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New type of traversable wormhole

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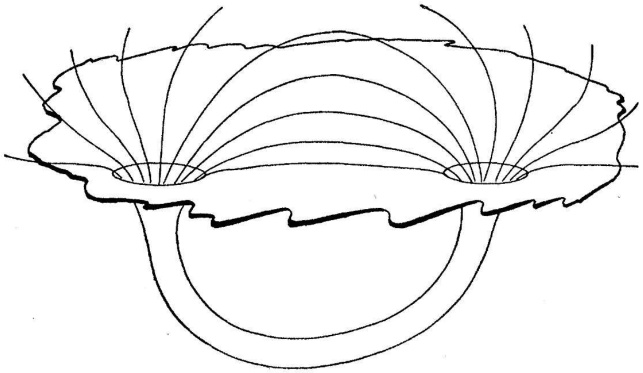
1. Introduction

BRIEF HISTORY:

- 1915: Einstein's general relativity (GR).
- 1935: Einstein-Rosen bridge connecting different parts of spacetime, but a traveller cannot go across, as he/she will run into a black hole.
- 1950's: Wheeler's spacetime foam teeming with (Euclidean) wormholes of Planckian length scales ($l_P \equiv \sqrt{\hbar G/c^3} \sim 10^{-35}$ m). Here, an emblematic picture from Wheeler's 1955 "Geons" paper:



- 1988: Morris&Thorne's result that certain Lorentzian wormholes may, in principle, be **traversable**, but at a high price: **exotic matter**.

1. Introduction

OUTLINE:

1. Introduction
2. Exotic-matter wormhole
3. New wormhole
4. New wormhole – Details
5. New wormhole – General Ansatz
6. New wormhole – Two final remarks
7. References

2. Exotic-matter wormhole

Morris and Thorne (MT) have discussed a simple metric in Box 2 of their 1988 paper. Specifically, this special case of a more general metric is given by (setting $c = 1$)

$$\begin{aligned} ds^2 \Big|^{\text{(EBMT-worm-spec)}} &\equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu \Big|^{\text{(EBMT-worm-spec)}} \\ &= -dt^2 + dl^2 + \left(b_0^2 + l^2\right) \left[d\theta^2 + \sin^2\theta\, d\phi^2\right], \quad (1) \end{aligned}$$

with a nonzero real constant b_0 (taken to be positive, for definiteness).

The coordinates t and l in (1) range over $(-\infty,\, \infty)$ and $\theta \in [0,\, \pi]$ and $\phi \in [0,\, 2\pi)$ are the standard spherical polar coordinates [strictly speaking, we should use *two* coordinate patches for the 2-sphere, for example, by stereographic projections from the North Pole and the South Pole].

Earlier discussions of this type of metric have appeared in Ellis-1973 and Bronnikov-1973. Hence, we have added “EB” to the suffix of (1).

2. Exotic-matter wormhole

The resulting Ricci and Kretschmann curvature scalars are

$$R\Big|^{\text{(EBMT-worm-spec)}} = -2\,\frac{b_0^2}{\left(b_0^2+l^2\right)^2}\,, \tag{2a}$$

$$K\Big|^{\text{(EBMT-worm-spec)}} = 12\,\frac{\left(b_0^2\right)^2}{\left(b_0^2+l^2\right)^4}\,, \tag{2b}$$

