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UHECR bounds on Lorentz violation in the photon sector

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[1] S. Bernadotte & FRK, PRD 75, 024028 (2007), hep-ph/0610216.

[2] C. Kaufhold & FRK, PRD 76, 025024 (2007), arXiv:0704.3255 [hep-th].

[3] FRK & M. Risse, PRD 77, 016002 (2008), arXiv:0709.2502 [hep-ph].

[4] FRK & M. Risse, PRD 77, 117901 (2008), arXiv:0806.4351 [hep-ph].

[5] FRK, arXiv:0807.2147 [hep-ph].

← REVIEW

Introduction

1. INTRODUCTION

Fundamental question: *does space remain smooth as one probes smaller and smaller distances?*

A conservative limit on the typical length scale ℓ of any small-scale structure of space:

$$\text{LEP/Tevatron: } \ell \lesssim 10^{-18} \text{ m} \approx \hbar c / (200 \text{ GeV}) . \quad (1)$$

Yet, **astrophysics** provides us with very much higher energies.

Outline of this talk:

- phenomenology of a simple photon-propagation model;
- bounds from ultra-high-energy cosmic rays (UHECRs);
- theoretical implications.

Phenomenology

2. PHENOMENOLOGY

2.1 Model

Action for a Lorentz-violating deformation of quantum electrodynamics:

$$S_{\text{modQED}} = S_{\text{modM}} + S_{\text{standD}}, \quad (2)$$

with **modified-Maxwell term** [Chadha & Nielsen, NPB 217, 125 (1983)]:

$$S_{\text{modM}} = \int_{\mathbb{R}^4} d^4x \left(-\frac{1}{4} (\eta^{\mu\rho}\eta^{\nu\sigma} + \kappa^{\mu\nu\rho\sigma}) F_{\mu\nu}(x) F_{\rho\sigma}(x) \right), \quad (3a)$$

and standard Dirac term for spin- $\frac{1}{2}$ particle with charge e and mass M :

$$S_{\text{standD}} = \int_{\mathbb{R}^4} d^4x \bar{\psi}(x) \left(\gamma^\mu (i \partial_\mu - e A_\mu(x)) - M \right) \psi(x). \quad (3b)$$

Theory is gauge-invariant, CPT-even, and power-counting renormalizable.

Phenomenology

$\kappa^{\mu\nu\rho\sigma}$ is a constant background tensor with the same symmetries as the Riemann curvature tensor and a double trace condition $\kappa^{\mu\nu}{}_{\mu\nu} = 0$,
 $\Rightarrow 20 - 1 = 19$ independent components.

10 birefringent parameters are already constrained at the 10^{-32} level [Kostelecky & Mewes, hep-ph/0205211].

Restrict the theory to the **nonbirefringent sector**:

$$\kappa^{\mu\nu\rho\sigma} = \frac{1}{2} \left(\eta^{\mu\rho} \tilde{\kappa}^{\nu\sigma} - \eta^{\mu\sigma} \tilde{\kappa}^{\nu\rho} - \eta^{\nu\rho} \tilde{\kappa}^{\mu\sigma} + \eta^{\nu\sigma} \tilde{\kappa}^{\mu\rho} \right), \quad (4)$$

for a symmetric and traceless matrix $\tilde{\kappa}^{\mu\nu}$ with $10 - 1 = 9$ components.

\Rightarrow **9 Lorentz-violating (LV) deformation parameters $\tilde{\kappa}^{\mu\nu}$.**

Phenomenology

Rewrite these parameters $\tilde{\kappa}^{\mu\nu}$ as follows:

$$(\tilde{\kappa}^{\mu\nu}) \equiv \text{diag}\left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \tilde{\kappa}^{00} + (\delta\tilde{\kappa}^{\mu\nu}), \quad \delta\tilde{\kappa}^{00} = 0, \quad (5)$$

with 1 independent parameter $\tilde{\kappa}^{00}$ for the spatially isotropic part of $\tilde{\kappa}^{\mu\nu}$ and 8 independent parameters $\delta\tilde{\kappa}^{\mu\nu}$.

