

# Dark matter from dark energy in $q$ -theory

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A constant (spacetime-independent)  $q$ -field may play a crucial role for the cancellation of Planck-scale contributions to the gravitating vacuum energy density. We now show that a small spacetime-dependent perturbation of the equilibrium  $q$ -field behaves gravitationally as a pressureless perfect fluid. This makes the fluctuating part of the  $q$ -field a candidate for the inferred dark-matter component of the present universe. For a Planck-scale oscillation frequency of the  $q$ -field perturbation, the implication would be that direct searches for dark-matter particles would remain unsuccessful in the foreseeable future.

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## 1. INTRODUCTION

A condensed-matter-type approach to the cosmological constant problem [1] is given by  $q$ -theory [2, 3, 4, 5]. The aim of the  $q$ -theory formalism is to describe the thermodynamics and dynamics of the deep quantum vacuum without detailed knowledge of the microscopic (Planck-scale) degrees of freedom. Instead, an effective theory is considered with one or more conserved  $q$ -fields. For constant (spacetime-independent)  $q$ -fields, the thermodynamics leads to an exact cancellation of the quantum-field zero-point-energies in equilibrium, which partly solves the cosmological constant problem.

It was already noted in Ref. [3] that a rapidly-oscillating  $q$ -field could give a significant contribution to the inferred dark-matter component of our present universe. Here, we expand on this dark-matter aspect of  $q$ -theory.

## 2. SETUP

Consider the particular realization of  $q$ -theory based on a 3-form gauge field  $A$  with a corresponding 4-form field strength  $F \propto q$  (see Refs. [2, 3] and further references therein). The action is now taken to include a kinetic term for the  $q$ -field [6],

$$S = - \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left( \frac{R}{16\pi G_N} + \epsilon(q) + \frac{1}{8} K(q) g^{\alpha\beta} \nabla_\alpha(q^2) \nabla_\beta(q^2) + \mathcal{L}^{\text{SM}} \right), \quad (1a)$$

$$q^2 \equiv -\frac{1}{24} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta}, \quad F_{\alpha\beta\gamma\delta} \equiv \nabla_{[\alpha} A_{\beta\gamma\delta]}, \quad (1b)$$

$$F_{\alpha\beta\gamma\delta} = q \sqrt{-g} \epsilon_{\alpha\beta\gamma\delta}, \quad F^{\alpha\beta\gamma\delta} = q \epsilon^{\alpha\beta\gamma\delta} / \sqrt{-g}, \quad (1c)$$

where the functions  $\epsilon(q)$  and  $K(q)$  in (1a) involve only even powers of  $q$ , as  $q$  is a pseudoscalar according to (1c) with the Levi-Civita symbol  $\epsilon_{\alpha\beta\gamma\delta}$ . In the 4-form realization, the mass dimension of  $q$  is 2. Here, and elsewhere, we use natural units with  $c = \hbar = 1$  and take the metric signature  $(-+++)$ . For the curvature tensors, we use the same conventions as in Ref. [3].

The Lagrange density  $\mathcal{L}^{\text{SM}}$  in the action (1a) involves the fields of the standard model (SM) of elementary particle physics. In principle, it is also possible to replace Newton's gravitational constant  $G_N$  by a function  $G(q)$ , but we will not do so in the present article as we already have explicit  $q$  derivatives in the action. Note that the energy density  $\epsilon(q)$  in the integrand of (1a) may contain a constant term  $\Lambda_{\text{bare}}$  of arbitrary sign.

With the definition

$$C(q) \equiv K(q) q^2, \quad (2)$$

the equations of motion for the 3-form gauge field can be written as a generalized Maxwell equation,

$$\nabla_\beta \left( \frac{d\epsilon(q)}{dq} - \frac{1}{2} \frac{dC(q)}{dq} \nabla_\alpha q \nabla^\alpha q - C(q) \square q \right) = 0. \quad (3)$$

The solution of this generalized Maxwell equation is given by

$$\frac{d\epsilon(q)}{dq} - \frac{1}{2} \frac{dC(q)}{dq} \nabla_\alpha q \nabla^\alpha q - C(q) \square q = \mu. \quad (4)$$

with an integration constant  $\mu$ .

The Einstein equation from (1a) reads

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -8\pi G_N \left( T_{\alpha\beta}^{(q)} + T_{\alpha\beta}^{(\text{SM})} \right). \quad (5)$$

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The contribution of the 3-form gauge field to the energy-momentum tensor is given by

$$T_{\alpha\beta}^{(q)} = -g_{\alpha\beta} \left( \epsilon(q) - \mu q + \frac{1}{2} C(q) \nabla_\alpha q \nabla^\alpha q \right) + C(q) \nabla_\alpha q \nabla_\beta q, \quad (6)$$

where the solution (4) with integration constant  $\mu$  has been used to simplify the expression.

