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Electroweak baryon number violation: basic mechanism

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1. INTRODUCTION

Conditions for baryogenesis [Sakharov, 1967]:

1. C and CP violation — Yes
2. Thermal nonequilibrium — Yes
3. Baryon number (B) violation — ?

Strictly speaking, we know of only one physical theory that is expected to have B violation:

the electroweak Standard Model (EWSM).

[Side remark: the *ultimate* fate of black holes is uncertain.]

But the relevant physical processes of the EWSM are only known at

$$T \ll M_W \approx 10^2 \text{ GeV} ,$$

and their rate is negligible,

$$\Gamma \propto \exp[\,-4\,\pi\,\sin^2\theta_w\,/\,\alpha\,] \approx 0\ .$$

Clearly, we should study electroweak baryon number violation for the conditions of the early universe,

$$T \gtrsim 10^2 \text{ GeV} \ .$$

This is a difficult problem, but entirely well-posed.

In this talk, we focus on the fundamental physics, i.e., the microscopic process.
That is, we must really deal with the fermions.

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2. CLASSIC RESULTS

Consider $SU(2)$ Yang–Mills–Higgs theory with vanishing Yukawa couplings. Actually, forget about the Higgs, which may be reasonable above the EW phase transition.

Triangle anomaly in the AAA-diagram, provided the VVV-diagram is anomaly-free [A69,BJ69].
[Side remark: this is Feynman *perturbation* theory.]

The gauge vertices of the EWSM are V–A and must be nonanomalous (gauge invariance is needed for unitarity). Instead, the $B+L$ current is anomalous [H76]:

$$
\begin{aligned}
\Delta(B-L) &= 0\,, \\
\underbrace{\Delta(B+L)}_{\text{fermion charges}} &= \underbrace{2\,N_{\mathsf{fam}}}_{\text{constant}} \times \underbrace{\Delta N_{\mathsf{CS}}}_{\text{gauge field characteristic}}\,.
\end{aligned}
$$

In the $A_0=0$ gauge, one has the Chern–Simons number

$$N_{\mathsf{CS}}(t) = N_{\mathsf{CS}}[\vec{A}(\vec{x},t)]$$

and

$$\Delta N_{\mathsf{CS}} \equiv N_{\mathsf{CS}}(t_{\mathsf{out}}) - N_{\mathsf{CS}}(t_{\mathsf{in}})\,.$$

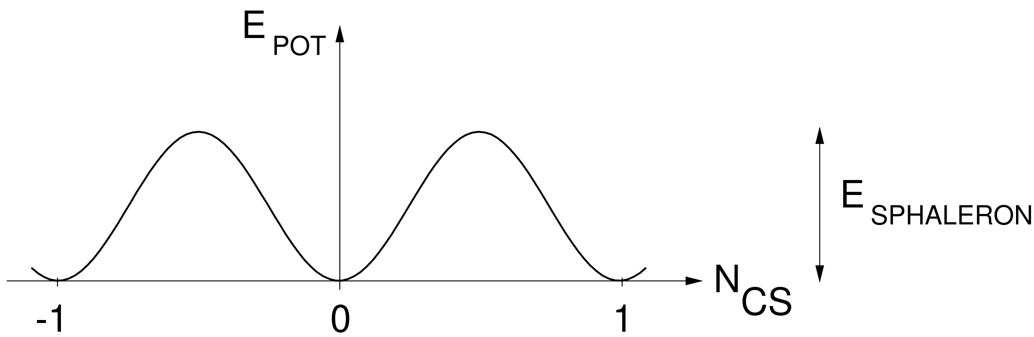

Figure 1: Potential energy surface over configuration space.

’t Hooft (1976) calculated the tunneling amplitude.
The BPST instanton, which is a finite action solution of the imaginary-time theory (Euclidean spacetime), gives

$$\Delta N_{\mathsf{CS}} = Q[A_{\mathsf{finite\ action}}] \in \mathbb{Z} \; ,$$

where the topological charge Q is the winding number of the map

$$S^3\Big|_{|x|=\infty} \to SU(2) \sim S^3 \; .$$

This holds only for transitions from near-vacuum to near-vacuum, i.e., at very low temperatures or energies. As mentioned above, the rate is then effectively zero, but, at least, $\Delta(B+L)$ is integer.

