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Cosmological constant problem: Revisiting the unimodular-gravity approach

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1. Introduction

The main **Cosmological Constant Problem** (CCP1) can be phrased as follows (Pauli, 1933; Bohr, 1948; Veltman, 1974; see [1, 2] for two reviews):

why do the quantum fields in the vacuum not produce naturally a large cosmological constant Λ *in the Einstein gravitational field equation?*

The magnitude of the problem is enormous:

$$|\mathbf{\Lambda}^{\text{theory}}|/|\mathbf{\Lambda}^{\text{experiment}}| \geq \mathbf{10^{54}}\,,$$

where the large number on the RHS will be explained on the next slide.

From now on, $c = 1$ and $\hbar = 1$.

1. Introduction

With the ATLAS and CMS results [3, 4] in support of the Higgs mechanism, it is clear that the EWSM in the laboratory involves a vacuum energy density of order

$$\left|\epsilon_V^{\text{(EWSM)}}\right| \sim (100 \text{ GeV})^4 \sim 10^{44} \text{ eV}^4 \,.$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_V^{\text{(EWSM)}} = \epsilon_V^{\text{(EWSM)}}(T)\,.$$

How can the Universe then end up with a vacuum energy density

$$\left|\Lambda^{\text{(obs)}}\right| < 10^{-28} \text{ g cm}^{-3} \sim 10^{-10} \text{ eV}^4 \,?$$

Here, there are 54 orders of magnitude to explain:

$$\left|\Lambda^{\text{(obs)}}/\epsilon_V^{\text{(EWSM)}}\right| \leq 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,001\,.$$

1. Introduction

In short, the main cosmological constant problem is

CCP1 – why $|\mathbf{\Lambda}| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{Planck}})^4$?

Still more CCPs after the discovery of the "accelerating Universe":

CCP2a – why $\mathbf{\Lambda} \neq \mathbf{0}$?

CCP2b – why $\mathbf{\Lambda} \sim \rho_{\text{matter}}\big|_{\text{present}} \sim +\mathbf{10^{-11}}\ \text{eV}^4$?

Hundreds of papers have been published on CCP2. But, most likely:

CCP1 needs to be solved first, before CCP2 can even be addressed.

1. Introduction

Here, a discussion of one particular approach to CCP1 by Volovik and the speaker, which goes under the name of **q–theory** [5, 6, 7, 8, 9, 10] (a brief review appears in App. A of [11]).

Originally, we considered four explicit realizations of q–theory using

1. a three-form gauge field [12, 13, 14, 15],
2. a massless vector-field [16, 17],
3. a spacetime 4D-brane [18],
4. an elasticity tetrad from a spacetime crystal [19].

The present talk will, however, focus on an entirely new and attractive realization.

1. Introduction

OUTLINE:

1. Introduction
2. Basics of q–theory $\longleftarrow$ original idea
3. Postulated three-form gauge field
4. Metric determinant $\longleftarrow$ new approach
5. Metric determinant: Cosmology
6. Conclusion
7. References

2. Basics of q–theory

Crucial insight [5]: there is vacuum energy and vacuum energy.

More specifically and introducing an appropriate notation:

the vacuum energy density $\boxed{\epsilon}$ appearing in the action

need not be the same as

the vacuum energy density $\boxed{\rho_V}$ in the Einstein field equation.

How could this happen concretely . . .

2. Basics of q–theory

Assume the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Study, then, the **macroscopic** equations of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy:

- Take the mass density ρ of a liquid, for example, liquid Argon.
- This ρ describes microscopic quantities ($\rho = m_{\text{Ar}}\, n_{\text{Ar}}$ with number density n_{Ar} and mass m_{Ar} of the atoms).
- Still, ρ obeys the macroscopic equations of hydrodynamics, because of particle-number conservation and mass conservation.

2. Basics of q–theory

However, is the quantum vacuum similar to a “normal” liquid?

No, the quantum vacuum behaves like a liquid but not like a “normal” liquid.

2. Basics of q–theory

In fact, the quantum vacuum is known to be **Lorentz invariant** (cf. experimental limits at the 10^{-15} level in the photon sector [20]).