Three remarks on the  $q$ -field energy-momentum tensor are in order. First, for constant (spacetime-independent)  $q$ -fields, the energy-momentum tensor (6) has a cosmological-constant-type term  $-g_{\alpha\beta} \Lambda_{\text{eff}}(q)$  with a gravitating vacuum energy density  $\rho_V(q)$  different from energy density  $\epsilon(q)$  of the action,

$$\Lambda_{\text{eff}}(q) \Big|_{q=\text{const.}} = \rho_V(q) = \epsilon(q) - q \frac{d\epsilon(q)}{dq}, \quad (7)$$

where  $\mu$  has been replaced by  $d\epsilon/dq$  according to (4).

Second, for nonconstant  $q$ -fields, we observe that terms with  $(dC/dq)(\nabla q)^2$  and  $C \square q$  have been absorbed completely by the constant  $\mu$  in (6) and that the two remaining terms with  $C(\nabla q)^2$  have the same structure as if  $q$  were a fundamental (pseudo)-scalar.

Third, in a cosmological context with  $q = q(t)$ , the  $q$ -derivative terms in the action (1a) allow the vacuum energy density  $\rho_V(q)$  to change with cosmic time  $t$  even for constant gravitational coupling,  $G(q) = G_N$  (compare with the discussion in the second paragraph of Sec. II in Ref. [3]). For the moment, we postpone the study of these relaxation effects.

### 3. EQUILIBRIUM Q-FIELD

In equilibrium, the cosmological constant  $\Lambda_{\text{bare}}$  (from quantum-field zero-point-energies, cosmological phase transitions, or other origins) is cancelled by a spacetime-independent  $q$ -field of appropriate magnitude  $q_0$ . This cancellation mechanism by the composite pseudoscalar  $q$ -field is essentially different from a possible cancellation mechanism by a fundamental pseudoscalar field  $\phi$ ; see, in particular, the discussion of Sec. 2 in Ref. [4].

Specifically, we have for the equilibrium state

$$q(x) = \text{constant} = q_0, \quad (8a)$$

$$\mu = \mu_0 = d\epsilon/dq \Big|_{q=q_0}, \quad (8b)$$

$$\epsilon(q_0) - \mu_0 q_0 = 0. \quad (8c)$$

According to (6) and (7) for constant  $q$ , the last equality (8c) corresponds to the nullification of the gravitating vacuum energy density [2],

$$\Lambda_{\text{eff}}(q_0) \equiv \rho_V(q_0) = 0. \quad (9)$$

A further stability condition is given by the positivity of the inverse isothermal vacuum compressibility [2],

$$(\chi_0)^{-1} \equiv \left[ q^2 \frac{d^2 \epsilon(q)}{dq^2} \right]_{q=q_0} > 0. \quad (10)$$

Without additional matter, the Einstein equation (5) for the equilibrium  $q$ -field (8) gives Minkowski spacetime with the metric

$$g_{\alpha\beta}(x) \Big|_{\text{equil.}} = \eta_{\alpha\beta} = [\text{diag}(-1, 1, 1, 1)]_{\alpha\beta}, \quad (11)$$

for standard Cartesian coordinates  $(x^0, x^1, x^2, x^3) = (t, x, y, z)$ .

The question is still what the orders of magnitude are of  $q_0$  and  $1/\chi_0$ . If we assume that the theory (1a) without the SM term only contains a single energy scale, then that scale must be of order of the Planck energy

$$E_P \equiv (G_N)^{-1/2} \approx 1.22 \times 10^{19} \text{ GeV}. \quad (12)$$

In that case, we have

$$q_0 \stackrel{?}{\sim} (E_P)^2, \quad (13a)$$

$$1/\chi_0 \stackrel{?}{\sim} (E_P)^4, \quad (13b)$$

where the question marks are to remind us that these estimates are based on an assumption. For definiteness, we will phrase the rest of the discussion in terms of the Planck-scale estimates (13), but the discussion can be readily adapted if the relevant energy scale is significantly different from the Planck energy scale  $E_P$ .

The outstanding dynamical question is how the equilibrium state is reached. It appears that energy exchange between the vacuum  $q$ -field and the matter fields plays a crucial role. For the special case of massless-particle production, it has been found that the equilibrium value  $q_0$  is reached dynamically and that the final metric corresponds to the one of Minkowski spacetime [5]. As the present article is explorative, we will simply start from the equilibrium state.