3. CRUCIAL QUESTION

For real-time processes (e.g., in Minkowski spacetime), the topological charge Q is, in general, noninteger.

Hence, the question

$$\Delta(B+L) \propto \text{which gauge field characteristic}\,?$$

In the following, we consider pure $SU(2)$ Yang–Mills theory with a single isodoublet of left-handed fermions.

(The fermion number $B+L$ of the EWSM follows by multiplying with $2\,N_{\mathsf{fam}}$. Note also that $B-L$ remains conserved in the EWSM.)

Furthermore, the gauge fields will be called dissipative if their energy density approaches zero uniformly as $t \to \pm\,\infty$.

4. PARTIAL ANSWER

Start from the eigenvalue equation of the time-dependent Dirac Hamiltonian:

$$H(\vec{x},t)\,\Psi(\vec{x},t) = E(t)\,\Psi(\vec{x},t)\ .$$

Then, fermion number violation is related to the so-called spectral flow $\mathcal{F}$. See, e.g., Refs. [C80,KR03].

Definition: $\mathcal{F}[\,t_f, t_i\,]$ is the number of eigenvalues of the Dirac Hamiltonian that cross zero from below minus the number of eigenvalues that cross zero from above, for the time interval $[\,t_i, t_f\,]$ with $t_i < t_f$.

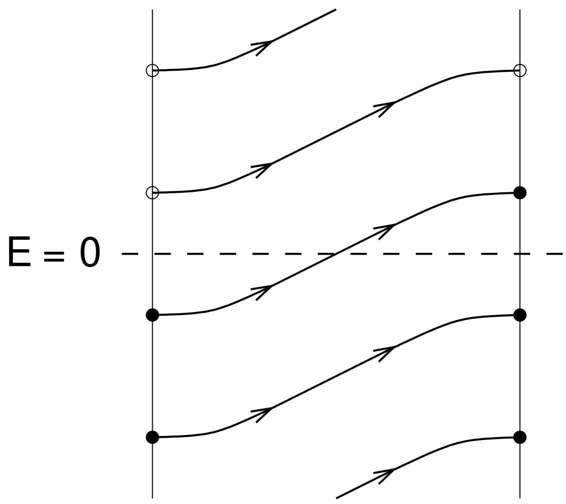

Figure 2: Spectral flow with $\mathcal{F}[\,t_f, t_i\,] = +1$.

Strongly dissipative gauge fields have [C80,GH95,K95]:

$$\mathcal{F} \;=\; \Delta N_{\mathsf{CS}}[A_{\,\mathsf{associated\ vacuum}}] \equiv \Delta N_{\mathsf{winding}} \;.$$

Now three weakly dissipative, spherically symmetric gauge field solutions [Lüscher & Schechter, 1977]:

1. (low energy) $\Delta N_{\mathsf{winding}} = 0$ and $\mathcal{F} = 0\,,$
2. (moderate energy) $\Delta N_{\mathsf{winding}} = 1$ and $\mathcal{F} = 1\,,$
3. (high energy) $\Delta N_{\mathsf{winding}} = 1$ and $\mathcal{F} = -1\,.$

$$\Rightarrow \;\; \left[\,\mathcal{F} \neq \Delta N_{\mathsf{winding}}\,\right]_{\mathsf{spherically\ symmetric\ fields}} \;.$$

In fact, there is another gauge field characteristic [KL01]:

$\Delta N_{\mathsf{twist}} = 0$ for case 1 and 2,

$\Delta N_{\mathsf{twist}} = -2$ for case 3.

$$\Rightarrow \;\; \left[\,\mathcal{F} = \Delta N_{\mathsf{winding}} + \Delta N_{\mathsf{twist}}\,\right]_{\mathsf{spherically\ symmetric\ fields}} \;.$$

For weakly dissipative gauge fields, one has thus

$$\Delta(B+L) =$$

$$2\,N_{\mathsf{fam}} \times \Big(\Delta N_{\mathsf{CS}}\,[A_{\,\mathsf{associated\ vacuum}}] + \underline{\mathsf{extra\ terms}}\Big).$$

But the “extra terms” are not known in general.

