Brief introduction to $q$–theory
and
a QCD-scale modified-gravity universe

Frans R. Klinkhamer

Institute for the Physics and Mathematics of the Universe,
University of Tokyo
and
Institute for Theoretical Physics, University of Karlsruhe,
Karlsruhe Institute of Technology
Email: frans.klinkhamer@kit.edu
0. Introduction

“Dark Energy” ("DE"):  
- effect is more or less established (accelerating Universe);  
- nature and origin remain unclear.

At this moment, there is a need for new physical mechanisms.

One mechanism [1,2] goes under the name of ‘$q$–theory.’

The $q$–theory approach to the main Cosmological Constant Problem gives an explanation of how the gravitating vacuum energy density $\rho_V(q)$ can be self-adjusted to zero in an equilibrium state $q = q_0$: $\rho_V(q_0) = 0$.

There may be perturbations of this equilibrium state resulting in a “small” positive value of the vacuum energy density $\rho_V(\delta q) > 0$.

Here, we consider one possible type of perturbation with energy scale set by QCD. This may lead to a particular modified-gravity universe.

Outline of the talk:
1. Brief introduction to $q$–theory [1,2];
2. QCD gluon condensate and $q$–theory [3];
3. QCD-scale modified-gravity universe [4,5];
4. Conclusions

1. $q$–theory

Crucial insight [2]: there is vacuum energy and vacuum energy.

More specifically and introducing an appropriate notation:

the vacuum energy density $\epsilon$ appearing in the action need not be the same as
the vacuum energy density $\rho_V$ in the Einstein field equations.

How can this happen concretely . . .
1. $q$–theory

One physical picture is to consider the full quantum vacuum as a type of **self-sustained medium** (similar to a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Then, consider **macroscopic equations** of this conserved **microscopic** variable (later called $q$), whose precise nature need not be known.

This quantity $q$ is similar to the mass density in liquids, which describes a microscopic quantity – the number density of atoms – but obeys the macroscopic equations of hydrodynamics, because of particle-number conservation.

However, is the quantum vacuum just like a normal fluid?
1. $q$–theory

No, as the vacuum is known to be Lorentz invariant (cf. experimental limits at the $10^{-15}$ level in the photon sector [6,7,8]).

The Lorentz invariance of the vacuum rules out the standard type of charge density which arises from the time component $j_0$ of a conserved vector current $j_\mu$.

Needed is a new type of relativistic conserved charge, called the vacuum variable $q$.

In other words, look for a relativistic generalization ($q$) of the number density ($n$) which characterizes the known material fluids.

1. $q$–theory

With such a variable $q$, the vacuum energy density of the effective action is given by a generic function

$$\epsilon = \epsilon(q),$$

which may include a constant term due to the zero-point energies of the fields of the Standard Model (SM), $\epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{var}}(q)$.

From (1) thermodynamics and (2) Lorentz invariance, it then follows that

$$P_V \overset{(1)}{=} - \left( \epsilon - q \frac{d\epsilon}{dq} \right) \overset{(2)}{=} -\rho_V \neq -\epsilon,$$

with the first equality corresponding to an integrated form of the Gibbs–Duhem equation (with chemical potential $\mu \equiv d\epsilon/dq$).

Recall GD-eq: $N \, d\mu = V \, dP - S \, dT \Rightarrow dP = (N/V) \, d\mu$ for $dT = 0$. 

1. $q$–theory

Both terms entering $\rho_V$ from (2) can be of order $(E_{UV})^4$, but they can cancel exactly for an appropriate value $q_0$ of the vacuum variable $q$.

Hence, for a generic function $\epsilon(q)$,

$$\exists q_0 : \Lambda \equiv \rho_V = \left[ \epsilon(q) - q \frac{d\epsilon(q)}{dq} \right]_{q=q_0} = 0,$$

with constant vacuum variable $q_0$ [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, main Cosmological Constant Problem solved, in principle....

However, is a relativistic vacuum variable $q$ possible at all?

Yes, there exist several theories which contain such a $q$. 

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1. $q$–theory

To summarize, $q$–theory approach to the main Cosmological Constant Problem provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics does have a $q$–type variable.

Still, better to have one possible solution than none.

Realizations of $q$ thought to be operative at UV (Planck) energy scales.