Express these parameters in terms of the so-called SME parameters:

$$\begin{pmatrix} \tilde{\kappa}^{00} \\ \delta\tilde{\kappa}^{01} \\ \delta\tilde{\kappa}^{02} \\ \delta\tilde{\kappa}^{03} \\ \delta\tilde{\kappa}^{11} \\ \delta\tilde{\kappa}^{12} \\ \delta\tilde{\kappa}^{13} \\ \delta\tilde{\kappa}^{22} \\ \delta\tilde{\kappa}^{23} \end{pmatrix} \equiv \begin{pmatrix} (3/2) \tilde{\kappa}_{\text{tr}} \\ -(\tilde{\kappa}_{\text{o}+})^{(23)} \\ -(\tilde{\kappa}_{\text{o}+})^{(31)} \\ -(\tilde{\kappa}_{\text{o}+})^{(12)} \\ -(\tilde{\kappa}_{\text{e}-})^{(11)} \\ -(\tilde{\kappa}_{\text{e}-})^{(12)} \\ -(\tilde{\kappa}_{\text{e}-})^{(13)} \\ -(\tilde{\kappa}_{\text{e}-})^{(22)} \\ -(\tilde{\kappa}_{\text{e}-})^{(23)} \end{pmatrix}. \quad (6)$$

Phenomenology

2.2 Possible spacetime origin

Calculations of standard photons and Dirac particles propagating in simple classical spacetime-foam models reproduce a restricted (isotropic) version of model (2):

$$2 \tilde{\kappa}_{\text{tr}} = -\tilde{\sigma}_2 \tilde{F} \equiv -\tilde{\sigma}_2 (\tilde{b} / \tilde{l})^4, \quad \delta \tilde{\kappa}^{\mu\nu} = 0, \quad (7)$$

in terms of the quadratic coefficient of modified photon dispersion relation given below.

Heuristics: Bethe holes (1944).



Phenomenology

For randomly oriented “defects” (size \bar{b}) embedded in Minkowski spacetime (separation \bar{l}), proton and photon dispersion relations are:

$$\omega_p^2 \equiv M_p^2 c_p^4 / \hbar^2 + c_p^2 k^2 + \mathbf{O}(k^4), \quad (8a)$$

$$\omega_\gamma^2 = \left[1 + \left(\tilde{\sigma}_2 \tilde{F} \right) \right] c_p^2 k^2 + \left(\tilde{\sigma}_4 \tilde{F} \tilde{b}^2 \right) c_p^2 k^4 + \mathbf{O}(k^6), \quad (8b)$$

with wave number $k \equiv |\mathbf{k}|$,

effective defect on/off factors $\tilde{\sigma}_2, \tilde{\sigma}_4 \in \{-1, 0, +1\}$,

effective size \tilde{b} , and effective excluded-volume factor $\tilde{F} \equiv (\tilde{b}/\tilde{l})^4 \ll 1$.

Specific results [1] for the relation between effective parameters (tilde) and fundamental spacetime parameters (bar):

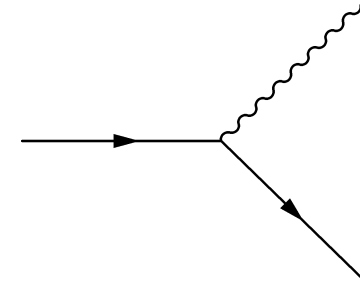
$$\tilde{b} = \beta \bar{b}, \quad \tilde{l} = \lambda \bar{l}, \quad \tilde{\sigma}_2 = -1, \quad \tilde{\sigma}_4 = 1, \quad (9)$$

with positive constants β and λ of order unity.

Phenomenology

2.3 Vacuum Cherenkov radiation

The decay process $p \rightarrow p + \gamma$ in model (2) has been studied classically by Altschul [hep-th/0609030] and quantum-mechanically by Kaufhold & FRK [2].



Radiated energy rate of a point particle with electric charge Ze , mass $M > 0$, momentum q , and ultrarelativistic energy $E \sim c|q|$:

$$\frac{dW_{\text{modQED}}}{dt} \sim Z^2 e^2 \xi(\hat{\mathbf{q}}) E^2 / \hbar \Big|_{E^2 \gg E_{\text{thresh}}^2}, \quad (10)$$

with direction-dependent coefficient $\xi \geq 0$ and **threshold energy**

$$E_{\text{thresh}}^2 = \frac{M^2 c^4}{R[2\tilde{\kappa}_{\text{tr}} + 2\delta\tilde{\kappa}^{0j}\hat{\mathbf{q}}^j + \delta\tilde{\kappa}^{jk}\hat{\mathbf{q}}^j\hat{\mathbf{q}}^k]} + \mathcal{O}(M^2 c^4), \quad (11)$$

for ramp function $R[x] \equiv (x + |x|)/2$.