5. SOME DETAILS

Chiral $SU(2)$ Yang–Mills theory over Minkowski spacetime (indices M, N running over 0, 1, 2, 3):

$$
\begin{aligned}
S &= S_G + S_F \;, \\
S_G &= -\frac{1}{2g^2}\int \mathrm{d}^4x \, \mathrm{tr}\left(F^{MN}F_{MN}\right) \,, \\
S_F &= \int \mathrm{d}^4x \, \sum_{f=1}^{N_F} \bar{\Psi}_f \, \Gamma^M D_M \Psi_f \;, \\
F_{MN} &\equiv \partial_M A_N - \partial_N A_M + [A_M, A_N] \;, \\
A_M &\equiv A_M^a \, \sigma^a/(2i) \;, \\
D_M &\equiv \partial_M + A_M \, \mathsf{P}_L \;, \quad \mathsf{P}_L \equiv (1-\Gamma_5)/2 \;.
\end{aligned}
$$

Spherical Ansatz: invariance under $SO(3)$ rotations, modulo $SU(2)$ gauge transformations.

For $N_F = 1$, this gives an effective (1+1)-dimensional $U(1)$ gauge field theory with:

- gauge field $a_\mu(t, r)$, $\quad \mu = 0, 1,$
- complex scalar $\chi(t, r)$,
- 2-component Dirac spinor $\psi(t, r)$.

Effective (1+1)-dimensional $U(1)$ gauge field theory (indices μ, ν running over 0, 1):

$$
\begin{aligned}
S &= \frac{4\pi}{g^2} \int_{-\infty}^{+\infty} \mathrm{d}t \int_0^{\infty} \mathrm{d}r \ \left(s_G + s_F \right) \ , \\
s_G &= \frac{1}{4}\, r^2 f_{\mu\nu} f^{\mu\nu} + |D_\mu \chi|^2 + \frac{1}{2\, r^2} \ \left(|\chi|^2 - 1 \right)^2 , \\
s_F &= g^2\, \bar{\psi} \left(\gamma^\mu D_\mu + \frac{1}{r} \left(\mathrm{Re}\chi + i\gamma_5\, \mathrm{Im}\chi \right) \right) \psi \ , \\
f_{\mu\nu} &\equiv \partial_\mu a_\nu - \partial_\nu a_\mu \ , \\
D_\mu \chi &\equiv \left(\partial_\mu - i a_\mu \right) \chi \ , \\
D_\mu \psi &\equiv \left(\partial_\mu + i/2\, a_\mu\, \gamma_5 \right) \psi \ , \\
\gamma^0 &= i\sigma^1, \quad \gamma^1 = -\sigma^3, \quad \gamma_5 = -\gamma^0 \gamma^1 = \sigma^2.
\end{aligned}
$$

The boundary conditions on the fields are for $r \to 0$:

$$
|\,\chi| \to 1, \quad D_\mu \chi \to 0, \quad \psi \to 0 \,,
$$

and for $r \to \infty$:

$$
\chi \to e^{i\omega}, \quad D_\mu \chi \to 0, \quad a_\mu \to \partial_\mu\, \omega, \quad \psi \to 0 \,.
$$

Taking the gauge condition

$$\chi(t,0) = \chi(t,\infty) = 1 \; ,$$

results in a <u>closed loop</u> in the χ-plane.

Now write χ in polar notation:

$$\begin{aligned} \chi(t,r) &= \rho(t,r)\; e^{i\varphi(t,r)} \; , \\ \rho(t,r) &\geq 0 \; . \end{aligned}$$

Then define the <u>WINDING NUMBER</u> at time t ,

$$N_{\text{winding}}(t) \equiv \left[\varphi(t,\infty) - \varphi(t,0)\right] / \, (2\pi),$$

and the <u>WINDING FACTOR</u> between t_i and t_f ,

$$\Delta N_{\text{winding}}[t_f, t_i] \equiv N_{\text{winding}}(t_f) - N_{\text{winding}}(t_i).$$

Note that $\Delta N_{\text{winding}} = \Delta N_{\text{CS}}$ for near-vacuum fields.