The Lorentz invariance of the vacuum rules out the standard type of charge density, which arises from the time component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material liquids.

2. Basics of q–theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{nonconstant}}(q)\,, \tag{1}$$

including a possible constant term Λ_{bare} from the zero-point energies of the fields of the Standard Model (SM).

From ① thermodynamics and ② Lorentz invariance follows that [5]

$$P_V \overset{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \overset{②}{=} -\rho_V\,, \tag{2}$$

where the first equality corresponds to an integrated form of the Gibbs–Duhem equation for chemical potential $\mu \equiv d\epsilon/dq$.

Recall GD eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

2. Basics of q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{Planck}})^4$, but they cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 = \text{const} \;\; : \quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\, \frac{d\,\epsilon(q)}{d\,q} \right]_{q=q_0} = 0\,, \qquad (3)$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle . . .

But, is such a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q variable and one example will be given in Sec. 3.

3 Postulated three-form gauge field

Vacuum variable q may arise from a 3–form gauge field A [12, 13].

Start from the effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = \int_{\mathbb{R}^4} d^4x \sqrt{-\det g}\,\Big(K_N\,R[g] + \Lambda_{\text{SM}} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \qquad (4)$$

with gravitational coupling constant $K_N \equiv 1/(16\pi G_N)$ and $c = 1 = \hbar$.

Add a 3–form gauge field A and get [6, 7]:

$$\widetilde{S}^{\text{eff}}[A,g,\psi] \;=\; \int_{\mathbb{R}^4} d^4x \sqrt{-\det g}\,\Big(K(q)\,R[g] + \epsilon(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \qquad (5a)$$

$$\boxed{q \;\equiv\; -\tfrac{1}{24}\,\epsilon^{\alpha\beta\gamma\delta}\,\nabla_\alpha A_{\beta\gamma\delta}\,/\,\sqrt{-g}}\,, \qquad (5b)$$

where $\epsilon(q)$ is a generic function of q, which arises from the 4-form field strength $F = d\,A$. The gravitational coupling $K(q)$ is a positive function.

Variational principle gives generalized Einstein and Maxwell equations:

3 Postulated three-form gauge field

$$\begin{aligned} 2K(q)\left(R_{\alpha\beta} - g_{\alpha\beta}\, R/2\right) &= -2\left(\nabla_\alpha \nabla_\beta - g_{\alpha\beta}\,\Box\right) K(q) \\ &\quad + \rho_V(q)\, g_{\alpha\beta} - T^M_{\alpha\beta}\,, \qquad \text{(6a)} \\ \frac{d\rho_V(q)}{dq} + R\,\frac{dK(q)}{dq} &= 0\,, \qquad \text{(6b)} \end{aligned}$$

with a vacuum energy density,

$$\rho_V = \epsilon - q\left(\frac{d\epsilon}{dq} + R\,\frac{dK}{dq}\right) = \epsilon - q\,\mu\,, \qquad (7)$$

for integration constant (chemical potential) μ. Eq. (7) is precisely of the Gibbs–Duhem form (2) in Minkowski spacetime ($R = 0$). Technically, the extra $g_{\alpha\beta}$ term on the RHS of (6a) appears because $q = q(A, \underline{\underline{g}})$.

The expression (5b) shows that q is a non-fundamental scalar field, which invalidates Weinberg's no-go theorem (see [7] for details).

4. Metric determinant – Preliminaries

Preliminaries:

We have several examples of potential q-fields (e.g., 4-form field strength and 4D-brane), but all were added by hand.

The idea, here, is to use only the known fields from GR+SM, but perhaps to reinterpret them differently.

In fact, we propose to use the metric determinant

$$g(x) \equiv \det\Big(g_{\alpha\beta}(x)\Big)\,. \tag{8}$$

Yet, $g(x)$ is not a scalar but only a scalar density. Still, it is a scalar if coordinate transformations are restricted to those of unit Jacobian:

$$\det\Big(\partial x'^{\,\alpha}/\partial x^{\beta}\Big) = 1\,. \tag{9}$$

4. Metric determinant – Preliminaries

This reminds us of the so-called unimodular-gravity approach to the CCP [21, 22, 2], which uses restricted coordinate invariance and eliminates g as a dynamical variable (Λ then arises as a constant of integration).

