Susskind, JMP 36, 6377 (1995), arXiv:hep-th/9409089.

Entropic gravity



Left panel: Spherical holographic screen Σ_{sph} with area $A = 4\pi R^2$ and test mass m. Space has emerged outside the screen Σ_{sph}, which has N microscopic degrees of freedom at an equilibrium temperature T with total equipartition energy $E = \frac{1}{2}\, N\, k_B T$.

Right panel: The gravitational effects of Σ_{sph} for the emergent space correspond, in leading order, to those of a point mass $M = E/c^2$ located at the center of a sphere with radius R (the Schwarzschild radius $R_{\text{Schw}} \equiv 2GM/c^2$ taken negligible compared to R).

Entropic gravity

With this spherical holographic screen Σ_{sph} [3], a different 'derivation' [5] may give a clue to the origin of the previous 'suggestive' formula (3):

$$\begin{aligned} A_{\text{grav}} &\overset{①}{=} 2\pi\, c\,\big(k_B T/\hbar\big) \\ &\overset{②}{=} 4\pi\, f c\,\big(\tfrac{1}{2}\, N\, k_B T/\hbar\big)\,\big(f^{-1}/N\big) \\ &\overset{③}{=} 4\pi\, f c\,\big(E/\hbar\big)\,\big(l^2/A\big) \\ &\overset{④}{=} f\, c\,\big(M c^2/\hbar\big)\,\big(l^2/R^2\big)\,, \end{aligned} \tag{4}$$

where step ① relies on the Unruh effect [6] and step ③ on the relation between the number N of d.o.f. and the area A of the holographic screen:

$$N = f^{-1}\, A/l^2\,. \tag{5}$$

---

[5] F.R. Klinkhamer, arXiv:1006.2094v3.

[6] W.G. Unruh, PRD 14, 870 (1976).

Entropic gravity

The several steps of (4) constitute, if confirmed, a **derivation** of Newton's gravitational coupling constant G in the form (2).

New insight from (5): given the "effective quantum of area" l^2, the inverse of the constant f entering Newton's constant (2) may be related to the nature of the microscopic d.o.f. on the holographic screen.

For example, an "atom of space" with "spin" s_{atom} may give $f^{-1} = 2\, s_{\text{atom}} + 1 \equiv d_{\text{atom}}$, but s_{atom} need not be half-integer.

Therefore, rewrite (5) as

$$N = d_{\text{atom}}\, N_{\text{atom}}\,, \quad d_{\text{atom}} \equiv f^{-1} \in \mathbb{R}^{+} \quad N_{\text{atom}} \equiv A/l^2 \in \mathbb{N}_1\,, \qquad (6)$$

where the "atoms of space" (total number N_{atom}) have no translational degrees of freedom but only internal degrees of freedom (d_{atom}).

G calculation

Next, calculate the factor $f \equiv (d_{\text{atom}})^{-1}$ entering formula (2) for G.

Consider a maximally-coarse-grained spherical surface (horizon) with area A. Entropy given by the Bekenstein–Hawking black-hole result [7]:

$$S_{\text{BH}}/k_B = (1/4)\,A/(f\,l^2) = (1/4)\,N\,. \tag{7}$$

Equating the number of configurations of the “atoms of space” from (6) with the exponential of the BH entropy (7) gives a **set of conditions** [5]:

$$(d_{\text{atom}})^{N_{\text{atom}}} = e^{(1/4)\,d_{\text{atom}}\,N_{\text{atom}}}\,, \tag{8}$$

which reduces to a **single transcendental equation** for d_{atom} :

$$4\,\ln d_{\text{atom}} = d_{\text{atom}}\,. \tag{9}$$

This equation has two solutions:

$$d_{\text{atom}}^{(+)} \approx 8.613\,169\,456\,, \quad d_{\text{atom}}^{(-)} \approx 1.429\,611\,825\,. \tag{10}$$

---

[7] J.D. Bekenstein, PRD 7, 2333 (1973); S.W. Hawking, CMP 43, 199 (1975).

