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Superluminal neutrino: Theoretical considerations

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1. Introduction

Let me commence with a personal remark.

Lorentz violation (LV) has been the main focus of my work over the last 12 years, starting with the so-called CPT anomaly.

Tentative conclusion of various investigations:

Lorentz invariance (LI) may be exact, even at the Planck energy scale.

All the more reason to be surprised by the announcement on September 23, 2011 $\longrightarrow$

1. Introduction

OPERA's claim of a superluminal neutrino velocity [1]:

$$\left.\frac{v_{\nu_\mu} - c}{c}\right|_{\langle c\,|\mathbf{p}|\rangle = 17\text{ GeV}} = \Big[2.4 \pm 0.3\,\text{(stat)} \pm 0.3\,\text{(sys)}\Big] \times 10^{-5}\,, \qquad (1)$$

with $c = 299\,792\,458$ m/s the velocity of light in vacuum.

Recall the earlier suggestive but inconclusive result by MINOS [2]:

$$\left.\frac{v_{\nu_\mu} - c}{c}\right|_{\langle c\,|\mathbf{p}|\rangle = 3\text{ GeV}} = \Big[5.1 \pm 1.3\,\text{(stat)} \pm 2.6\,\text{(sys)}\Big] \times 10^{-5}\,. \qquad (2)$$

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[1] T. Adam et al. [OPERA Collaboration], arXiv:1109.4897v1 (22Sep2011), v2 (17Nov2011).
[2] P. Adamson et al. [MINOS Collaboration], PRD 76, 072005 (2007), arXiv:0706.0437.

1. Introduction

OPERA's claim (1) needs, of course, independent confirmation.

Awaiting that, the following question can be asked:

Is LV as suggested by (1) theoretically acceptable?

Short answer:

Yes, acceptable fundamentally, but border-line phenomenologically.

Task of theory is to find an explanation that is both consistent with all experimental facts and convincing, i.e., with not too many assumptions.

Now, first, discuss the main challenges for model builders
and, then, two relatively simple models.

2. Challenges

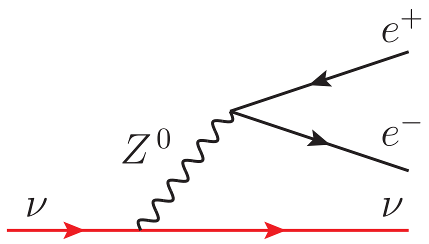
Start with the following (incomplete) list of experimental "facts" :

(i) OPERA result [1]: $[v_{\nu_\mu}(10\ \text{GeV}) - c]/c \sim 10^{-5}$.

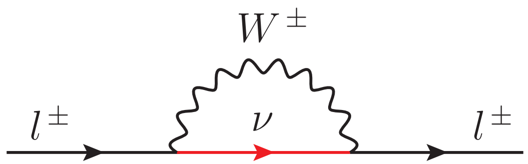
(ii) Supernova SN1987a bound [3]: $|v_{\overline{\nu}_e}(10\ \text{MeV}) - c|/c \lesssim 10^{-9}$.

(iii) Coherent mass-difference neutrino oscillations, requiring [4] equal maximum velocity of neutrino flavors $f = e, \mu, \tau$.

(iv) Absence of catastrophic energy losses for CNGS neutrinos from tree-level vacuum-Cherenkov-type process [5].



(v) Negligible leakage of LV from the neutrino sector into the charged-lepton sector by quantum effects [6].



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[3] M.J. Longo, PRD 36, 3276 (1987).

[4] S.R. Coleman and S.L. Glashow, PRD 59, 116008 (1999), arXiv:hep-ph/9812418.

[5] A.G. Cohen and S.L. Glashow, PRL 107, 181803 (2011), arXiv:1109.6562.

[6] G.F. Giudice, S. Sibiryakov, and A. Strumia, arXiv:1109.5682.

2. Challenges

Upshot:

(i)+(ii) [OPERA+SN1987a]

$\Rightarrow$ energy dependence of v_{group}, possible to implement.

(iii) [Neutrino oscillations]

$\Rightarrow$ possible to implement.

(iv) [Cherenkov losses]

$\Rightarrow$ difficult, 3 options:

(a) “relativity,” (b) sterile neutrino, and (c) reduced rate.