For us, g is not eliminated from the dynamics, but plays an essential role in the cancellation of the cosmological constant.

In short, the metric determinant is a dynamical variable.

4. Metric determinant – Motivation

Motivation (cond-mat inspired, courtesy of G.E. Volovik):

It is possible that the metric field $g_{\alpha\beta}(x)$ arises from a spacetime crystal with elasticity tetrads [19]. Then, the density of lattice points $n(x)$ $\left[\text{with dimension of } 1/\text{length}^4\right]$ would be proportional to the metric determinant,

$$M^{-4}\,n(x) = \sqrt{-g(x)}\,, \tag{10}$$

where the crystal has a fundamental length scale $\ell \equiv 1/M$.

The total number of lattice points is given by

$$N = \int d^4x\, n(x)\,, \tag{11}$$

and it is natural to assume that this number is conserved.

4. Metric determinant – Motivation

Then there is a Lagrange multiplier in the action:

$$S_N = -\mu\, N = -\mu \int d^4x\, n(x)\,, \tag{12}$$

where μ is the corresponding (dimensionless) chemical potential.

The crucial observation is that n can enter the matter action, provided coordinate invariance is restricted by (9). (The possibility of adding extra $\sqrt{-g}$ terms in the matter action was already noted in, e.g., Ref. [22], but was not pursued further.)

We, next, simplify the theory to the bare minimum. In fact, we only need a standard real scalar $X(x)$ for the appropriate expansion of the FRW-type model later on.

4. Metric determinant – Action

Action [23]:

$$S = S_G + S_M + S_{\Lambda-\text{plus}} + S_N \,, \tag{13a}$$

$$S_G = \int d^4x \, \sqrt{-g} \, \frac{R}{16\pi G_N} \,, \tag{13b}$$

$$S_M = \int d^4x \, \sqrt{-g} \, \left[\frac{1}{2} \, g^{\alpha\beta} \, \partial_\alpha X \, \partial_\beta X + \frac{1}{2} \, g_2 \, M^2 \, X^2 \right] \,, \tag{13c}$$

$$S_{\Lambda-\text{plus}} = \int d^4x \, \sqrt{-g} \, \epsilon(\Lambda, \, n) = \int d^4x \, \sqrt{-g} \, \Big[\Lambda + \zeta \, n \Big] \,, \tag{13d}$$

$$S_N = -\mu \int d^4x \, n(x) \,, \tag{13e}$$

$$n(x) = \sqrt{-g(x)} \, M^4 \,, \tag{13f}$$

where we have used the simplest possible *Ansätze* in (13c) and (13d), with real parameters $\zeta > 0$ and $g_2 \geq 0$.

4. Metric determinant – Action

Strictly speaking, the only new input is the single term $n \propto \sqrt{-g}$ in the potential (13d), consistent with having coordinate invariance restricted by (9).

The resulting gravitational field equation reads:

$$\frac{1}{8\pi G_N}\left(R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta}\, R\right) = \rho_{\text{vac}}\, g_{\alpha\beta} + T^{M}_{\alpha\beta}\,, \qquad \text{(14a)}$$

$$\rho_{\text{vac}} = \Lambda + 2\,\zeta\, n - \mu\, M^4\,, \qquad \text{(14b)}$$

$$n = \sqrt{-g}\; M^4\,, \qquad \text{(14c)}$$

$$\Lambda = \lambda\, M^4\,, \qquad \text{(14d)}$$

where the chemical potential μ traces back to the action term (13e).

4. Metric determinant – Action

Taking the covariant divergence of (14a) and using the contracted Bianchi identities, the following combined energy-momentum conservation relation is obtained:

$$\left(\rho_{\text{vac}}\, g_{\alpha\beta} + T^{M}_{\alpha\beta}\right)^{;\,\beta} = 0\,, \tag{15}$$

where the semicolon stands for a covariant partial derivative (the colon stands for a standard partial derivative).

If the matter component is separately conserved, $\left(T^{M}_{\alpha\beta}\right)^{;\,\beta} = 0$, then equally so for the vacuum component, so that $\rho_{\text{vac}}{}^{,\,\beta} = 0$.