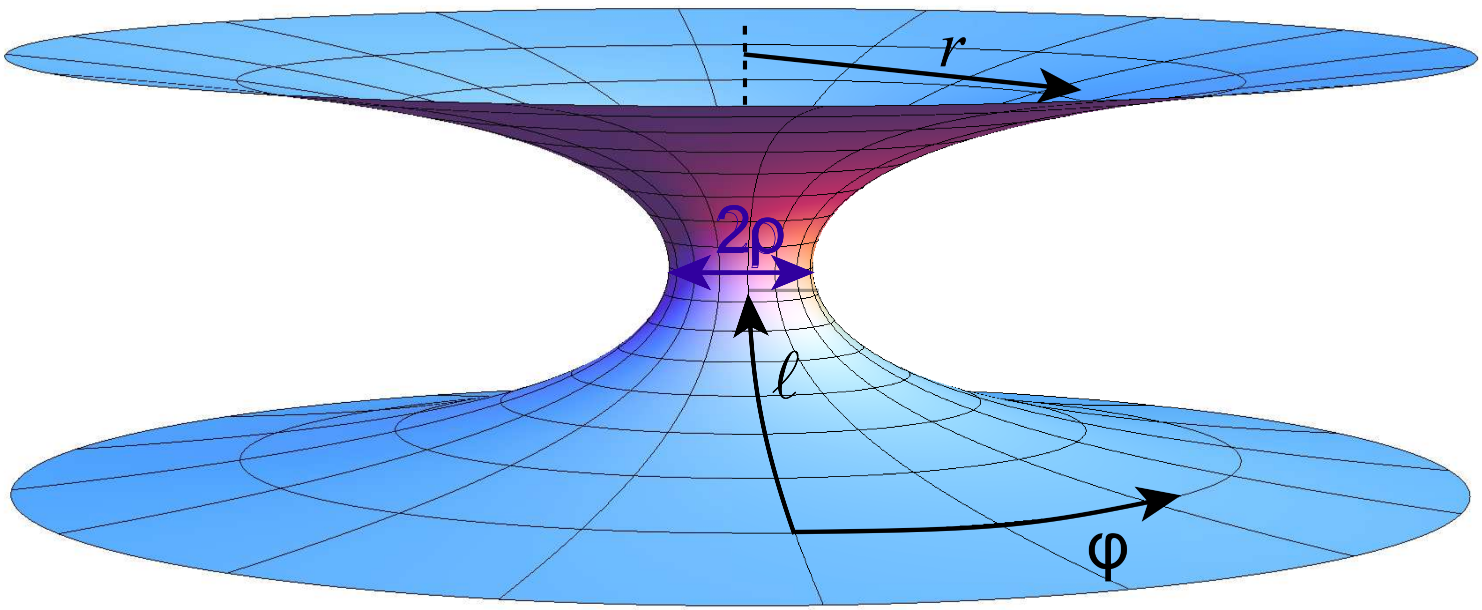
both of which are seen to vanish as $l \to \pm\infty$.

Indeed, two distinct flat Minkowski spacetimes are approached for $l \to \pm\infty$. $\rightarrow$ Figure

This EBMT wormhole is traversable, as shown by items (d) and (e) of Box 2 in Morris–Thorne-1988 (see also Fig. 6 in Ellis-1973).

2. Exotic-matter wormhole

Embedding diagram ($t =$ const, $\theta = \pi/2,\ 2\,\rho \equiv 2\,b_0 > 0,\ r \equiv \sqrt{b_0^2 + l^2}\,$):



[Image credit: O. James, E. von Tunzelmann, P. Franklin, and K. S. Thorne, “Visualizing Interstellar’s Wormhole, Am. J. Phys. **83**, 486 (2015), arXiv:1502.03809]

2. Exotic-matter wormhole

The crucial question, however, concerns the **dynamics**:

can this wormhole metric be a solution of the Einstein equation?

Morris and Thorne's brilliant idea was to use an **engineering approach**:

fix the desired specifications and see what it takes.

2. Exotic-matter wormhole

Fixing the metric to (1) for a traversable wormhole, the Einstein equation, $R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi G\, T_{\mu\nu}$, then requires the following components of the energy-momentum tensor [MT-1988]:

$$T^{\,t}{}_{t}\,\Big|^{\text{(EBMT-worm-spec)}} = \frac{1}{8\pi G}\,\frac{b_0^2}{\left(b_0^2+l^2\right)^2}\,, \tag{3a}$$

$$T^{\,l}{}_{l}\,\Big|^{\text{(EBMT-worm-spec)}} = -\frac{1}{8\pi G}\,\frac{b_0^2}{\left(b_0^2+l^2\right)^2}\,, \tag{3b}$$

$$T^{\,\theta}{}_{\theta}\,\Big|^{\text{(EBMT-worm-spec)}} = \frac{1}{8\pi G}\,\frac{b_0^2}{\left(b_0^2+l^2\right)^2}\,, \tag{3c}$$

$$T^{\,\phi}{}_{\phi}\,\Big|^{\text{(EBMT-worm-spec)}} = \frac{1}{8\pi G}\,\frac{b_0^2}{\left(b_0^2+l^2\right)^2}\,, \tag{3d}$$

with all other components vanishing.