Phenomenology

Exact tree-level result [C. Kaufhold, FRK, & M. Schreck, 2007] for the radiation rate of a spin- $\frac{1}{2}$ Dirac particle (charge Ze , mass $M > 0$, and $E \geq E_{\text{thresh}}$) in the restricted isotropic model (7) with only $\alpha_0 \equiv 2\tilde{\kappa}_{\text{tr}} > 0$:

$$\begin{aligned} \frac{dW_{\text{modQED}}^{\text{isotropic case}}}{dt} &= \frac{Z^2 e^2}{4\pi} \frac{1}{3\alpha_0^3 E \sqrt{E^2 - M^2}} \left(\sqrt{\frac{2 - \alpha_0}{2 + \alpha_0}} E - \sqrt{E^2 - M^2} \right)^2 \\ &\times \left\{ 2(\alpha_0^2 + 4\alpha_0 + 6) E^2 - (2 + \alpha_0) \right. \\ &\times \left. \left(3(1 + \alpha_0) M^2 + 2(3 + 2\alpha_0) \sqrt{\frac{2 - \alpha_0}{2 + \alpha_0}} E \sqrt{E^2 - M^2} \right) \right\}, \end{aligned} \quad (14a)$$

with

$$E_{\text{thresh}}^{\text{modQED, isotropic case}} = \frac{Mc^2}{\sqrt{\alpha_0}} \sqrt{1 + \alpha_0/2}. \quad (14b)$$

UHECR bounds

3. UHECR BOUNDS

3.1 Basic idea

A remarkably simple observation [a,b]:

- if vacuum Cherenkov radiation has a **threshold** $E_{\text{thresh}}(\tilde{b}, \tilde{l}, \tilde{\kappa})$, then UHECRs with $E > E_{\text{thresh}}$ cannot travel far, as they rapidly radiate away their energy;
- **observing** an UHECR of energy E implies that this energy is at or below threshold,

$$E \leq E_{\text{thresh}}(\tilde{b}, \tilde{l}, \tilde{\kappa}), \quad (13)$$

which then gives **bounds** on combinations of \tilde{b} , \tilde{l} , and $\tilde{\kappa}$.

[a] Beall, PRD 1, 961 (1970).

[b] Coleman & Glashow, PLB 405, 249 (1997).

UHECR bounds

3.2 Bounds on LV photon parameters

Take the following **29 selected events**:

- 27 from Auger [arXiv:0712.2843],
- 1 from Fly's Eye [astro-ph/9410067],
- 1 from AGASA [PRL 73, 3491 (1994)].

Table 1 on the next page gives their

- arrival time (year and Julian day);
- primary energy E in EeV, where $1 \text{ EeV} \equiv 10^{18} \text{ eV}$;
- arrival direction with right ascension and declination in degrees.

Uncertainties in the energies are of the order of 25 % and in the pointing directions of the order of 1 deg.

UHECR bounds

Table 1: UHECR events from Auger (2004–2007), Fly’s Eye (1991), and AGASA (1993).

year	day	E	RA	DEC	year	day	E	RA	DEC
1991	288	320	85.2	48.0	2006	81	79	201.1	−55.3
1993	337	210	18.9	21.1	2006	185	83	350.0	9.6
2004	125	70	267.1	−11.4	2006	296	69	52.8	−4.5
2004	142	84	199.7	−34.9	2006	299	69	200.9	−45.3
2004	282	66	208.0	−60.3	2007	13	148	192.7	−21.0
2004	339	83	268.5	−61.0	2007	51	58	331.7	2.9
2004	343	63	224.5	−44.2	2007	69	70	200.2	−43.4
2005	54	84	17.4	−37.9	2007	84	64	143.2	−18.3
2005	63	71	331.2	−1.2	2007	145	78	47.7	−12.8
2005	81	58	199.1	−48.6	2007	186	64	219.3	−53.8
2005	295	57	332.9	−38.2	2007	193	90	325.5	−33.5
2005	306	59	315.3	−0.3	2007	221	71	212.7	−3.3
2005	306	84	114.6	−43.1	2007	234	80	185.4	−27.9
2006	35	85	53.6	−7.8	2007	235	69	105.9	−22.9
2006	55	59	267.7	−60.7					

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UHECR bounds

Pierre Auger Observatory (Pampa Amarilla, Argentina)

Left: Water Cherenkov surface detector (Andes mountains to the west).
Right: Fluorescence detector telescopes (bay windows open).