5. Metric determinant: Cosmology

With diffeomorphisms restricted to those of unit Jacobian, the appropriate spatially-flat Robertson–Walker (RW) metric is given by [22]

$$ds^2 = g_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta = -\widetilde{A}(t)\, dt^2 + \widetilde{R}^{\,2}(t)\, \delta_{ij}\, dx^i\, dx^j\,, \qquad (16)$$

where t is the cosmic time coordinate from $x^0 = c\,t = t$ and $\widetilde{A}(t) > 0$ an additional *Ansatz* function.

For $\widetilde{A}(t) = \text{const} > 0$, we recover the standard spatially-flat RW metric.

Remark that the extended RW metric (16) gives the vacuum variable

$$n \propto \sqrt{-g} = (\widetilde{A}\,)^{1/2}\, |\widetilde{R}\,|^3. \qquad (17)$$

5. Metric determinant: Cosmology

Henceforth, we set

$$E_{\text{Planck}} \equiv 1/\sqrt{G_N} = M\,, \tag{18}$$

and introduce the following dimensionless quantities (the chemical potential μ is already dimensionless):

$$t \to \tau\,, \qquad \rho_X(t) \to r_\chi(\tau)\,, \qquad \widetilde{A}(t) \to a(\tau)\,, \tag{19a}$$

$$X(t) \to \chi(\tau)\,, \qquad P_X(t) \to p_\chi(\tau)\,, \qquad \widetilde{R}(t) \to r(\tau)\,, \tag{19b}$$

$$n(t) \to n(\tau)\,, \qquad \Lambda \to \lambda\,, \tag{19c}$$

where $n(\tau)$ is dimensionless and equal to $\sqrt{a(\tau)}\,|r(\tau)|^3$.

5. Metric determinant: Cosmology

From the field equations of the action (13) for the RW metric (16) and using the homogeneous perfect fluid from the χ scalar, we obtain the following ODEs:

$$\dot{r}_\chi + 3\,(1 + w_M)\,\left(\frac{\dot{r}}{r}\right)\,r_\chi = 0\,, \tag{20a}$$

$$3\,\left(\frac{\dot{r}}{r}\right)^2 = 8\,\pi\,a\,\Big(r_\chi + r_{\text{vac}}\Big)\,, \tag{20b}$$

$$\frac{2\,\ddot{r}}{r} + \left(\frac{\dot{r}}{r}\right)^2 - \left(\frac{\dot{a}}{a}\right)\,\left(\frac{\dot{r}}{r}\right) = -8\,\pi\,a\,\Big(w_M\,r_\chi - r_{\text{vac}}\Big)\,, \tag{20c}$$

$$r_{\text{vac}} = \lambda + 2\,\zeta\,\sqrt{a}\,|r|^3 - \mu\,, \tag{20d}$$

where the overdot stands for differentiation with respect to τ. These ODEs have three real parameters: the matter equation-of-state parameter $w_M \equiv p_\chi/r_\chi > -1$ and two parameters, $\zeta > 0$ and $\mu \neq 0$, entering the vacuum energy density r_{vac}.

5. Metric determinant: Cosmology

We can get analytic Friedmann-type and deSitter-type solutions from the following *Ansatz* functions:

$$a(\tau) = \alpha\, \tau^{-2\,p}\,, \tag{21a}$$

$$r(\tau) = \alpha^{-1/6}\, \widehat{r}\, \tau^{p/3}\,, \tag{21b}$$

$$r_\chi(\tau) = \alpha^{-1}\, \widehat{\chi}\, \tau^{-m}\,, \tag{21c}$$

with positive parameters α, p, $\widehat{r}$, $\widehat{\chi}$, and m.