2. Exotic-matter wormhole

As the energy density is given by $\rho = T^{\,tt} = -T^{\,t}_{\ \ t}$, we have $\rho < 0$ from (3a), which definitely corresponds to exotic matter.

For the radial null vector $\overline{k}^{\,\mu} = (1,\,1,\,0,\,0)$, we obtain the inequality

$$T^{\,\mu}_{\ \ \nu}\,\overline{k}_{\mu}\,\overline{k}^{\,\nu}\,\Big|^{\text{(EBMT-worm-spec)}} = \frac{1}{8\pi G}\,\frac{b_0^2}{\left(b_0^2+l^2\right)^2}\,\Big[-1-1\Big] < 0\,, \qquad (4)$$

which corresponds to a violation of the Null-Energy-Condition (NEC).

Problem: not clear if the needed exotic matter exists.

3. New wormhole

We now propose a somewhat different metric:

$$ds^2\Big|^{\text{(K-worm-spec)}} = -dt^2 + \frac{\xi^2}{\xi^2+\lambda^2}\,d\xi^2 + \left(b_0^2+\xi^2\right)\,\left[d\theta^2 + \sin^2\theta\,d\phi^2\right], \quad (5)$$

with nonzero real constants λ and b_0 (both taken to be positive, for definiteness) and coordinates t and ξ ranging over $(-\infty,\,\infty)$.

The resulting Ricci and Kretschmann curvature scalars are

$$R\Big|^{\text{(K-worm-spec)}} = -2\,\frac{b_0^2-\lambda^2}{\left(b_0^2+\xi^2\right)^2}\,, \quad (6a)$$

$$K\Big|^{\text{(K-worm-spec)}} = 12\,\frac{\left(b_0^2-\lambda^2\right)^2}{\left(b_0^2+\xi^2\right)^4}\,, \quad (6b)$$

both of which are finite, smooth, and vanishing as $\xi \to \pm\infty$.

3. New wormhole

The metric $g_{\mu\nu}(x)$ from (5) is **degenerate** with a vanishing determinant $g(x) \equiv \det[g_{\mu\nu}(x)]$ at $\xi = 0$.[‡]

In physical terms, this 3-dimensional hypersurface at $\xi = 0$ corresponds to a "spacetime defect" and the Einstein equation is defined at $\xi = 0$ by continuous extension from its limit $\xi \to 0$.

The terminology "spacetime defect" is by analogy with crystallographic defects in an atomic crystal (these crystallographic defects are typically formed during a rapid crystallization process).

The new wormhole metric (5) did not fall out of the sky but is a direct follow-up of earlier work on a particular "time defect" that regularizes the big bang [Klinkhamer, 2019].

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‡ The metric from (1) is nondegenerate, as its determinant vanishes nowhere, provided two coordinate patches are used for the 2-sphere.

3. New wormhole

Using Morris and Thorne's engineering approach, the Einstein equation, $R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi G\, T_{\mu\nu}$, now requires for this new metric:

$$T^{\,t}_{\;\;t}\Big|^{\text{(K-worm-spec)}} = \frac{1}{8\pi G}\,\frac{b_0^2-\lambda^2}{\left(b_0^2+\xi^2\right)^2}\,, \tag{7a}$$

$$T^{\,\xi}_{\;\;\xi}\Big|^{\text{(K-worm-spec)}} = -\frac{1}{8\pi G}\,\frac{b_0^2-\lambda^2}{\left(b_0^2+\xi^2\right)^2}\,, \tag{7b}$$

$$T^{\,\theta}_{\;\;\theta}\Big|^{\text{(K-worm-spec)}} = \frac{1}{8\pi G}\,\frac{b_0^2-\lambda^2}{\left(b_0^2+\xi^2\right)^2}\,, \tag{7c}$$

$$T^{\,\phi}_{\;\;\phi}\Big|^{\text{(K-worm-spec)}} = \frac{1}{8\pi G}\,\frac{b_0^2-\lambda^2}{\left(b_0^2+\xi^2\right)^2}\,. \tag{7d}$$

3. New wormhole

Compared to the previous results (3), we see that the previous factors b_0^2 in the numerators have been replaced by new factors $(b_0^2 - \lambda^2)$ in (7). Starting from $\lambda^2 = 0^+$, these new numerator factors then change sign as λ^2 increases above b_0^2 and we no longer require exotic matter.

