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Brief introduction to q–theory
and
a QCD-scale modified-gravity universe

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0. Introduction

"Dark Energy" ("DE"):

- effect is more or less established (accelerating Universe);
- nature and origin remain unclear.

At this moment, there is a need for <u>new physical mechanisms</u>.

One mechanism [1,2] goes under the name of 'q–theory.'

The q–theory approach to the main Cosmological Constant Problem gives an explanation of how the gravitating vacuum energy density $\rho_V(q)$ can be self-adjusted to zero in an equilibrium state $q = q_0$: $\rho_V(q_0) = 0$.

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[1] F.R. Klinkhamer and G.E. Volovik, PRD 77, 085015 (2008),arXiv:0711.3170.

[2] F.R. Klinkhamer and G.E. Volovik, JETPL 91, 259 (2010), arXiv:0907.4887.

0. Introduction

There may be perturbations of this equilibrium state resulting in a "small" positive value of the vacuum energy density $\rho_V(\delta q) > 0$.

Here, we consider one possible type of perturbation with energy scale set by QCD. This may lead to a particular modified-gravity universe.

Outline of the talk:

1. Brief introduction to q–theory [1,2];
2. QCD gluon condensate and q–theory [3];
3. QCD-scale modified-gravity universe [4,5];
4. Conclusions

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[3] F.R. Klinkhamer and G.E. Volovik,, PRD 79, 063527 (2009), arXiv:0811.4347.
[4] F.R. Klinkhamer, PRD 81, 043006 (2010), arXiv:0904.3276.
[5] F.R. Klinkhamer, arXiv:1005.2885.

1. q–theory

Crucial insight [2]: *there is vacuum energy and vacuum energy.*

More specifically and introducing an appropriate notation:

the vacuum energy density ϵ appearing in the action
need not be the same as
the vacuum energy density ρ_V in the Einstein field equations.

How can this happen concretely . . .

1. q–theory

One physical picture is to consider the full quantum vacuum as a type of **self-sustained medium** (similar to a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Then, consider **macroscopic equations** of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

This quantity q is similar to the mass density in liquids, which describes a microscopic quantity – the number density of atoms – but obeys the macroscopic equations of hydrodynamics, because of particle-number conservation.

However, is the quantum vacuum just like a normal fluid?

1. q–theory

No, as the vacuum is known to be Lorentz invariant
(cf. experimental limits at the 10^{-15} level in the photon sector [6,7,8]).

The Lorentz invariance of the vacuum rules out the standard type of charge density which arises from the time component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material fluids.

[6] A. Kostelecký and M. Mewes, PRD 66, 056005 (2002), arXiv:hep-ph/0205211.

[7] F.R. Klinkhamer and M. Risse, PRD 77, 117901 (2008), arXiv:0709.2502

[8] F.R. Klinkhamer and M. Schreck, PRD 78, 085026 (2008), arXiv:0809.3217.

1. q–theory

With such a variable q, the vacuum energy density of the effective action is given by a generic function

$$\epsilon = \epsilon(q)\,, \tag{1}$$

which may include a constant term due to the zero-point energies of the fields of the Standard Model (SM), $\epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{var}}(q)$.

From ① thermodynamics and ② Lorentz invariance, it then follows that

$$P_V \overset{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \overset{②}{=} -\rho_V \neq -\epsilon\,, \tag{2}$$

with the first equality corresponding to an integrated form of the Gibbs–Duhem equation (with chemical potential $\mu \equiv d\epsilon/dq$).

Recall GD-eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

1. q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{UV}})^4$, but they can cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\, \frac{d\,\epsilon(q)}{d\,q} \right]_{q=q_0} = 0\,, \tag{3}$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, main Cosmological Constant Problem solved, in principle....

However, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q.

1. q–theory

To summarize, q–theory approach to the main Cosmological Constant Problem provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics does have a q–type variable.

Still, better to have one possible solution than none.

Realizations of q thought to be operative at UV (Planck) energy scales.

Here, consider, instead, the well-established theory of the strong interactions, but in an unusual context.

2. Gluon condensate

Gluon condensate [9] from quantum chromodynamics (QCD):

$$\widetilde{q} \equiv \left\langle \frac{1}{4\pi^2} G^{a\,\mu\nu}\, G^{a}{}_{\mu\nu} \right\rangle = \left\langle \frac{1}{4\pi^2} G_{a\,\kappa\lambda}\, g^{\kappa\mu} g^{\lambda\nu}\, G^{a}{}_{\mu\nu} \right\rangle , \tag{4}$$

with Yang–Mills field strength $G^{a}{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + f^{abc} A^b{}_\mu A^c{}_\nu$ for $su(3)$ structure constants f^{abc}.

particle physics experiments: $\widetilde{q} \sim (300\ \text{MeV})^4$

observational cosmology: $\rho_V \sim (2\ \text{meV})^4$

$\Rightarrow$ how to reconcile the typical QCD vacuum energy density $\epsilon_{\text{QCD}} \sim 10^{34}$ eV^4 with the observed value $\rho_V \sim 10^{-11}$ eV^4 ?