Here, consider, instead, the well-established theory of the strong interactions, but in an unusual context.
2. Gluon condensate

Gluon condensate [9] from quantum chromodynamics (QCD):

\[ \tilde{q} \equiv \left\langle \frac{1}{4\pi^2} G^{a\mu\nu} G^{a\mu\nu} \right\rangle = \left\langle \frac{1}{4\pi^2} G^{\kappa\mu} g^{\kappa\mu} g^{\lambda\nu} G^{a\mu\nu} \right\rangle, \tag{4} \]

with Yang–Mills field strength \( G^{a\mu\nu} = \partial_\mu A^{a\nu} - \partial_\nu A^{a\mu} + f^{abc} A^{b\mu} A^{c\nu} \) for \( su(3) \) structure constants \( f^{abc} \).

particle physics experiments: \( \tilde{q} \sim (300 \text{ MeV})^4 \)

observational cosmology: \( \rho_V \sim (2 \text{ meV})^4 \)

\[ \Rightarrow \text{how to reconcile the typical QCD vacuum energy density} \]
\[ \epsilon_{\text{QCD}} \sim 10^{34} \text{ eV}^4 \text{ with the observed value} \rho_V \sim 10^{-11} \text{ eV}^4 ? \]

2. Gluon condensate

General $q$–theory argument [1]:

1. there exists a conserved microscopic variable $q$ whose macroscopic behavior can be studied;

2. the vacuum energy density ($\epsilon_{\text{vac}}$) of the effective action differs from the one ($\rho_V$) that enters the gravitational equations;

3. in equilibrium, $q$ has self-adjusted to the value $q_0$ with $\rho_V(q_0) = 0$.

Now, $q$ is given by (4), which can be shown as follows.
2. Gluon condensate

Effective action for the gluon condensate \( q \) from (4) [dropping the tilde]:

\[
S_{\text{eff}} = S_{\text{grav}} + S_{\text{vac}} = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_N} R[g] + \epsilon_{\text{vac}}(q) \right).
\] (5)

Energy-momentum tensor for the gravitational field equations:

\[
T^\text{vac}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{vac}}}{\delta g_{\mu\nu}} = \epsilon_{\text{vac}}(q) g_{\mu\nu} - 2 \frac{d\epsilon_{\text{vac}}(q)}{dq} \frac{\delta q}{\delta g_{\mu\nu}}
\]

\[
= \left( \epsilon_{\text{vac}}(q) - q \frac{d\epsilon_{\text{vac}}(q)}{dq} \right) g_{\mu\nu} \equiv \rho_V(q) g_{\mu\nu}
\] (6)

⇒ equilibrium state: \( q = q_0, \rho_V(q_0) = 0, \) and \( g_{\mu\nu}(x) = \eta^\text{Minkowski}_{\mu\nu} \).
2. Gluon condensate

In a nonequilibrium state such as the expanding Universe (with Hubble parameter $H \neq 0$), there is a perturbation of the vacuum:

$$ q = q_0 + \delta q(H) \neq q_0 \Rightarrow \rho_V(q) \sim \frac{d\rho_V}{dq} \delta q(H) \neq 0. \quad (7) $$

For QCD, this is a difficult IR problem (cf. [10abc]). A priori, can have

$$ \rho_V(H) \sim 0 + H^2 \Lambda_{QCD}^2 + H^4 + \cdots $$

$$ + |H| \Lambda_{QCD}^3 + |H|^3 \Lambda_{QCD} + \cdots $$

(8)

Linear term in $H$ gives the correct order of magnitude for $\rho_V$, a.k.a. the cosmological “constant.”

3. Modified-gravity model

Flat FRW universe has Ricci curvature scalar \(R = 6 \left(2H^2 + \dot{H}\right)\) and, from \(4\), \(q_0 \sim (\Lambda_{\text{QCD}})^4\).

So, previous \(|H| \Lambda_{\text{QCD}}^3\) term suggests modified-gravity action \([5]\):

\[
S_{\text{eff,0}}[\psi, g] = \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left(K_0 R - \eta |R|^{1/2} |q_0|^{3/4} + \mathcal{L}^M[\psi, g]\right), \quad (9)
\]

with flat-spacetime equilibrium value \(q_0\) of gluon condensate \(4\), gravitational coupling \(K_0 \equiv \left[16\pi G_0\right]^{-1} \equiv \left[16\pi G(q_0)\right]^{-1} > 0\), and dimensionless coupling constant \(\eta > 0\) [standard GR has \(\eta = 0\)].

Modified-gravity model \(9\) has:
- one unknown (in principle, calculable) coupling constant \(\eta\);
- two energy scales, \(E_{\text{QCD}} = O(10^2 \text{ MeV})\) and \(E_{\text{Planck}} = O(10^{18} \text{ GeV})\).
3. Modified-gravity model

Solve resulting FRW cosmological equations in the scalar-tensor formalism [Brans–Dicke (BD) scalar $\phi(\tau) < 1$] and consider a single pressureless matter component (cold dark matter, CDM).

Also calculate the linear growth parameter $\beta$ for sub-horizon density perturbations and the gravity estimator [11]

$$E_G^{\text{theo}}(z) = \frac{\Omega_{\text{CDM}}(\tau_p)}{\phi(z) \beta(z)},$$

with present matter-energy-density parameter

$$\Omega_{\text{CDM}}(\tau_p) \equiv \frac{\rho_{\text{CDM}}(\tau_p)}{\rho_{\text{crit}}(\tau_p)},$$

$$\rho_{\text{crit}}(\tau_p) \equiv \frac{3H^2(\tau_p)}{8\pi G_N},$$

where $G_0 = G_N$ has been used (see below).