(v) [LV leakage]

$\Rightarrow$ very difficult, but solution perhaps possible.

2. Challenges

(a): Problem (iv) solved by a new realization of “relativity” [7]?

The vacuum-Cherenkov-type process $\nu_\mu \to \nu_\mu + Z^0 \to \nu_\mu + e^- + e^+$ is a preferred-frame effect: it occurs for energies above a certain threshold in a preferred frame defined by the LV.

If theory still has “relativity” but now with deformed nonlinear Lorentz transformations (LTs), then there is no Cherenkov-type radiation at all.

A purely kinematic calculation in a toy-model [7] shows indeed that there is no Cherenkov-type threshold.

But there exists no interacting “relativistic” theory which shows this. My guess is that such an interacting theory does not exist.

Conclusion: no definite solution yet of (iv) via deformed LTs.

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[7] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, and L. Smolin, arXiv:1110.0521..

2. Challenges

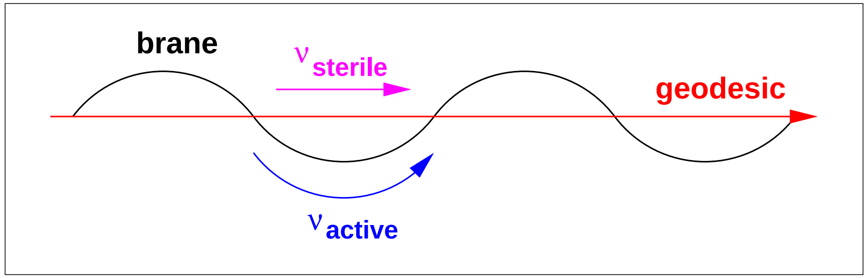
(b): Problem (iv) solved by a light (eV-scale) sterile neutrino [6,8,9]?

By definition, light sterile neutrino does not couple to Z^0 at tree level and the vacuum-Cherenkov-type process is simply absent.

But how does the sterile neutrino acquire a superluminal velocity?

Currently fashionable explanation: extra dimensions and a warped braneworld [8,9,10].

Basic idea in a figure from [10]:



[8] S. Hannestad and M.S. Sloth, arXiv:1109.6282.
[9] A. Nicolaidis, arXiv:1109.6354.
[10] H. Päs, S. Pakvasa, and T.J. Weiler, PRD 72, 095017 (2005), arXiv:hep-ph/0504096.

2. Challenges

However, is an OPERA explanation appealing to extra dimensions, to a braneworld, and to an appropriate warping really convincing?

For the moment, not to the present speaker.

For the moment, he prefers a simpler theory with four spacetime dimensions and spontaneously broken Lorentz invariance (SBLI) [11] in the sterile-neutrino sector [12].

Option (b) with light sterile neutrino and SBLI will be discussed in Chap. 3.

Option (c) with reduced rate and SBLI will be discussed in Chap. 4.

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[11] F.R. Klinkhamer and G.E. Volovik, JETP Lett. 94, 673 (2011), arXiv:1109.6624.
[12] F.R. Klinkhamer, arXiv:1111.4931v3.

3. Theory

Start at the phenomenological level and then work down [12].

Model dispersion relations of neutrino mass eigenstates ($n = 1, 2, 3, 4$):

$$E^2 = c^2 p^2 + \left(m_n\, c^2\right)^2, \quad \text{for } n = 1,\ 2,\ 3, \tag{3a}$$

$$E^2 \sim c^2 p^2 + \left(m_4\, c^2\right)^2 + (b^0)^4\, M^{-2}\, p^4, \quad \text{for } n = 4, \tag{3b}$$

for $p \equiv |\mathbf{p}|$ and with dimensionless constant $b^0 \in \mathbb{R}$ and mass scale M, both coming from a fermion condensate (see below).
Also assume equal active-sterile mixing:

$$|U_{e4}| = |U_{\mu 4}| = |U_{\tau 4}| \neq 0\,, \tag{3c}$$

perhaps natural if sterile-neutrino and active-neutrino sectors are completely decoupled, except for indirect gravitational interactions [13].

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[13] E.J. Chun, A.S. Joshipura, and A.Y. Smirnov, PLB 357, 608 (1995), arXiv:hep-ph/9505275.