[9] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, NPB 147, 385 (1979).

2. Gluon condensate

General q–theory argument [1]:

1. there exists a conserved microscopic variable q whose macroscopic behavior can be studied;

2. the vacuum energy density (ϵ_{vac}) of the effective action differs from the one (ρ_V) that enters the gravitational equations;

3. in equilibrium, q has self-adjusted to the value q_0 with $\rho_V(q_0) = 0$.

Now, q is given by (4), which can be shown as follows.

2. Gluon condensate

Effective action for the gluon condensate q from (4) [dropping the tilde]:

$$S_{\text{eff}} = S_{\text{grav}} + S_{\text{vac}} = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G_N} R[g] + \epsilon_{\text{vac}}(q) \right) . \tag{5}$$

Energy-momentum tensor for the gravitational field equations:

$$\begin{aligned} T^{\text{vac}}_{\mu\nu} &= -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{vac}}}{\delta g^{\mu\nu}} = \epsilon_{\text{vac}}(q)\, g_{\mu\nu} - 2\, \frac{\mathrm{d}\epsilon_{\text{vac}}(q)}{\mathrm{d}q} \frac{\delta q}{\delta g^{\mu\nu}} \\ &= \left(\epsilon_{\text{vac}}(q) - q\, \frac{\mathrm{d}\epsilon_{\text{vac}}(q)}{\mathrm{d}q} \right) g_{\mu\nu} \equiv \rho_V(q)\, g_{\mu\nu} \end{aligned} \tag{6}$$

$\Rightarrow$ equilibrium state: $q = q_0$, $\rho_V(q_0) = 0$, and $g_{\mu\nu}(x) = \eta^{\text{Minkowski}}_{\mu\nu}$.

2. Gluon condensate

In a nonequilibrium state such as the expanding Universe (with Hubble parameter $H \neq 0$), there is a perturbation of the vacuum:

$$q = q_0 + \delta q(H) \neq q_0 \quad \Rightarrow \quad \rho_V(q) \sim \frac{\mathrm{d}\rho_V}{\mathrm{d}q}\, \delta q(H) \neq 0\,. \tag{7}$$

For QCD, this is a difficult IR problem (cf. [10abc]). *A priori*, can have

$$\begin{aligned} \rho_V(H) \quad &\sim \quad 0 + H^2\, \Lambda^2_{\text{QCD}} + H^4 + \cdots \\ &\qquad + |H|\, \Lambda^3_{\text{QCD}} + |H|^3\, \Lambda_{\text{QCD}} + \cdots \end{aligned} \tag{8}$$

Linear term in H gives the correct order of magnitude for ρ_V, a.k.a. the cosmological "constant."

[10a] R. Schützhold, PRL 89, 081302 (2002).
[10b] J.D. Bjorken, arXiv:astro-ph/0404233.
[10c] F.R. Urban and A.R. Zhitnitsky, arXiv:0906.2162; arXiv:0909.2684.

3. Modified-gravity model

Flat FRW universe has Ricci curvature scalar $R = 6\big(2\,H^2 + \dot{H}\big)$ and, from (4), $q_0 \sim (\Lambda_{\text{QCD}})^4$.

So, previous $|H|\,\Lambda^3_{\text{QCD}}$ term suggests modified-gravity action [5]:

$$S_{\text{eff},0}[\psi, g] = \int_{\mathbb{R}^4} d^4x\,\sqrt{-g}\,\Big(K_0\,R - \eta\,|R|^{1/2}\,|q_0|^{3/4} + \mathcal{L}^{\text{M}}[\psi, g]\Big)\,, \tag{9}$$

with flat-spacetime equilibrium value q_0 of gluon condensate (4), gravitational coupling $K_0 \equiv \big[16\pi G_0\big]^{-1} \equiv \big[16\pi G(q_0)\big]^{-1} > 0$, and dimensionless coupling constant $\eta > 0$ [standard GR has $\eta = 0$].

Modified-gravity model (9) has:

- one unknown (in principle, calculable) coupling constant η;
- two energy scales, $E_{\text{QCD}} = \text{O}(10^2\ \text{MeV})$ and $E_{\text{Planck}} = \text{O}(10^{18}\ \text{GeV})$.

3. Modified-gravity model

Solve resulting FRW cosmological equations in the scalar-tensor formalism [Brans–Dicke (BD) scalar $\phi(\tau) < 1$] and consider a single pressureless matter component (cold dark matter, CDM).