G calculation

Given l^2, there are then two possible values for the gravitational coupling constant (2):

$$G_\pm = \left(d_{\text{atom}}^{(\pm)}\right)^{-1} c^3\, l^2/\hbar\,. \tag{11}$$

The detailed microscopic theory must tell which of the two d_{atom} values from (10) enters (11).

It could, for example, be that the microscopic theory demands $d_{\text{atom}} \geq 2$, selecting the value $d_{\text{atom}}^{(+)} \approx 8.6$ and giving

$$G_+ \approx \left(8.613\,169\,456\right)^{-1} c^3\, l^2/\hbar \approx \left(0.116\,101\,280\,1\right)\, c^3\, l^2/\hbar\,. \tag{12}$$

G calculation

But the experimental value of Newton's gravitational coupling constant is already known (to 100 ppm [1]): $G_N = 6.6743(7)\ 10^{-11}$ m^3 kg^{-1} s^{-2}.

A more practical interpretation of result (10) for d_{atom} is, therefore, to calculate two possible values for the "effective quantum of area":

$$(l_{\pm})^2 \quad = \quad d_{\text{atom}}^{(\pm)}\, \big(l_P\big)^2 \approx \begin{cases} 2.2498 \times 10^{-69}\ \text{m}^2\,, \\ 3.7343 \times 10^{-70}\ \text{m}^2\,, \end{cases} \tag{13}$$

with $l_P \equiv (\hbar\, G_N)^{1/2}/c^{3/2} \approx 1.6162 \times 10^{-35}$ m.

The microscopic theory would, again, have to choose between these alternative values.

For either choice, the implication would be that l and l_P are of the same order of magnitude.

G calculation

The crucial question, now, is if l^2 can be measured directly.

Possible experiments:

- Cosmic-ray particle-propagation experiments (e.g., Auger) can search for Lorentz-violating effects from a nontrivial small-scale structure of spacetime [2] and may determine the ratio $f = (l_P/l)^2$ **if** the size of spacetime defects is set by l_P and their separation by l.
- A *Gedankenexperiment* can measure quantum modifications [8] of Newton's gravitational acceleration (3) by a multiplicative factor $\left[1 - \widetilde{a}\, l^2/R^2\right]$ and determine l^2 **if** $\widetilde{a} > 0$ is known from theory.

But the **if**'s make these experiments inconclusive, for the moment.

Perhaps further examples from the conceptual remark below Table 2.

---

[8] L. Modesto and A. Randono, arXiv:1003.1998v1.

G calculation

Finally, two remarks on the numerical value of G_N.

First, the order of magnitude is given by (using mks units):

$$G_N \sim c^3\,\frac{l^2}{\hbar} \sim \left(3\times 10^8\right)^3\,\frac{3\times 10^{-70}}{1\times 10^{-34}} \sim 10^{-10}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}\,, \tag{14}$$

Second, note that the accurate measurement of one of the values of l^2 in (13) allows for an equally accurate calculation of G from (11).

For example, measuring for l^2 the larger value in (13) with a relative uncertainty of 100 ppb would give G also with an uncertainty of approximately 100 ppb from (12):

$$G_N \stackrel{?}{=} G_+ \approx \left(0.116\,101\,280\,1\right)\,c^3\,l^2/\hbar\,. \tag{15a}$$

If, instead, the smaller value for l^2 would be measured to 100 ppb, then

$$G_N \stackrel{?}{=} G_- \approx \left(0.699\,490\,576\,9\right)\,c^3\,l^2/\hbar\,. \tag{15b}$$

Conclusion

Two interesting results:

- 'derivation' of $G = f\, c^3\, l^2/\hbar$ via Unruh temperature & holography;
- calculation of $f \equiv \left(d_{\text{atom}}\right)^{-1} = (l_P/l)^2$ from BH black-hole entropy.

Many outstanding questions:

- Are space and gravity really emergent phenomena?
- If so, really from a holographic theory?
- Also, is Newton's gravitational force really an entropic force ?
- Independently, is there a new fundamental constant l^2 ?
- If so, what is the value of f in the relation $G = f\, c^3\, l^2/\hbar$?
- Also, how can l^2 be measured, in principle and in practice?