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Brief Report: Linearized Einstein gravity from four-form q-theory at equilibrium

F.R. Klinkhamer*

Institute for Theoretical Physics,
Karlsruhe Institute of Technology (KIT),
76128 Karlsruhe, Germany

Abstract

We consider localized perturbations of four-form q-theory [1–3] at the equilibrium point ($\mu = \mu_0$ or $q = q_0$) and find the standard linearized Einstein equation. The on-shell graviton mass, in particular, vanishes.

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*Electronic address: frans.klinkhamer@kit.edu

The main equations to be used have been derived in Ref. [2] and will be referred to explicitly, but the notation of the composite pseudoscalar F will be changed to q.

The action is given by Eq. (2.1) of Ref. [2] with the *Ansatz* $G(q) = G_N =$ constant. Recall that the 4-form field strength can be written solely in terms of partial derivatives ∂_κ instead of covariant derivatives ∇_κ,

$$F_{\kappa\lambda\mu\nu} \equiv \nabla_{[\kappa} A_{\lambda\mu\nu]} = \partial_{[\kappa} A_{\lambda\mu\nu]}\,, \tag{1}$$

where a pair of square brackets around spacetime indices stands for complete anti-symmetrization (without additional normalization factor). Now, q is defined by

$$q(x) \equiv -\frac{1}{24}\,\frac{1}{\sqrt{-g(x)}}\,F_{\kappa\lambda\mu\nu}(x)\,\epsilon^{\kappa\lambda\mu\nu}\,, \tag{2}$$

with Levi–Civita symbol $\epsilon^{\kappa\lambda\mu\nu}$.

The only term in the action involving the q-field is a potential-type term:

$$I_\text{pot} = \int_{\mathbb{R}^4} d^4x\,\sqrt{-g}\;\epsilon(q)\,, \tag{3}$$

with energy density

$$\epsilon(q) = \Lambda_\text{bare} + \epsilon_\text{dyn}(q)\,. \tag{4}$$

The following *Ansatz* for the function ϵ_dyn of the dynamic q-field was used in Ref. [3]:

$$\epsilon_\text{dyn}(q) = q^2 + (E_P)^8/q^2\,, \tag{5}$$

with the Planck energy $E_P \equiv \sqrt{1/G_N} \approx 1.22 \times 10^{19}\,\text{GeV}$. Other choices for $\epsilon_\text{dyn}(q)$ are certainly possible.

The gravitational field equation, given by Eq. (2.3) of Ref. [2] for $G(q) = G_N$,

$$\frac{1}{8\pi G_N}\left(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}\right) = \rho_V(q)\,g_{\mu\nu} - T^\text{M}_{\mu\nu}\,, \tag{6}$$

has a source term with the standard energy-momentum tensor $T^\text{M}_{\mu\nu}$ of the matter fields and a cosmological-constant-type term with gravitating energy density

$$\rho_V(q) \equiv \epsilon(q) - q\,d\epsilon(q)/dq\,. \tag{7}$$

The equilibrium configuration has a constant value q_0 of the q-field, so that

$$\rho_V(q_0) = 0\,. \tag{8}$$

Using Cartesian spacetime coordinates $(x^0,\, x^1,\, x^2,\, x^3) = (t,\, x,\, y,\, z)$, the equilibrium configuration has the following fields:

$$g_{\mu\nu}(x) = \overline{g}_{\mu\nu}(x) = \eta_{\mu\nu} \equiv [\text{diag}(-1,1,1,1)]_{\mu\nu}\,, \tag{9a}$$

$$A_{\lambda\mu\nu}(x) = \overline{A}_{\lambda\mu\nu}(x)\,, \tag{9b}$$

$$q(x) = \overline{q}(x) = q_0\,, \tag{9c}$$

with, for example, the 3-form gauge field

$$\overline{A}_{\lambda\mu\nu} = \begin{cases} -4\,q_0\,t\,\epsilon_{\lambda\mu\nu}\,, & \text{for } \{\lambda\mu\nu\} = P\{1,2,3\}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{10}$$

with P standing for any permutation.

