KA–TP–23–2025 (v1; September 10, 2025)

Defect cosmology: Four-leaf-clover universe

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Abstract

We present a new cosmological model without curvature singularities but with spacetime defects. Specifically, we combine a cosmological time-defect and a wormhole space-defect. The resulting universe has four segments, which explains the second part of the title. These four segments can be interpreted as two pairs of CPT-conjugated worlds.

PACS numbers: 04.20.Cv, 98.80.Bp, 98.80.Jk

Keywords: general relativity, big bang theory, mathematical and relativistic aspects of cosmology

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I. INTRODUCTION

CPT-conjugated worlds with a Big Bang in between have been discussed by Sakharov [1, 2] and, more recently, by Boyle, Finn, and Turok [3, 4]. Here, we present a new cosmological model without curvature singularities but with spacetime defects instead. This model is based on previous work with a particular time-defect for a Friedmann-like cosmology [5] and a particular space-defect for a traversable vacuum wormhole [6, 7]. A recent review [8] gives an extensive discussion of this type of spacetime defects and contains further references.

The new defect-cosmology model, with a "tamed" Big Bang and a traversable wormhole, consists, in fact, of *two* pairs of CPT-conjugated worlds: W–$\overline{W}$ and $\mathfrak{w}$–$\overline{\mathfrak{w}}$. We call this the *four-leaf-clover universe* and give two sketches in Fig. 1. (Note that the plane curve of the four-leaf clover is known as quadrifolium in mathematics.)

II. METRIC AND TETRADS

The new four-leaf-clover (f-l-c) metric combines a cosmological defect at $t = 0$ and a vacuum-type wormhole defect at $\xi = 0$:

$$
\begin{aligned}
ds^2\Big|_{\text{f-l-c}} &\equiv g_{\mu\nu}(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu\Big|_{\text{f-l-c}} \\
&= -\frac{t^2}{b^2+t^2}\,\mathrm{d}t^2 + a^2(t)\left(\frac{\xi^2}{\lambda^2+\xi^2}\,\mathrm{d}\xi^2 + \left(\lambda^2+\xi^2\right)\left[\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right]\right), \qquad \text{(1a)}
\end{aligned}
$$

$$b\,,\lambda > 0\,, \qquad \text{(1b)}$$

$$a(t) > 0\,, \quad a(-t) = a(t)\,, \qquad \text{(1c)}$$

$$t\,,\xi \in (-\infty,\,\infty)\,, \quad \theta\in[0,\,\pi]\,, \quad \phi\in[0,\,2\pi)\,. \qquad \text{(1d)}$$

The appropriate choice of tetrads $e^a_{\ \mu}$ is given by the following dual basis (1-forms defined by $e^a \equiv e^a_{\ \mu}\,\mathrm{d}x^\mu$) :

$$e^0\Big|_{\text{f-l-c}} = \frac{t}{\sqrt{b^2+t^2}}\,\mathrm{d}t\,, \qquad \text{(2a)}$$

$$e^1\Big|_{\text{f-l-c}} = a(t)\,\frac{\xi}{\sqrt{\lambda^2+\xi^2}}\,\mathrm{d}\xi\,, \qquad \text{(2b)}$$

$$e^2\Big|_{\text{f-l-c}} = a(t)\,\sqrt{\lambda^2+\xi^2}\,\mathrm{d}\theta\,, \qquad \text{(2c)}$$

$$e^3\Big|_{\text{f-l-c}} = a(t)\,\sqrt{\lambda^2+\xi^2}\,\sin\theta\,\mathrm{d}\phi\,, \qquad \text{(2d)}$$