The corresponding dimensionless Ricci and Kretschmann curvature scalars read:

$$\mathcal{R} = \frac{2}{3}\, p\, (5\, p - 3)\, \frac{1}{\alpha}\, \tau^{-2\,(1-p)}\,, \tag{22a}$$

$$\mathcal{K} = \frac{4}{27}\, p^2\, (9 - 24\, p + 17\, p^2)\, \frac{1}{\alpha^2}\, \tau^{-4\,(1-p)}\,. \tag{22b}$$

5. Metric determinant: Cosmology

Assuming $\mu > 0$ and $\lambda < \mu$, the analytic Friedmann-type solution with $r_{\text{vac}} = 0$ has parameters:

$$\alpha_{\text{F-sol}} > 0\,, \quad p_{\text{F-sol}} = \frac{2}{3 + w_M}\,, \quad \widehat{r}_{\text{F-sol}} = \left[\frac{1}{2\,\zeta}\,(\mu - \lambda)\right]^{1/3}\,, \tag{23a}$$

$$m_{\text{F-sol}} = \frac{2\,(1 + w_M)}{3 + w_M}\,, \quad \widehat{\chi}_{\text{F-sol}} = \frac{1}{6\,\pi\,(3 + w_M)^2}\,, \tag{23b}$$

so that the Ricci and Kretschmann scalars (22) drop to 0 asymptotically.

Assuming $\mu > 0$ and $0 < \lambda < \mu$, a particular analytic deSitter-type solution with $r_{\text{vac}} = \lambda$ has parameters:

$$\alpha_{\text{deS-sol}} = \frac{1}{24\,\pi\,\lambda}\,, \quad p_{\text{deS-sol}} = 1\,, \quad \widehat{r}_{\text{deS-sol}} = \sqrt[3]{\frac{\mu}{2\,\zeta}}\,, \tag{24a}$$

$$\widehat{\chi}_{\text{deS-sol}} = 0\,, \tag{24b}$$

so that the Ricci and Kretschmann curvature scalars (22) are constant.

5. Metric determinant: Cosmology

The ODEs (20) give a constant vacuum energy density, $\dot{r}_{\text{vac}} = 0$. But, with $r_{\text{vac}} > 0$ initially, particle creation by the spacetime curvature [24] will result in a decrease of r_{vac} and an increase of r_χ.

The modified ODEs with vacuum-matter energy exchange are given by

$$\dot{r}_\chi + 4 \left(\frac{\dot{r}}{r} \right) r_\chi = \Gamma \,, \tag{25a}$$

$$\dot{r}_{\text{vac}} = -\Gamma \,, \tag{25b}$$

$$3 \left(\frac{\dot{r}}{r} \right)^2 = 8\,\pi\, a \left(r_\chi + r_{\text{vac}} \right) , \tag{25c}$$

$$\frac{1}{8\,\pi\, a} \left[\frac{2\,\ddot{r}}{r} + 2 \left(\frac{\dot{r}}{r} \right)^2 - \left(\frac{\dot{r}}{r} \right) \left(\frac{\dot{a}}{a} \right) \right] = \frac{4}{3}\, r_{\text{vac}} \,, \tag{25d}$$

$$r_{\text{vac}} = \lambda + 2\,\zeta\, \sqrt{a}\, |r|^3 - \mu \,. \tag{25e}$$

5. Metric determinant: Cosmology

As the left-hand side of (25d) is proportional to the Ricci scalar, we have $\mathcal{R} \propto r_{\text{vac}}$. Therefore, we can write the Zeldovich–Starobinsky-type [24, 8] source term ($\Gamma \propto \mathcal{R}^2$) as the following simpler expression:

$$\Gamma = \widetilde{\gamma}\, |\dot{r}/r|\, \left(r_{\text{vac}}\right)^2 , \tag{26a}$$

$$\widetilde{\gamma}(\tau) = \gamma \left[\frac{\tau^2 - \tau_{\text{bcs}}^2}{\tau^2 + 1}\right]^2 , \tag{26b}$$

$$\gamma \geq 0\,, \tag{26c}$$

where we have added a smooth switch-on function $\widetilde{\gamma}(\tau)$ for initial boundary conditions at $\tau = \tau_{\text{bcs}}$, in order to ease the numerical evaluation of the ODEs.

5. Metric determinant: Cosmology

We have obtained numerical solutions of the ODEs (25) with source term (26), for initial boundary conditions at or near the analytic Friedmann-type solution, which also holds for nonzero positive γ.