UHECR bounds

With these 29 primary energies and directions, we obtain the following two- σ (95% CL) **Cherenkov bounds** on the nine isolated SME parameters of nonbirefringent modified-Maxwell theory [4]:

$$(ij) \in \{(23), (31), (12)\} : |(\tilde{\kappa}_{o+})^{(ij)}| < 2 \times 10^{-18}, \quad (14a)$$

$$(kl) \in \{(11), (12), (13), (22), (23)\} : |(\tilde{\kappa}_{e-})^{(kl)}| < 4 \times 10^{-18}, \quad (14b)$$

$$\tilde{\kappa}_{tr} < 1.4 \times 10^{-19}, \quad (14c)$$

for the Sun-centered celestial equatorial coordinate system.

Here, we have set $M_{\text{prim}} = 56 \text{ GeV}/c^2$ and, for (14c), used the 148 EeV Auger event which has a reliable energy calibration.

From a 202 EeV Auger event, a preliminary bound at the two- σ level is:

$$-2 \times 10^{-19} < \tilde{\kappa}_{tr}^{\text{univ}} < 8 \times 10^{-20},$$

but involves some assumptions for the lower bound [FRK & Schreck, 2008].

UHECR bounds

The Cherenkov bounds (14abc) depend only on the measured energies and flight directions of the charged cosmic-ray primaries at the top of the Earth atmosphere.

Leverage factor for the UHECR Cherenkov bounds is

$$\left(E_{\text{prim}}/M_{\text{prim}}c^2\right)^2 = 10^{18} \left(\frac{E_{\text{prim}}}{50 \text{ EeV}}\right)^2 \left(\frac{50 \text{ GeV}}{M_{\text{prim}}}\right)^2 .$$

May perhaps be increased to a value of 10^{21} in the future:

- M_{prim} reduced by a factor 10 from the use of further information such as the shower-maximum atmospheric depth X_{max} [3];
- average energy increased to $E_{\text{prim}} \sim 150 \text{ EeV}$.

UHECR bounds

Hence, **UHECR bounds** for nonbirefringent mod-Maxwell theory give nine parameters bounded at the 10^{-18} level.

Current **laboratory bounds** (complete set of references in [5]):

- direct bounds at the 10^{-12} level
for the three parity-odd nonisotropic parameters in $\tilde{\kappa}_{o+}$;
- direct bounds at the 10^{-14} to 10^{-16} levels
for the five parity-even nonisotropic parameters in $\tilde{\kappa}_{e-}$;
- direct bound at the 10^{-7} level
for the single parity-even isotropic parameter $\tilde{\kappa}_{tr}$;
- indirect bound at the 10^{-8} level for $\tilde{\kappa}_{tr}$ (from measured $g_e - 2$).

Interestingly, the UHECR Cherenkov bounds are the strongest where the laboratory bounds are the weakest, they are truly complementary.

UHECR bounds

From the 148 EeV Auger event, we also get a bound on the general coefficient of the quartic photon term in (8b):

$$|\tilde{F}\tilde{b}^2| \lesssim (1.4 \times 10^{-35} \text{ m})^2, \quad (15)$$

based on the analysis of Gagnon & Moore [hep-ph/0404196] but scaling their result to $M_{\text{prim}} = 56 \text{ GeV}/c^2$ and $E_{\text{prim}} = 148 \text{ EeV}$. (#)

Taking $\tilde{F} = 10^{-19}$ from (14c), this bound becomes

$$\tilde{b} \lesssim 4 \times 10^{-26} \text{ m}, \quad (16)$$

which is still a very small length.