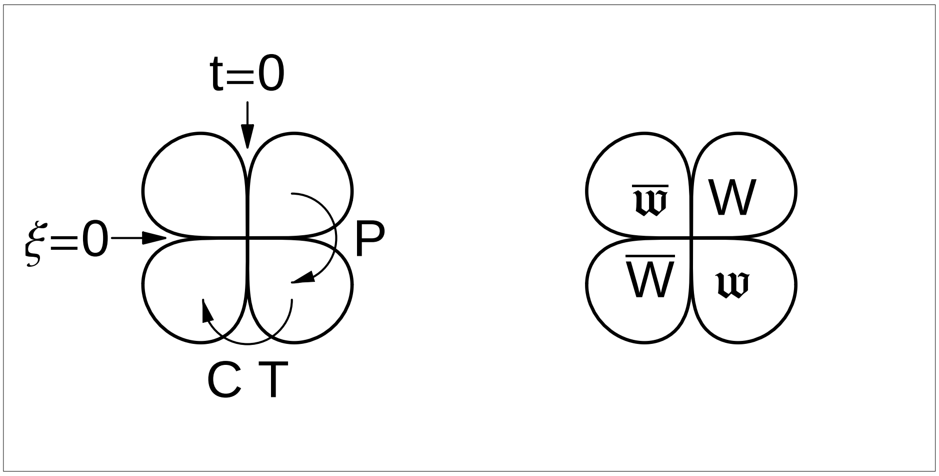

FIG. 1. Two sketches of the four-leaf-clover-universe, where each sketch has the cosmological time coordinate $t \in \mathbb{R}$ running horizontally to the right and the radial coordinate $\xi \in \mathbb{R}$ running vertically upwards. Left: defects at $t = 0$ and $\xi = 0$, with exemplary P and CT transformations. Right: two pairs of CPT-conjugated worlds $\big(W$–$\overline{W}$ and $\mathfrak{w}$–$\overline{\mathfrak{w}}\big)$.

which reproduce the above metric from the definition $g_{\mu\nu} \equiv e^a{}_\mu \, e^b{}_\nu \, \eta_{ab}$ for Minkowski metric $\eta_{ab} = [\text{diag}(-1,\, 1,\, 1\,,\, 1)]_{ab}$. Observe that the tetrads from (2a) are T-odd and those from (2b) P-odd (cf. the left panel in Fig. 1).

We close this section with two remarks on possible generalizations of the metric, but it is perfectly possible to skip ahead to Sec. III in a first reading.

First, we remark that the metric *Ansatz* (1) is consistent with having a homogeneous isotropic perfect fluid, as will be used in Sec. III. But we can easily generalize the metric *Ansatz* to allow for an inhomogeneous anisotropic perfect fluid.

Second, the metric (1) describes a single vacuum-type wormhole evolving with time. It is not difficult to consider multiple vacuum-type wormholes; we refer to Sec. 3 of Ref. [7] for further details on the construction. Very briefly, the idea is to use an embedding space M_embed, which consists of the union of two copies of Euclidean 3-space $E_3^{(\pm)}$, one copy being labeled by '+' (the "upper" world) and the other by '−' (the "lower" world). Each of these 3-spaces (with opposite orientability) has standard Cartesian coordinates, $(X_\pm,\, Y_\pm,\, Z_\pm) \in \mathbb{R}^3$. In these Euclidean 3-spaces, we then excise N ball-interiors and identify the ball-boundaries antipodally. Near these ball-boundaries we have better coordinates, $\xi_n \in (-\infty,\, \infty)$, $\theta_n \in [0,\, \pi]$, and $\phi_n \in [0,\, 2\pi)$, for wormhole label $n \in \{1,\, 2,\, 3,\, \ldots\,,\, N\}$.

Now, the metrics for these coordinates are simple generalizations of the metrics given by

(11) and (12) in Ref. [7]. Specifically, we take

$$ds^2\Big|_{\text{f-l-c},\,N\geq 1}^{\text{(outside WH mouths)}} = -\frac{t^2}{b^2+t^2}\,\mathrm{d}t^2 + a^2(t)\,\Big(\,(\mathrm{d}X_\pm)^2+(\mathrm{d}Y_\pm)^2+(\mathrm{d}Z_\pm)^2\Big), \tag{3a}$$

$$ds^2\Big|_{\text{f-l-c},\,N\geq 1}^{\text{(near WH mouths)}} = -\frac{t^2}{b^2+t^2}\,\mathrm{d}t^2 + a^2(t)\sum_{n=1}^{N}\left(\frac{\xi_n^2}{\lambda_n^2+\xi_n^2}\,\mathrm{d}\xi_n^2 + \left(\lambda_n^2+\xi_n^2\right)\left[\mathrm{d}\theta_n^2+\sin^2\theta_n\,\mathrm{d}\phi_n^2\right]\right), \tag{3b}$$

with possibly different wormhole sizes $\lambda_n > 0$. The sketches of Fig. 1 are still relevant, as long as the single ξ coordinate there is understood symbolically as an assemblage of all ξ_n. The top two petals in Fig. 1 belong to the Euclidean 3-space $E_3^{(+)}$, now for $t<0$ and $t>0$, and the bottom two petals belong to the Euclidean 3-space $E_3^{(-)}$, again for $t<0$ and $t>0$.