3. Theory (details)

Now set $c = \hbar = 1$ and use Minkowski metric $g_{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1)$.

Theory without LV is defined by the standard-model Lagrange density, to which are added the standard Dirac term for the sterile neutrino with mass m_4 and similar terms for the masses m_1, m_2, and m_3 :

$$\mathcal{L}_{\text{LI}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{m_1,m_2,m_3} + \mathcal{L}_{m_4} \tag{4}$$

Modified dispersion relation (3b) may then come from an additional Lagrange-density term for the sterile-neutrino Dirac field $\psi_s(x)$

$$\mathcal{L}_{s-\text{LV}} = -M^{-1}\,\overline{\psi}_s\left(+i\,\overleftarrow{\nabla}_\alpha\right)b^\alpha\,b^\beta\left(-i\,\overrightarrow{\nabla}_\beta\right)\psi_s\,, \tag{5}$$

with a purely timelike background vector

$$(b^\alpha) = (b^0,\,0,\,0,\,0)\,. \tag{6}$$

3. Theory (details)

This background vector b_α can arise as a fermion condensate,

$$b^\alpha = M^{-4}\, g^{\alpha\beta}\ < \overline{\chi}_S\, (-i\, \nabla_\beta)\, \chi_S > , \tag{7}$$

where $\chi_S(x)$ is the field of a heavy sterile neutrino with mass scale $M \gg m_4$. Dynamically, (12) can come from the interaction term

$$\mathcal{L}_{S-\text{int}} = -\lambda\, M^4 \left(X - h^2 \right)^2 , \tag{8a}$$

$$X \equiv M^{-8}\, g^{\alpha\beta} \left[\overline{\chi}_S\, (-\, i\, \nabla_\alpha)\, \chi_S \right] \left[\overline{\chi}_S\, (-\, i\, \nabla_\beta)\, \chi_S \right] , \tag{8b}$$

with real coupling constants λ and h. Vanishingly small symmetry-breaking perturbations pick out a purely timelike condensate vector with time-component

$$b^0 = \pm\, h\, . \tag{9}$$

3. Theory (details)

An appropriate interaction term $\mathcal{L}_{s-S-\text{int}}$ then produces the momentum-dependent mass term (5) for the light sterile neutrino propagator.

All in all, one possible <u>relativistic</u> theory for the model dispersion relations (3ab) has Lagrange density

$$\mathcal{L} = \mathcal{L}_{\text{LI}} + \mathcal{L}_{S-\text{int}} + \mathcal{L}_{s-S-\text{int}} + \cdots \;, \tag{10a}$$

$$\mathcal{L}_{\text{LI}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{m_1,m_2,m_3} + \mathcal{L}_{m_4} + \mathcal{L}_{M} \;, \tag{10b}$$

$$\mathcal{L}_{S-\text{int}} = -\lambda \left(\frac{1}{M^6} \big(-i\,\overline{\chi}_S\, \nabla^\alpha \chi_S \big) \big(-i\,\overline{\chi}_S\, \nabla_\alpha\, \chi_S \big) - h^2 M^2 \right)^2, \tag{10c}$$

$$\mathcal{L}_{s-S-\text{int}} = \frac{1}{M^9} \big(\nabla_\alpha\, \overline{\psi}_s \big) \big(\overline{\chi}_S\, \nabla^\alpha \chi_S \big) \big(\overline{\chi}_S\, \nabla^\beta \chi_S \big) \big(\nabla_\beta\, \psi_s \big) \;, \tag{10d}$$

where the dots in (10a) contain mixing terms (cf. [13,14]).

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[14] V. Berezinsky, M. Narayan, and F. Vissani, NPB 658, 254 (2003), arXiv:hep-ph/0210204.

3. Theory

Checklist:

(i) [OPERA] $\Rightarrow$ OK, if $M \sim 3$ TeV for $|b^0| \sim 1$ or $M \gg$ TeV for $|b^0| \gg 1$.

(ii) [SN1987a] $\Rightarrow$ OK, as group velocity $v_s(E) \sim c + O(E^2)$ for $E \ll Mc^2$.

(iii) [Neutrino oscillations] $\Rightarrow$ OK, as (3c) can be expected to hold.

(iv) [Cherenkov losses] $\Rightarrow$ OK, as only the sterile particle is superluminal.