Also calculate the linear growth parameter β for sub-horizon density perturbations and the gravity estimator [11]

$$E_G^{\text{theo}}(z) = \frac{\Omega_{\text{CDM}}(\tau_p)}{\phi(z)\,\beta(z)}\,, \tag{10}$$

with present matter-energy-density parameter

$$\Omega_{\text{CDM}}(\tau_p) \equiv \rho_{\text{CDM}}(\tau_p)/\rho_{\text{crit}}(\tau_p)\,, \tag{11a}$$

$$\rho_{\text{crit}}(\tau_p) \equiv 3\,H^2(\tau_p)/(8\pi\,G_N)\,, \tag{11b}$$

where $G_0 = G_N$ has been used (see below).

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[11] P. Zhang et al., PRL 99, 141302 (2007), arXiv:0704.1932.

3. Modified-gravity model

Dimensionless variables t, h, r_{CDM}, and s (for dimensionless predictions):

$$\tau \equiv t\, K_0 \big/ \left(\eta\, (q_0)^{3/4}\right), \qquad H(\tau) \equiv h(t)\, \eta\, (q_0)^{3/4} \big/ K_0\,, \qquad \text{(12a)}$$

$$\rho_{\text{CDM}}(\tau) \equiv r_{\text{CDM}}(t)\, \eta^2\, (q_0)^{3/2} \big/ K_0\,, \qquad \phi(\tau) \equiv s(t)\,, \qquad \text{(12b)}$$

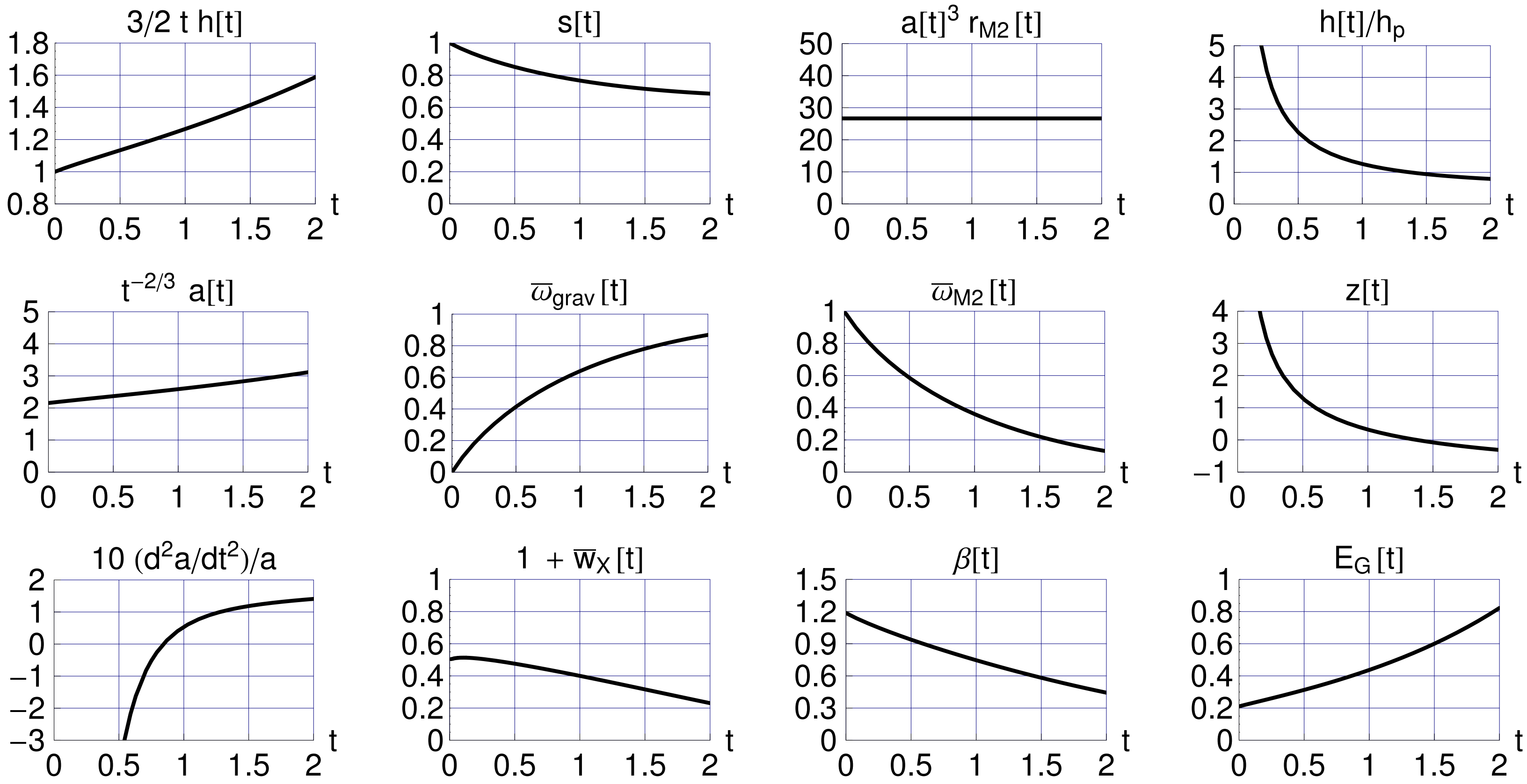
Model parameters (only important for dimensional predictions):