We have also obtained numerical solutions for initial boundary conditions from the analytic deSitter-type solution, which is a solution only for the $\gamma = 0$ case.

Full numerical results are given in Ref. [23]. Here, we only present some results with a start from the deSitter-type solution of the unmodified ODEs (20). $\rightarrow$ Figs. 1–2

5. Metric determinant: Cosmology

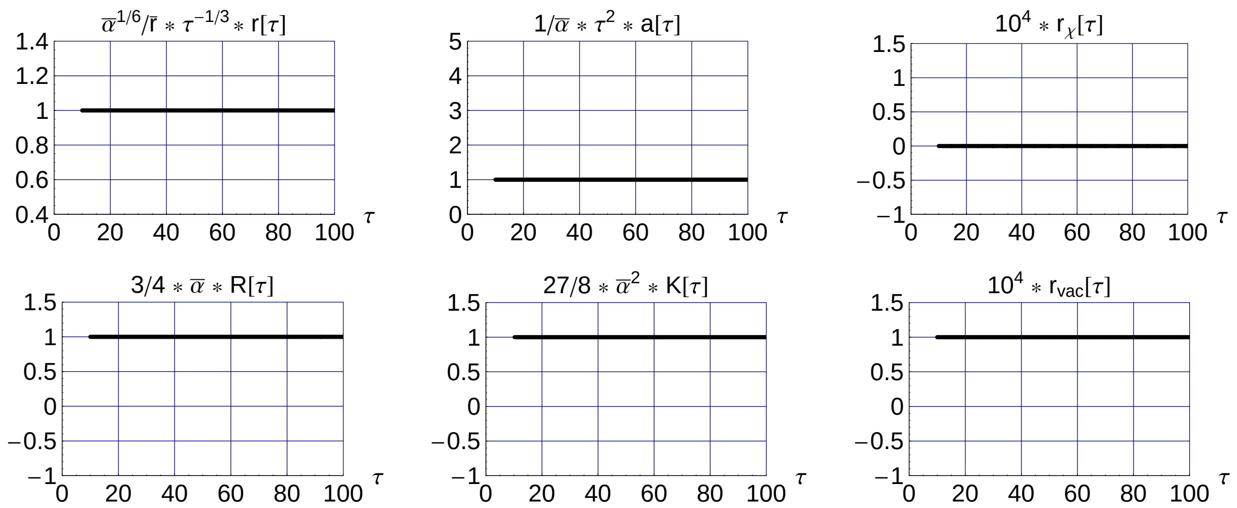

Fig. 1: Numerical solution of the modified ODEs (25) with source term (26) and parameters $w_M = 1/3$, $\zeta = 1$, $\mu = 3$, $\lambda = 10^{-4}$, and $\gamma = 0$ (quantum-dissipative effects turned off). The initial boundary conditions are taken from the analytic de-Sitter-type solution (21) and (24), having $\overline{\alpha} \equiv \alpha_{\text{deS-sol}} = 132.629$ and $\overline{r} \equiv r_{\text{deS-sol}} = 1.14471$. The top row shows the three basic variables: the metric functions $r(\tau)$ and $a(\tau)$ and the dimensionless energy density r_χ. The bottom row shows three derived quantities: the dimensionless Ricci curvature scalar $\mathcal{R}$, the dimensionless Kretschmann curvature scalar $\mathcal{K}$, the dimensionless gravitating vacuum energy density r_{vac} from (25e).