III. REDUCED FIELD EQUATIONS AND NONSINGULAR SOLUTION

If we insert the metric (1) and the energy-momentum tensor of a homogeneous isotropic perfect fluid into the standard Einstein equation without cosmological constant, we get the same ordinary differential equations (ODEs) as before [5], but now in *two* copies, one for $\xi<0$ and another for $\xi>0$. Using appropriate limits for $t=0$ and $\xi=0$ (cf. Sec. 4.4 in Ref. [8]), we indeed obtain the following modified spatially-flat Friedmann equations:

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\,\rho_M\,, \tag{4a}$$

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\ddot a}{a}+\frac{1}{2}\left(\frac{\dot a}{a}\right)^2\right) - \frac{b^2}{t^3}\,\frac{\dot a}{a} = -4\pi G\,P_M\,, \tag{4b}$$

$$\dot\rho_M + 3\,\frac{\dot a}{a}\,\Big(\rho_M+P_M\Big) = 0\,, \tag{4c}$$

$$P_M = P_M\big(\rho_M\big)\,, \tag{4d}$$

where the overdot stands for differentiation with respect to t and (4d) corresponds to the equation of state of the perfect fluid.

Heuristically, it is easy to understand why the wormhole structure does not make any difference for these ODEs. On constant-t slices [i.e., $\mathrm{d}t=0$ in the line element (1a)], we can change the spatial coordinates, so that the total 3-space corresponds to two Euclidean 3-spaces with two ball-interiors excised and two ball-boundaries antipodally identified (see Sec. 2.2 of Ref. [7]). We then have essentially two copies of the previous defect cosmology (with Euclidean hypersurfaces at constant t), as discussed in the original paper [5] and the review [8]. Incidentally, the same argument applies to the case of the four-leaf-clover universe with multiple vacuum-type wormholes, as discussed in the last two paragraphs of Sec. II.

For a homogeneous relativistic perfect fluid [with a constant equation-of-state parameter $w_M(t) \equiv P_M(t)/\rho_M(t) = 1/3$], we obtain the following nonsingular solution:

$$a(t)\Big|_{\text{f-l-c}} = \sqrt[4]{\left(b^2+t^2\right)/\left(b^2+t_0^2\right)}\,, \tag{5a}$$

$$\rho_M(t)\Big|_{\text{f-l-c}} = \rho_{M0}/a^4(t)\Big|_{\text{f-l-c}} = \rho_{M0}\left(b^2+t_0^2\right)/\left(b^2+t^2\right), \tag{5b}$$

where the energy density and the Kretschmann curvature scalar K are *finite* at $t = 0$, respectively of order E_{Planck}^2/b^2 and $1/b^4$, with $E_{\text{Planck}} \equiv \sqrt{\hbar\, c^5/G} \approx 1.22\times 10^{19}$ GeV.

IV. DISCUSSION

Future work on the proposed four-leaf-clover universe (1) can proceed in, at least, two directions.

First, it needs to be rigorously established by a comprehensive quantum-field-theory (QFT) calculation that the pairs of emerging worlds are truly *CPT-conjugate* to each other. Some work in this direction has been presented in Refs [3, 4] but, as the authors acknowledge themselves, this QFT work needs to be consolidated.

Second, it needs to be understood how the double defects *originate* from the underlying fundamental theory. Some toy-model calculations in the context of the IIB matrix model (as a nonperturbative realization of superstring theory) have been reviewed in the last four paragraphs of Sec. 6 in Ref. [8].

Ultimately, the hope is that these calculations, or others, can explain the emergence of a smooth spacetime with appropriate defects inserted. If somehow small matter perturbations arise near $t \sim 0$, then these perturbations can grow [9] with a power of t^2, hence symmetrically with coordinate time t. This gives rise to a thermodynamic time $\mathcal{T}$ and a corresponding arrow-of-time (in the direction of growing entropy). Specifically, this thermodynamic time is given by $\mathcal{T} \equiv +t$ for $t \geq 0$ and $\mathcal{T} \equiv -t$ for $t \leq 0$. Then, the $t = 0$ and $\xi = 0$ defects give rise to *four* expanding worlds (that is, expanding with respect to the thermodynamic time $\mathcal{T} = |t|$).

Communication between the worlds W and $\mathfrak{w}$ in the terminology of Fig. 1 (or between the worlds $\overline{W}$ and $\overline{\mathfrak{w}}$) is, in principle, possible, the wormhole being traversable after all (see

Sec. 4 in Ref. [7] for some further discussion on vacuum-type wormhole phenomenology, also for the case of multiple vacuum-type wormholes). But classical communication between the $t > 0$ worlds (W and $\mathfrak{w}$) and $t < 0$ worlds ($\overline{W}$ and $\overline{\mathfrak{w}}$) appears to be impossible, as explained in Sec. 5.2 of Ref. [8], leaving the remote possibility of quantum communication.

ACKNOWLEDGMENTS

It is a pleasure to thank Z.L. Wang for comments on the manuscript.

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