### 4. PRESSURELESS PERFECT FLUID

Now consider a small spacetime-dependent change of the equilibrium  $q$ -field,

$$q(x) = q_0 + q_0 \xi(x), \quad (14)$$

in terms of a dimensionless real scalar field  $\xi(x)$  with  $|\xi(x)| \ll 1$ . Also take

$$K(q) = \text{constant} = (q_0)^{-3}, \quad (15a)$$

$$q_0 > 0, \quad (15b)$$

so that  $C(q_0)$  from (2) is positive.

The reduced Maxwell equation (4) for  $\mu = \mu_0$  gives the following Klein–Gordon equation [6]:

$$\square \xi - \frac{1}{q_0} \left[ q^2 \frac{d^2 \epsilon(q)}{dq^2} \right]_{q=q_0} \xi = 0, \quad (16)$$

where higher-order  $\xi$  terms have been omitted. Neglecting, at first, the spatial derivatives of  $\xi$ , the solution of (16) is a rapidly-oscillating homogeneous function,

$$\xi(t) = a_\xi \sin(\omega t + \varphi_\xi), \quad (17a)$$

$$\omega^2 = (q_0)^{-1} (\chi_0)^{-1} \stackrel{?}{\sim} (E_P)^2, \quad (17b)$$

where the small amplitude  $a_\xi$  and the phase  $\varphi_\xi$  in (17a) are determined by the boundary conditions and where the last estimate in (17b) follows from (13).

For a time-dependent homogeneous perturbation  $\xi(t)$  in (14), the energy-momentum tensor (6) becomes

$$T_{00}^{(q)} = \frac{1}{2} q_0 (\partial_t \xi)^2 + \frac{1}{2} (\chi_0)^{-1} \xi^2, \quad (18a)$$

$$T_{11}^{(q)} = \frac{1}{2} q_0 (\partial_t \xi)^2 - \frac{1}{2} (\chi_0)^{-1} \xi^2. \quad (18b)$$

Note that the structure of (18) is the same as that of a fundamental scalar field  $\phi(t)$ , which agrees with the second remark at the end of Sec. 2. With the rapidly-oscillating solution (17), we get

$$T_{00}^{(q)} = \frac{1}{2} (\chi_0)^{-1} (a_\xi)^2, \quad (19a)$$

$$T_{11}^{(q)} = T_{22}^{(q)} = T_{33}^{(q)} = 0, \quad (19b)$$

$$T_{01}^{(q)} = T_{02}^{(q)} = T_{03}^{(q)} = 0, \quad (19c)$$

where we have added the results for the other components.

At this moment, recall that a perfect fluid has an energy-momentum tensor of the form

$$T_{\alpha\beta} = P g_{\alpha\beta} + (\rho + P) U_\alpha U_\beta, \quad (20)$$

with  $g_{\alpha\beta} = \eta_{\alpha\beta}$  and  $U^\alpha = (1, 0, 0, 0)$  in Minkowski spacetime. From (19), we then conclude that the homogeneous  $\xi$  perturbation of the vacuum  $q$ -field behaves as a perfect fluid with the following values for the energy density and pressure:

$$\rho^{(q\text{-perturbation})} = \frac{1}{2} (\chi_0)^{-1} (a_\xi)^2, \quad (21a)$$

$$P^{(q\text{-perturbation})} = 0, \quad (21b)$$

where  $a_\xi$  with  $|a_\xi| \ll 1$  is determined by the initial boundary conditions [in a cosmological context, taken

at the moment when the homogeneous vacuum energy density  $\rho_V(t)$  has reached its final near-zero value; see Sec. 5 for further discussion].

Next, consider additional space-dependence of the  $\xi$  field with a typical length-scale

$$L \gg c/\omega \stackrel{?}{\sim} \hbar c/E_P \sim 10^{-35} \text{ m}, \quad (22)$$

where  $\hbar$  and  $c$  have been temporarily reinstated and the estimate (17b) for  $\omega$  has been used. (For applications in a cosmological context, the length-scale  $L$  must be less than the cosmological length-scale  $c/H_0 \sim 10^{26}$  m.) The corrections to (19) will then be small, namely of order  $q_0 \xi^2/L^2$ , which is a factor  $(L\omega)^{-2} \ll 1$  times the leading term (19a). For such large-scale perturbations, there will be, to high precision, a pressureless perfect fluid and this fluid will cluster gravitationally, just as dark matter with standard Newtonian gravitation and dynamics.