5. Metric determinant: Cosmology

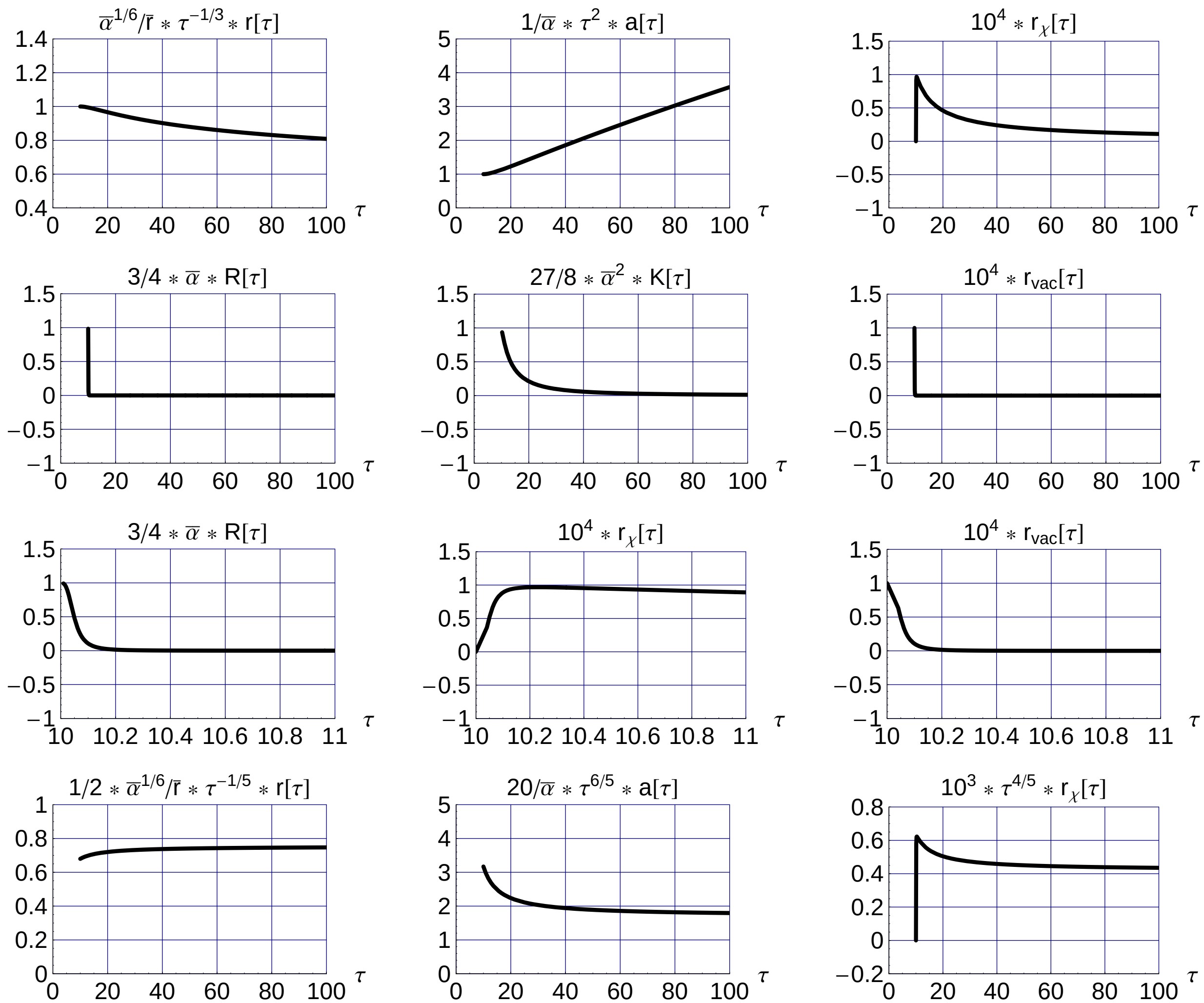

Fig. 2: Same as Fig. 1, but now with $\gamma = 2 \times 10^{11}$ (quantum-dissipative effects turned on).

6 Conclusions

To summarize, the q–theory approach to the main Cosmological Constant Problem (CCP1) provides a solution. For the moment, this is only a *possible solution*, because it is not known for sure that the "beyond-the-Standard-Model" physics contains such a q–type variable.

GENERAL REMARK: it is clear that the SM harbors huge vacuum energy densities, which somehow need to be cancelled by new d.o.f., possibly related to the fundamental theory of spacetime and gravity.

BAD NEWS: nothing is known for sure about these fundamental d.o.f.

GOOD NEWS: even though the detailed (high-energy) microphysics is unknown, it may be possible to describe the macroscopic (low-energy) effects along the lines of q–theory, just as for the hydrodynamics of water.

GOOD NEWS (cont.): it is also possible that gravity is an *emerging phenomenon* and that the metric determinant can play the role of a q-type variable and cancel the cosmological constant.

8. References

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