Indeed, we have from (7a) that $\rho = -T^{\,t}_{\;\;t} > 0$ for $\lambda^2 > b_0^2$. Moreover, we readily obtain, for any null vector $k^{\,\mu}$ and parameters $\lambda^2 \geq b_0^2$, the inequality

$$T^{\,\mu}_{\;\;\nu}\, k_\mu\, k^{\,\nu}\,\Big|^{\text{(K-worm-spec)}}_{\lambda^2 \geq b_0^2} \geq 0\,, \tag{8}$$

which verifies the NEC.

$\boxed{\text{New wormhole with degenerate metric (5) for } \lambda^2 \geq b_0^2 \text{ does not need exotic matter.}}$

3. New wormhole

In fact, the special case $\lambda^2 = b_0^2$ has the energy-momentum tensor vanishing altogether,

$$T^{\mu}{}_{\nu}\Big|^{\text{(K-worm-spec)}}_{\lambda^2=b_0^2} = 0\,, \tag{9}$$

and so do the curvature scalars (6).

In other words, we have:

$$\boxed{\text{an exact wormhole-type solution of the vacuum Einstein equation.}}$$

The corresponding spacetime is flat but different from Minkowski spacetime. How can that be? The short answer: different topology.

Time permitting, here are some details . . .

4.1 New wormhole – Other coordinate

Changing the spatial ξ coordinate to

$$\widetilde{l} = \xi\,\sqrt{1+\lambda^2/\xi^2} \in (-\infty,\,-\lambda] \cup [\lambda,\,\infty) \tag{10}$$

gives a metric similar to (1),

$$ds^2 = -dt^2 + d\widetilde{l}^{\,2} + \left(b_0^2 + \widetilde{l}^{\,2} - \lambda^2\right)\,\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]. \tag{11}$$

But this coordinate transformation $\xi \to \widetilde{l}$ is <u>not</u> a diffeomorphism.

Remark also that the coordinate $\widetilde{l}$ is unsatisfactory for a proper description of the whole spacetime manifold, because, for given values of $\{t,\,\theta,\,\phi\}$, <u>both</u> coordinates $\widetilde{l} = -\lambda$ and $\widetilde{l} = \lambda$ correspond to a <u>single</u> point of the manifold (with the single coordinate $\xi = 0$).

Still, we can get a useful picture as an “amputated” version of the surface on slide 6 with l replaced by $\widetilde{l}$ and b_0^2 by $b_0^2 - \lambda^2$.

4.2 New wormhole – Topology/orientability

With the coordinates $\{\widetilde{l},\, \theta,\, \phi\}$ in the metric (11) for general $\lambda > 0$ and $b_0 > 0$, we get the following two sets of Cartesian coordinates [one for the "upper" (+) universe and another for the "lower" (-) universe]:

$$\left\{ \begin{array}{c} Z_+ \\ Y_+ \\ X_+ \end{array} \right\} = \widetilde{l} \left\{ \begin{array}{l} \cos\theta \\ \sin\theta\, \sin\phi \\ \sin\theta\, \cos\phi \end{array} \right\}, \quad \text{for } \widetilde{l} \geq \lambda > 0\,, \tag{12a}$$

$$\left\{ \begin{array}{c} Z_- \\ Y_- \\ X_- \end{array} \right\} = \widetilde{l} \left\{ \begin{array}{l} \cos\theta \\ \sin\theta\, \sin\phi \\ \sin\theta\, \cos\phi \end{array} \right\}, \quad \text{for } \widetilde{l} \leq -\lambda < 0\,, \tag{12b}$$

$$\{Z_+,\, Y_+,\, X_+\} \;\hat{=}\; \{Z_-,\, Y_-,\, X_-\}\,, \quad \text{for } |\,\widetilde{l}\,| = \lambda\,, \tag{12c}$$

where the last relation implements the identification of "antipodal" points on the two 2-spheres $S^2_\pm$ with $|\,\widetilde{l}\,| = \lambda$.

4.2 New wormhole – Topology/orientability

Note that the two coordinates sets $\{Z_\pm,\, Y_\pm,\, X_\pm\}$ from (12a) and (12b) have **different orientation**.

The spatial topology of our degenerate-wormhole spacetime (5) is that of two copies of the Euclidean space E_3 with the interior of two balls removed and “antipodal” identification (12c) of their two surfaces.