3. Modified-gravity model

Dimensionless variables $t$, $h$, $r_{\text{CDM}}$, and $s$ (for dimensionless predictions):

\[
\tau \equiv t \frac{K_0}{(\eta (q_0)^{3/4})}, \quad H(\tau) \equiv h(t) \frac{\eta (q_0)^{3/4}}{K_0}, \quad (12a)
\]
\[
\rho_{\text{CDM}}(\tau) \equiv r_{\text{CDM}}(t) \frac{\eta^2 (q_0)^{3/2}}{K_0}, \quad \phi(\tau) \equiv s(t), \quad (12b)
\]

Model parameters (only important for dimensional predictions):

$G_0 = G_N$ [chamelecon effect for Cavendish experiments on Earth];

$q_0 = (300 \text{ MeV})^4 \equiv (E_{\text{QCD}})^4$ [gluon condensate from particle physics];

$\eta = 2.4 \times 10^{-4}$ [for age of the Universe equal to 13.2 Gyr, see below].
3. Modified-gravity model

Numerical solution [5] of cosmological ODEs from QCD-modified-gravity model (9). Boundary conditions from approx. solution at $t_{\text{start}} = 10^{-5}$. Label ‘$M2$’ stands for CDM.
3. Modified-gravity model

Defining the “present universe” to be at $\Omega_{\text{CDM}}(t_p) = 0.25$, there are, first, these two dimensionless results:

$$\overline{w_X}(t_p) \equiv -\frac{2}{3} \left( \frac{\ddot{a} a}{(\dot{a})^2} + \frac{1}{2} \right) \frac{1}{1 - \Omega_{\text{CDM}}} \bigg|_{t=t_p} \approx -0.662 , \quad (13a)$$

$$z_{\text{inflect}}(t_i, t_p) \equiv \frac{a(t_p)}{a(t_i)} - 1 \approx 0.523 . \quad (13b)$$

With chosen values for $q_0$, $G_0$, and $\eta$, also get three dimensional results:

$$\tau_p = t_p \eta^{-1} (16\pi G_N)^{-1} (E_{\text{QCD}})^{-3} \sim 13.2 \text{ Gyr} , \quad (14a)$$

$$H_p = h(t_p) \eta (16\pi G_N) (E_{\text{QCD}})^3 \sim 68.1 \text{ km/s/Mpc} , \quad (14b)$$

$$\rho_{V,p}^{(\text{BD})} = \frac{1}{4} \eta^2 / (1 - s(t_p)) (16\pi G_N) (E_{\text{QCD}})^6 \sim (2 \times 10^{-3} \text{ eV})^4 . \quad (14c)$$

$\Rightarrow$ model values in the same ballpark as the observed values.
3. Modified-gravity model

Further model prediction and first experimental result [12]:

\[ E_G^{\text{theo}} \big|_{z=0.32} \approx 0.437, \tag{15a} \]
\[ E_G^{\exp} \big|_{<z>=0.32} = 0.392 \pm 0.065, \tag{15b} \]

Redshift dependence of QCD-modified-gravity model vs. $\Lambda$CDM model:

<table>
<thead>
<tr>
<th>$z$</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>$10^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_G$</td>
<td>0.554</td>
<td>0.456</td>
<td>0.399</td>
<td>0.335</td>
<td>0.301</td>
<td>0.281</td>
<td>0.267</td>
<td>0.257</td>
<td>0.211</td>
</tr>
<tr>
<td>$E_G</td>
<td>_{\Lambda\text{CDM}}$</td>
<td>0.541</td>
<td>0.418</td>
<td>0.355</td>
<td>0.298</td>
<td>0.275</td>
<td>0.265</td>
<td>0.259</td>
<td>0.256</td>
</tr>
</tbody>
</table>

⇒ future surveys may distinguish between these theoretical models [5,11].

4. Conclusions

- self-adjustment of a conserved microscopic variable $q$ can give:
  \[ \rho_V = 0 \] for an equilibrium state,
  \[ \rho_V \neq 0 \] for a perturbed state (e.g., by the Hubble expansion);

- gravitational effects of the QCD gluon condensate can be described by $q$–theory;

- if \[ \rho_V \geq |H| \Lambda_{QCD}^3 \], then a QCD-induced modified-gravity model may give a satisfactory description of the present Universe, both qualitatively and quantitatively;

- if \[ \rho_V \not\geq |H| \Lambda_{QCD}^3 \], then some other mechanism must generate the remnant vacuum energy density corresponding to the observed cosmological constant, perhaps electroweak physics . . .