$G_0 = G_N$ [chameleon effect for Cavendish experiments on Earth];

$q_0 = (300\ \text{MeV})^4 \equiv (E_{\text{QCD}})^4$ [gluon condensate from particle physics];

$\eta = 2.4 \times 10^{-4}$ [for age of the Universe equal to 13.2 Gyr, see below].

3. Modified-gravity model



Numerical solution [5] of cosmological ODEs from QCD-modified-gravity model (9). Boundary conditions from approx. solution at $t_{\text{start}} = 10^{-5}$. Label '$M2$' stands for CDM.

3. Modified-gravity model

Defining the "present universe" to be at $\Omega_{\text{CDM}}(t_p) = 0.25$, there are, first, these two dimensionless results:

$$\overline{w}_{\text{X}}(t_p) \;\equiv\; -\frac{2}{3}\left(\frac{\ddot{a}\,a}{(\dot{a})^2}+\frac{1}{2}\right)\frac{1}{1-\Omega_{\text{CDM}}}\bigg|_{t=t_p} \approx\; -0.662\,, \qquad \text{(13a)}$$

$$z_{\text{inflect}}(t_{\text{i}},\,t_p) \;\equiv\; a(t_p)/a(t_i) - 1 \qquad \approx\; 0.523\,. \qquad \text{(13b)}$$

With chosen values for q_0, G_0, and η, also get three dimensional results:

$$\tau_p \;=\; t_p\,\eta^{-1}\,(16\pi G_N)^{-1}\,(E_{\text{QCD}})^{-3} \qquad \sim\; 13.2\ \text{Gyr}\,, \qquad \text{(14a)}$$

$$H_p \;=\; h(t_p)\,\eta\,(16\pi G_N)\,(E_{\text{QCD}})^{3} \qquad \sim\; 68.1\ \text{km/s/Mpc}\,, \qquad \text{(14b)}$$

$$\rho_{\text{V, p}}^{\text{(BD)}} \;=\; \tfrac{1}{4}\,\eta^2\big/\big(1-s(t_p)\big)\,(16\pi G_N)\,(E_{\text{QCD}})^6 \sim \big(2\times 10^{-3}\,\text{eV}\big)^4\,. \qquad \text{(14c)}$$

$\Rightarrow$ model values in the same ballpark as the observed values.

3. Modified-gravity model

Further model prediction and first experimental result [12]:

$$E_G^{\text{theo}}\Big|_{z=0.32} \approx 0.437\,, \tag{15a}$$

$$E_G^{\text{exp}}\Big|_{<z>=0.32} = 0.392 \pm 0.065\,, \tag{15b}$$

Redshift dependence of QCD-modified-gravity model vs. ΛCDM model:

z	0	0.25	0.5	1	1.5	2	2.5	3	10^2
E_G	0.554	0.456	0.399	0.335	0.301	0.281	0.267	0.257	0.211
$E_G\big\|_{\Lambda\text{CDM}}$	0.541	0.418	0.355	0.298	0.275	0.265	0.259	0.256	0.250

$\Rightarrow$ future surveys may distinguish between these theoretical models [5,11].

[12] R. Reyes et al., Nature 464, 256 (2010), arXiv:1003.2185.

4. Conclusions

- self-adjustment of a conserved microscopic variable q can give:
  $\rho_V = 0$ for an equilibrium state,
  $\rho_V \neq 0$ for a perturbed state (e.g., by the Hubble expansion);
- gravitational effects of the QCD gluon condensate can be described by q–theory;
- **if** $\rho_V \supseteq |H|\,\Lambda^3_{\text{QCD}}$, then a QCD-induced modified-gravity model may give a satisfactory description of the present Universe, both qualitatively and quantitatively;
- **if** $\rho_V \not\supseteq |H|\,\Lambda^3_{\text{QCD}}$, then some other mechanism must generate the remnant vacuum energy density corresponding to the observed cosmological constant, perhaps electroweak physics . . .