It can be verified that the defect-wormhole spacetime from (5) and (12) is **simply connected** (all loops in space are contractible to a point).

The defect-wormhole topology is different from that of the original exotic-matter EBMT wormhole, which is multiply connected (there are noncontractible loops in space, for example, a loop in the upper universe encircling the wormhole mouth).

For the moment, we stick to the “antipodal” identification from (12c), with the corresponding change of orientation at $\xi = 0$.

4.3 New wormhole – Radial geodesics

We can get explicitly the radial geodesics $\xi(t)$ passing through the vacuum-wormhole throat at $\xi = 0$:

$$\theta(t)\Big|^{\text{(K-worm-spec)}}_{\text{vacuum sol; rad-geod}} = \pi/2\,, \tag{13a}$$

$$\phi(t)\Big|^{\text{(K-worm-spec)}}_{\text{vacuum sol; rad-geod}} = 0\,, \tag{13b}$$

$$\xi(t)\Big|^{\text{(K-worm-spec)}}_{\text{vacuum sol; rad-geod}} = \begin{cases} \pm\sqrt{(B\,t)^2 + 2\,B\,\lambda\,t}\,, & \text{for}\;\; t \geq 0\,, \\ \mp\sqrt{(B\,t)^2 - 2\,B\,\lambda\,t}\,, & \text{for}\;\; t \leq 0\,, \end{cases} \tag{13c}$$

with a dimensionless constant $B \in (0,\,1]$ and different signs (upper or lower) in front of the square roots for motion in opposite directions.

4.3 New wormhole – Radial geodesics

The radial geodesic (13) with the upper signs has the following trajectory in terms of the Cartesian coordinates (12):

$$Z_{\pm}(t)\Big|^{\text{(K-worm-spec)}}_{\text{vacuum sol ; rad-geod}} = 0\,, \qquad \text{for} \;\; t \in (-\infty,\, \infty)\,, \tag{14a}$$

$$Y_{\pm}(t)\Big|^{\text{(K-worm-spec)}}_{\text{vacuum sol ; rad-geod}} = 0\,, \qquad \text{for} \;\; t \in (-\infty,\, \infty)\,, \tag{14b}$$

$$X_{-}(t)\Big|^{\text{(K-worm-spec)}}_{\text{vacuum sol ; rad-geod}} = -\lambda + B\,t\,, \qquad \text{for} \;\; t \leq 0\,, \tag{14c}$$

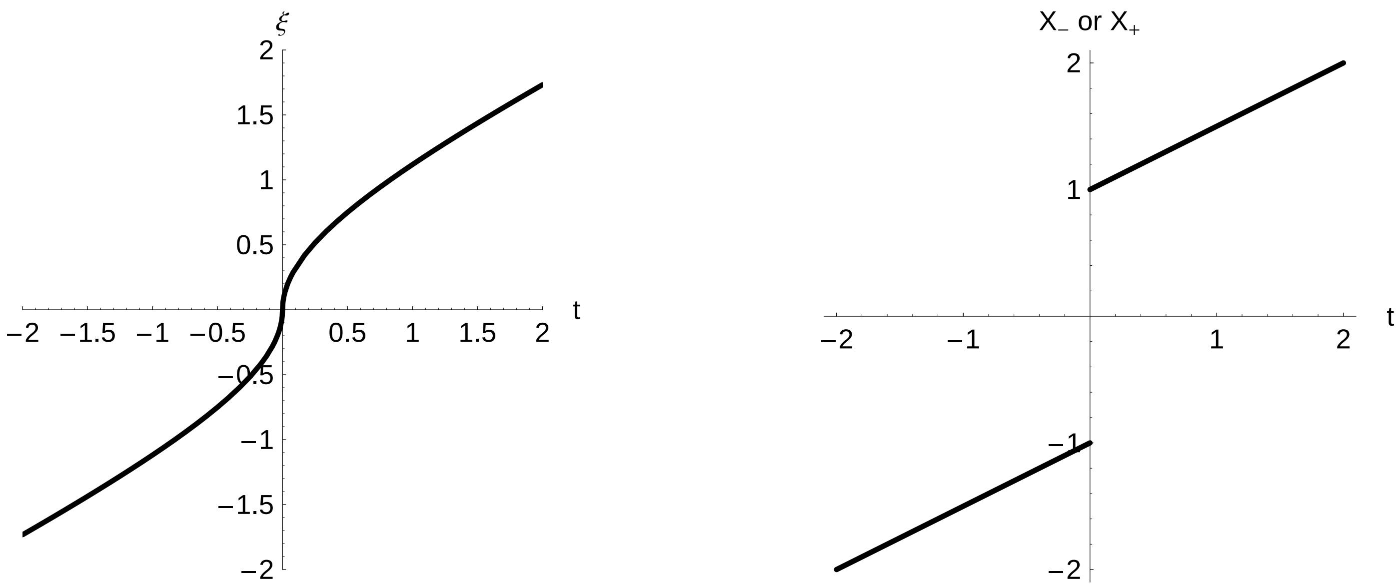
$$X_{+}(t)\Big|^{\text{(K-worm-spec)}}_{\text{vacuum sol ; rad-geod}} = +\lambda + B\,t\,, \qquad \text{for} \;\; t \geq 0\,, \tag{14d}$$