(#) Bound (15) disagrees, by 16 orders of magnitude, with a claimed “quantum-gravity” effect in a gamma-ray flare from Mkn 501 as observed by the MAGIC telescope [arXiv:0708.2889v1]; see [5].

UHECR bounds

3.3 Discussion

Cherenkov-type bounds have been obtained for combinations of the effective defect size (\tilde{b}) and separation (\tilde{l}):

$$\tilde{F} \equiv (\tilde{b}/\tilde{l})^4 \lesssim 10^{-19}, \quad (17a)$$

$$\tilde{b} \lesssim 4 \times 10^{-26} \text{ m} \approx \hbar c / (5 \times 10^9 \text{ GeV}). \quad (17b)$$

Bound (17b) is already quite remarkable (cf. LEP/Tevatron/LHC) and, moreover, severely constrains (read: rules out) TeV-gravity models [cf. Arkani-Hamed, Dimopoulos, & Dvali, hep-ph/9803315]:

any such theory with, for example, a nonperturbative gravity scale $E_{ADD} = \hbar c / L_{ADD} \sim 5 \text{ TeV}$ needs to explain the origin of a very small numerical factor f in the quartic photon term (setting $c = 1$):

$$\omega_\gamma^2 = k^2 + f L_{ADD}^2 k^4 + \mathcal{O}(k^6), \quad |f| \lesssim 10^{-12}. \quad (18)$$

UHECR bounds

More generally, also the Lorentz-violating deformation parameters of modified-Maxwell theory are strongly bounded:

$$|\kappa^{\mu\nu\rho\sigma}| \lesssim 10^{-18}, \quad (19)$$

where, for the sake of argument, the “one-sided” Cherenkov bound on the isotropic parameter $\tilde{\kappa}_{\text{tr}}$ has also been made “two-sided.”

Bounds (17a) and (19) imply that **a single-scale ($\tilde{b} \sim \tilde{l}$) classical spacetime foam is ruled out altogether.**

This conclusion holds, in fact, for arbitrarily small defect size \tilde{b} , as long as a classical spacetime makes sense.

That is, down to distances at which the **classical-quantum transition** occurs, possibly of order $l_{\text{Planck}} \approx 10^{-35}$ m ...

Outlook

4. OUTLOOK

Experimental result from astrophysics (UHECRs, in particular):

quantum spacetime foam must have “crystalized” to a classical spacetime manifold which is **remarkably smooth**, as quantified by

the defect excluded-volume factor $\tilde{F} \equiv (\tilde{b}/\tilde{l})^4 \lesssim 10^{-19} \ll 1$ and Lorentz-violating parameters $|\kappa^{\mu\nu\rho\sigma}| \lesssim 10^{-18} \ll 1$.

Clearly, this is a **null experiment** and there is an analogy with the Michelson–Morley experiment (1887):

theorists expect novel effects which are not seen by experimentalists.

This suggests the need for **radically new concepts** (cf. SR in 1905).

For example, a self-tuning Lorentz-invariant vacuum variable [6] may play a crucial role for the flatness of spacetime by resolving the so-called cosmological constant problem (see talk tomorrow).

[6] FRK & G. Volovik, PRD 77, 085015 (2008), arXiv:0711.3170 [gr-qc].

Conjectures – Fundamental length

A. TWO CONJECTURES

A.1 Fundamental length

In view of the conclusions from Sec. 3.3, the following question arises: theoretically, are we really sure that quantum spacetime effects *only* show up at distances of the order of the Planck length?

Conjecture 1a: *quantum spacetime has a **fundamental length scale** l , which is conceptually different from the Planck length,*

$$l \stackrel{?}{\neq} l_{\text{Planck}} \equiv \sqrt{\hbar G/c^3} \approx 1.6 \times 10^{-35} \text{ m}. \quad (20)$$

HEURISTICS: a quantum spacetime foam could arise from gravitational self-interactions which need not involve Newton's constant G describing the gravitational coupling of matter (similar to the case of a gas of instantons in Yang–Mills theory).

Conjectures – Fundamental length

Consider a generalized **quantum phase** for spacetime dynamics [6],

$$\mathcal{I}_{\text{grav}}^{\text{general}} = \frac{-1}{16\pi l^2} \int d^4x \sqrt{|g|} (R + 2\lambda) + \frac{G/c^3}{l^2} \int d^4x \sqrt{|g|} \mathcal{L}_{\text{matter}}^{\text{class.}}, \quad (21)$$

which reproduces the classical Einstein equations but contains a new fundamental length l .