(v) [LV leakage]$\Rightarrow$ TBA?, new quantum number for ‘sterile’ neutrino [13] ?

3. Theory

Challenges by and predictions for experiment/cosmology:

- Reactor neutrino anomaly $\Rightarrow m_{\text{sterile}} \sim 1$ eV ?
- Big bang nucleosynthesis $\Rightarrow N_{\text{sterile}} \lesssim 1$ and $m_{\text{sterile}} \lesssim 0.1$ eV ?
- Narrow symmetric pulse of nearly mono-energetic muon-neutrinos produced at CERN (or Fermilab) would give a broadened pulse of muon-neutrinos to be detected by OPERA (or MINOS)?

4. Alternative theory

Now no light sterile neutrino but only a heavy one to give SBLI (for details, see App. in [12]). Again, start at the phenomenological level.

Model dispersion relations of neutrino mass eigenstates ($n = 1, 2, 3$):

$$E^2 \;=\; c^2\,p^2 + \left(m_n\,c^2\right)^2 + (b^0)^4\,M^{-2}\,p^4\,, \quad \text{for } n = 1,\,2,\,3, \tag{11}$$

for $p \equiv |\mathbf{p}|$ and with dimensionless constant $b^0 \in \mathbb{R}$ and mass scale M, both coming from a fermion condensate as before:

$$b^\alpha = M^{-4}\,g^{\alpha\beta}\;<\overline{\chi}_S\,(-i\,\nabla_\beta)\,\chi_S>, \tag{12}$$

where $\chi_S(x)$ is the field of a heavy sterile neutrino with mass scale $M \gg \text{max}(m_1,\,m_2,\,m_3)$.

4. Alternative theory

Skipping the details, a gauge-invariant and relativistic theory for the model dispersion relations (11) has Lagrange density

$$\mathcal{L}' = \mathcal{L}'_{\text{LI}} + \mathcal{L}_{S-\text{int}} + \mathcal{L}_{\nu-S-\text{int}} \,, \tag{13a}$$

$$\mathcal{L}'_{\text{LI}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{m_1,m_2,m_3} + \mathcal{L}_{M} \,, \tag{13b}$$

$$\mathcal{L}_{S-\text{int}} = -\lambda \left(\frac{1}{M^6} \big(-i\, \overline{\chi}_S\, \nabla^\alpha \chi_S \big) \big(-i\, \overline{\chi}_S\, \nabla_\alpha\, \chi_S \big) - h^2 M^2 \right)^2, \tag{13c}$$

$$\mathcal{L}_{\nu-S-\text{int}} = \sum_{f=e,\mu,\tau} \Big[\overline{L}_f \cdot \Phi^* \left(\overline{\chi}_S\, \nabla^\alpha \chi_S \right) \left(\overline{\chi}_S\, \nabla^\beta \chi_S \right) \nabla_\alpha \nabla_\beta\, \nu_{R,f} + \text{H.c.} \Big], \tag{13d}$$

with the left-handed lepton isodoublets L_f and Higgs isodoublet Φ of the standard model, and right-handed sterile neutrinos $\nu_{R,f}$

4. Alternative theory

Checklist:

(i), (ii), and (iii)
$\Rightarrow$ same as before.

(iv) [Cherenkov losses]
$\Rightarrow$ do occur but at a significantly reduced rate [15], by a factor
$\left(1/\sqrt{3}\,\right)^5 \approx 1/16$ as can be understood heuristically [12,16].

(v) [LV leakage]
$\Rightarrow$ as difficult as before.

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[15] S. Mohanty and S. Rao, arXiv:1111.2725v3.
[16] C. Kaufhold and F.R. Klinkhamer, NPB 734, 1 (2006), arXiv:hep-th/0508074.

4. Alternative theory

Predictions for experiment:

- Narrow symmetric pulse of nearly mono-energetic muon-neutrinos produced at CERN (or Fermilab) would give an essentially equal pulse of muon-neutrinos to be detected by OPERA (or MINOS)?
- LV in the Higgs sector from (13d).

5. Conclusions

- OPERA’s claim (1) needs independent confirmation.
- SBLI-sterile-neutrino theories must confront phenomenology and, if successful, need better understanding of the dynamics.
- Other theories must also confront phenomenology and, if successful, need convincing physics motivation.