## 5. CONCLUSION

In the present article, we have shown that a small perturbation of the equilibrium  $q$ -field behaves gravitationally as a pressureless perfect fluid. As such, the fluctuating part of the  $q$ -field is a candidate for the inferred dark-matter component of the present universe (see, e.g., Sec. 26 of Ref. [7] for a review).

There are, however, many open questions. The main question concerns the amount of dark matter (DM) vs. that of dark energy (DE). The presently observed universe is very close to equilibrium,  $\rho_{V,\text{obs}} \ll (E_P)^4$ , but still somewhat away from it as  $\rho_{V,\text{obs}} \neq 0$ , which gives in our framework  $q_{\text{obs}} = q_0 + \delta q$  with  $0 < |\delta q| \ll |q_0|$ . We, therefore, need to find a mechanism which results in the small constant perturbation  $\delta q$  of the  $q$ -field for the present universe. Several possible mechanisms have been explored, including effects from TeV-mass particle decays [8, 9, 10] or from eV-mass neutrinos [11].

A final, definitive calculation of the appropriate  $\delta q$  and the corresponding  $\rho_V = -P_V > 0$  (to be interpreted as the inferred “dark-energy” component of the present universe) is not yet available. The same can be said of a further perturbative component  $\xi(x)$  in the  $q$ -field,  $q(x) = q_0 + \delta q + q_0 \xi(x)$ , which would determine the present amount of dark matter. Experimentally, we have  $\rho_{\text{DE}}/\rho_{\text{DM}} \sim 3$  and  $\rho_{\text{DE}} + \rho_{\text{DM}} \sim \rho_{\text{critical}} \equiv 3H_0^2/8\pi G_N$  (see, e.g., Secs. 26 and 27 of Ref. [7]).

Another open question concerns the effects on the clustering of subleading terms in the  $q$ -field energy-momentum tensor coming from the spatial derivatives of the perturbation  $\xi(x)$ . A further issue is the ultimate origin of the required perturbations in the dark-matter energy density with length scales obeying (22), which may or may not involve inflation-type processes.

To conclude, even though we still need to work out many details, we do have a clear prediction. Our proposal is to identify dark matter with an oscillating component of the  $q$ -field, most likely having a Planck-scale frequency. If correct, this implies that direct detection of dark-matter particles will fail, at least in the foreseeable future with the currently available energies.

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1. S. Weinberg, "The cosmological constant problem," *Rev. Mod. Phys.* **61**, 1 (1989).
  2. F.R. Klinkhamer and G.E. Volovik, "Self-tuning vacuum variable and cosmological constant," *Phys. Rev. D* **77**, 085015 (2008), arXiv:0711.3170.
  3. F.R. Klinkhamer and G.E. Volovik, "Dynamic vacuum variable and equilibrium approach in cosmology," *Phys. Rev. D* **78**, 063528 (2008), arXiv:0806.2805.
  4. F.R. Klinkhamer and G.E. Volovik, "Brane realization of  $q$ -theory and the cosmological constant problem," *JETP Lett.* **103**, 627 (2016), arXiv:1604.06060.
  5. F.R. Klinkhamer and G.E. Volovik, "Dynamic cancellation of a cosmological constant and approach to the Minkowski vacuum," *Mod. Phys. Lett. A* **31**, 1650160 (2016), arXiv:1601.00601.
  6. F.R. Klinkhamer and G.E. Volovik, "Propagating  $q$ -field and  $q$ -ball solution," arXiv:1609.03533.
  7. C. Patrignani *et al.* (Particle Data Group), "The review of particle physics (2016)," *Chin. Phys. C*, **40**, 100001 (2016) [available from <http://pdg.lbl.gov/>].
  8. N. Arkani-Hamed, L.J. Hall, C. Kolda, and H. Murayama, "New perspective on cosmic coincidence problems," *Phys. Rev. Lett.* **85**, 4434 (2000), arXiv:astro-ph/0005111.
  9. F.R. Klinkhamer and G.E. Volovik, "Vacuum energy density kicked by the electroweak crossover," *Phys. Rev. D* **80**, 083001 (2009), arXiv:0905.1919.
  10. F.R. Klinkhamer, "Effective cosmological constant from TeV-scale physics: Simple field-theoretic model," *Phys. Rev. D* **84**, 023011 (2011), arXiv:1101.1281.
  11. F.R. Klinkhamer and G.E. Volovik, "Dynamics of the quantum vacuum: Cosmology as relaxation to the equilibrium state," *J. Phys. Conf. Ser.* **314**, 012004 (2011), arXiv:1102.3152.