After a unitary transformation, put the resulting real Dirac spinor $\tilde{\psi}(t,r)$ in polar notation:

$$\tilde{\psi}(t,r) \equiv |\tilde{\psi}(t,r)|\, e^{i\sigma^2 \Theta(t,r)} \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

Fermion zero-mode equation $H(\vec{x},t)\,\Psi(\vec{x},t) = 0$, at fixed t, becomes:

$$\partial_r \Theta = -\lambda \, \sin 2\Theta + \mathcal{R} \ ,$$

$$\partial_r |\tilde{\psi}| = |\tilde{\psi}|\, \lambda \, \cos 2\Theta \ ,$$

with boundary conditions

$$\Theta(t,0) = 0, \quad |\tilde{\psi}(t,0)| = 0 \ ,$$

and definitions

$$\lambda \equiv \rho/r, \quad \mathcal{R} \equiv (a_1 - \partial_r \varphi)/2 \ .$$

With $Y(t,r) = \tan\Theta(t,r)$, the first ODE transforms into a Riccati equation:

$$\partial_r Y - \mathcal{R}\left(1 + Y^2\right) + 2\,\lambda\, Y = 0 \ .$$

Then define the SPINOR TWIST NUMBER at time t ,

$$N_{\mathsf{twist}}(t) \equiv [\Theta(t,\infty) - \Theta(t,0)] \,/\, \pi \ ,$$

and the TWIST FACTOR between t_i and t_f ,

$$\Delta N_{\mathsf{twist}}[t_f, t_i] \equiv N_{\mathsf{twist}}(t_f) - N_{\mathsf{twist}}(t_i) \ .$$

Remark that the twist factor $\Delta N_{\mathsf{twist}}$ measures an intrinsic property of the gauge field configuration:

$$\Delta N_{\mathsf{twist}}[t_f, t_i] = \frac{1}{\pi} \int_0^\infty \mathrm{d}r \int_{t_i}^{t_f} \mathrm{d}t \; \partial_t \, \partial_r \Theta(t,r) \ ,$$

with

$$\Theta = \Theta[\, \chi, a_1 \,] \ .$$

Whether or not there exists a more direct way to obtain $\Delta N_{\mathsf{twist}}$ remains an open question.

Now consider a fermion zero-mode at $t = t^*$.

A careful study of the generic time dependence of the zero-eigenvalue equation of the Dirac Hamiltonian, then gives *locally*

$$\text{sign}\left[\left.\frac{\mathrm{d}E}{\mathrm{d}t}\right|_{t=t^*}\right] = \delta N_{\text{winding}}\big|_{t=t^*} + \delta N_{\text{twist}}\big|_{t=t^*} \,.$$

For a *finite* time interval $[\,t_i, t_f\,]$ with $t_i < t_f$, this results in the *over-all* spectral flow:

$$\mathcal{F}[\,t_f, t_i\,] = \Delta N_{\text{winding}}[\,t_f, t_i\,] + \Delta N_{\text{twist}}[\,t_f, t_i\,] \,.$$

Note that this relation has the form of an <u>index theorem</u>:

LHS = property of the fermions;

RHS = characteristic of the gauge fields.

6. OUTLOOK

We know of only one physical theory with baryon number violation, the electroweak Standard Model.

Most discussions of electroweak baryogenesis have been based on the 't Hooft selection rule $\Delta(B+L) \propto \Delta N_{\mathrm{CS}}$.

But this relation has been found to be invalid for gauge field backgrounds that are weakly- or non-dissipative.

These are, of course, precisely the fields relevant to the physics of the early universe.

At this moment, we have only a partial result for the correct selection rule, namely for spherically symmetric fields.

To generalize this result to arbitrary gauge fields will be difficult, but is absolutely necessary for a serious discussion of electroweak baryon number violation in the early universe.