Now consider localized perturbations $h_{\mu\nu}(x)$ and $a_{\lambda\mu\nu}(x)$:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)\,, \tag{11a}$$

$$A_{\lambda\mu\nu}(x) = \overline{A}_{\lambda\mu\nu}(x) + a_{\lambda\mu\nu}(x)\,. \tag{11b}$$

Then, the Maxwell-type field equation, given by Eq. (2.5) of Ref. [2] for $G(q) = G_N$,

$$\partial_\mu \left(\frac{d\epsilon(q)}{dq} \right) = 0\,, \tag{12}$$

requires q to stay unchanged. Specifically, the relevant combination $(-g)^{-1/2} F_{\kappa\lambda\mu\nu}$ from (2) must have a linear perturbation proportional to a constant C_1,

$$(-\tfrac{1}{2}\, h_\lambda^{\ \lambda})\, q_0\, \epsilon_{\kappa\lambda\mu\nu} + \partial_{[\kappa}\, a_{\lambda\mu\nu]} \equiv \delta_1 q\; \epsilon_{\kappa\lambda\mu\nu} = C_1\, \epsilon_{\kappa\lambda\mu\nu}\,. \tag{13}$$

For localized perturbations, this constant C_1 vanishes and

$$\delta_1 q = 0\,, \tag{14}$$

for

$$\partial_{[\kappa}\, a_{\lambda\mu\nu]} = \tfrac{1}{2}\, q_0\, h_\lambda^{\ \lambda}\, \epsilon_{\kappa\lambda\mu\nu}\,. \tag{15}$$

Hence, $a_{\lambda\mu\nu}$ is determined by $h_{\mu\nu}$, which, in turn, is determined by the Einstein equation (6).

In fact, the remaining equation is precisely the standard linearized Einstein equation for $h_{\mu\nu}$, possibly with matter-fields perturbations but still with $\rho_V(q) = \rho_V(q_0) = 0$. There is, thus, no on-shell graviton mass. Physically, we can have a single binary-star system emitting gravitational waves, which propagate in the same way as standard GR without cosmological constant ($\Lambda = 0$). In our linearized theory with dynamic vacuum energy (needed to cancel Λ_{bare}), the binary-star system generates outgoing gravitational waves but no outgoing waves of q-fields (consistent with the fact that the 3-form gauge field has no propagating degrees of freedom [8, 9]).

Final remarks:

1. In equilibrium, the appropriate constant value q_0 of the dynamic q-field [3] allows for the cancellation of a bare cosmological constant Λ_{bare}. The resulting spacetime is Minkowskian. The introduction of a sufficiently small amount of matter with a localized distribution then perturbs the metric by a contribution $h_{\mu\nu}(x)$. But the 3-form gauge field perturbation $a_{\lambda\mu\nu}(x)$ can adjust itself, according to (15), in order to keep the q-field at the constant value q_0 needed for the perfect cancellation of Λ_{bare}.

2. The t-behavior of (10) may be compared with that of Dolgov's theory [4, 5] having an asymptotic vector-field component $A_0 \sim (Q_0/2)\, t$. But the latter theory ruins the local Newtonian gravitational dynamics [6], which can only be restored by the introduction of a further vector field B_μ with a special structure of the potential term of the action density [7]. How does four-form q-theory manage to keep the Newton-Einstein gravity theory unharmed to leading order? The short answer appears to be gauge invariance. Indeed, the Dolgov theory with a single massless vector field $A^\nu(x)$ has no gauge invariance and the basic dynamic variable is $A_\mu^{\ \nu} \equiv \nabla_\mu A^\nu \equiv \partial_\mu A^\nu + \Gamma^\nu_{\mu\rho}\, A^\rho$. The four-form q-theory with a three-form field $A_{\lambda\mu\nu}(x)$ does have gauge invariance and the basic gauge-invariant dynamic variable (1) has a curl-type structure, with the affine connection terms $\Gamma^\nu_{\mu\rho}$ cancelling out.

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