This suggests that, as far as spacetime is concerned, the role of Planck's constant \hbar would be replaced by the squared length l^2 , which might loosely be called the **quantum of area**.

Planck's constant \hbar would continue to play a role in the description of the matter quantum fields.

But, with \hbar and l^2 being **logically independent**, it is possible to consider the “limit” $\hbar \rightarrow 0$ while keeping l^2 fixed.

[7] FRK, JETPL **86**, 73 (2007), gr-qc/0703009.

Conjectures – Fundamental length

Table 2: Fundamental dimensionful constants of nature, including the hypothetical quantum of area l^2 .

quantum matter	classical relativity	quantum spacetime
\hbar	c, G	l^2

Here, we have considered only the 2nd and 3rd columns of Table 2 and leave the unified treatment of all columns to a future theory.

In that theory, “classical gravitation” may perhaps **emerge** from the combined quantum effects of matter and spacetime, giving the “large” Newton gravitational constant

$$G = f c^3 l^2 / \hbar, \tag{22}$$

as ratio of “small” quantum constants, with calculable numerical factor f .

Conjectures – Fundamental length

Return to the generalized action (21), possibly relevant for quantum spacetime as probed by classical matter.

Conjecture 1b: *the quantum spacetime length scale l is related to a nonvanishing cosmological constant or vacuum energy density.*

For the case of the early universe, with a vacuum energy density $\rho_{\text{vac}} \equiv E_{\text{vac}}^4$, it can be argued [7] that the following holds ($c = \hbar = 1$):

$$l \stackrel{?}{\sim} E_{\text{Planck}}/E_{\text{vac}}^2 \approx 2 \times 10^{-29} \text{ m} \left(\frac{E_{\text{Planck}}}{10^{19} \text{ GeV}} \right) \left(\frac{10^{16} \text{ GeV}}{E_{\text{vac}}} \right)^2, \quad (23)$$

where the Planck energy scale is given by $E_{\text{Planck}} \equiv 1/l_{\text{Planck}}$ and the numerical value for E_{vac} has been identified with the “grand-unification” scale suggested by elementary particle physics.

Conjectures – Fundamental length

If (23) holds true with $l_{\text{Planck}}/l \sim 10^{-6}$, it is perhaps possible to have sufficiently rare defects left-over from the crystallization process of classical spacetime from the initial quantum spacetime foam.

With average spacetime defect size \tilde{b} set by l_{Planck} (matter related) and average defect separation \tilde{l} set by l (vacuum related), these spacetime defects would give the following excluded-volume factor in the modified photon dispersion relation (8b):

$$\tilde{F} \equiv (\tilde{b}/\tilde{l})^4 \stackrel{?}{\sim} 10^{-24}, \quad (24)$$

which is close to saturating the current UHECR bound,

$$(\tilde{b}/\tilde{l})^4 \Big|_{\text{Fly's Eye}} \lesssim 3 \times 10^{-23}. \quad (25)$$

Conjectures – Cosmological constant

A.2 Cosmological constant

A different line of reasoning (motivated by “emerging symmetries” ideas) tries to explain the three cosmological constant problems:

1. why is $|\rho_{\text{vac}}| \ll (E_{\text{Planck}})^4$?
2. why is $\rho_{\text{vac}} \neq 0$?
3. why is now $\rho_{\text{vac}} \sim \rho_{\text{matter}}$?

Taking Lorentz-invariance seriously (cf. UHECR discussion in Sec. 3.3), a new idea on this famous problem is as follows [6]:

Conjecture 2: *The perfect quantum vacuum behaves as a self-sustained Lorentz-invariant medium with a new type of conserved charge.*

Argument is based solely on thermodynamics (cf. Einstein 1907) and has an analog in condensed-matter physics, the Larkin–Pikin effect (1969). Work in progress on the expanding (and accelerating!) universe [8].

[6] FRK & G. Volovik, PRD 77, 085015 (2008), arXiv:0711.3170 [gr-qc].

[8] FRK, arXiv:0803.0281 [gr-qc]; FRK & G. Volovik, arXiv:0803.0281 [gr-qc].