with $X_{-} = -\lambda$ and $X_{+} = +\lambda$ identified at $t = 0$.

4.3 New wormhole – Radial geodesics

The curves in the $(t,\, X_-)$ and $(t,\, X_+)$ planes, have two parallel straight-line segments, shifted at $t = 0$, with equal constant positive slope $B \leq 1$ (velocity magnitude in units with $c = 1$).

Plot of the radial geodesic (13)–(14) with $\lambda = 1$ and $B = 1/2$:



4.3 New wormhole – Radial geodesics

This equal Minkowski-space velocity before and after the defect crossing is the main argument for using the “antipodal” identification in (12), rather than a “direct” identification on the two 2-spheres $S^2_\pm$, which would correspond to replacing the prefactors $\widetilde{l}$ in (12a) and (12b) by $|\,\widetilde{l}\,|$ and would have a unique spatial orientation (but apparently nonsmooth motion along defect-crossing geodesics).

Such a “direct” identification of the points on the two 2-spheres at $|\,\widetilde{l}\,| = \lambda$ would match the original exotic-matter wormhole from (1), with surgery to remove the open l interval $(-\lambda,\, +\lambda)$.

As said before, we stick to the “antipodal” identification from (12c), with the corresponding change of orientation at $\xi = 0$.

4.4 New wormhole – Travel advisory

WARNING:

If the “advanced civilization” of Morris-Thorne1988 has access to our type of defect-wormhole, then it should perhaps start exploration by sending in parity-invariant machines or robots.

The reason is that humans of finite size and with right-handed DNA may not be able to pass safely through this particular wormhole-throat defect at $\xi = 0$, which separates two universes with different 3-space orientation.

5.1 New wormhole – General Ansatz

The special degenerate metric (5) can be generalized as follows:

$$ds^2\,\Big|^{\text{(K-worm-gen)}} = -e^{2\,\widetilde{\phi}(\xi)}\;dt^2 + \frac{\xi^2}{\xi^2+\lambda^2}\;d\xi^2 + \widetilde{r}^{\;2}(\xi)\,\Big[d\theta^2 + \sin^2\theta\,d\phi^2\Big]\,, \tag{15}$$

with a positive length scale λ and real functions $\widetilde{\phi}(\xi)$ and $\widetilde{r}(\xi)$. Again, the coordinates t and ξ range over $(-\infty,\,\infty)$, while $\theta \in [0,\,\pi]$ and $\phi \in [0,\,2\pi)$ are the standard spherical polar coordinates [as mentioned before, we should really use two coordinate patches for the 2-sphere].

If we assume that $\widetilde{\phi}(\xi)$ remains finite everywhere and that $\widetilde{r}(\xi)$ is positive with $\widetilde{r}(\xi) \sim |\xi|$ for $\xi \to \pm\infty$, then the spacetime from (15) corresponds to a wormhole.

5.1 New wormhole – General Ansatz

If the global minimum of the function $\widetilde{r}(\xi)$ has the value $b_0 > 0$ at $\xi = \xi_0 \equiv 0$ and if the function $\widetilde{\phi}(\xi)$ is essentially constant near $\xi = 0$, then we expect interesting behavior for λ^2 of the order of b_0^2 or larger.

In fact, using power series in ξ^2 for $\widetilde{\phi}(\xi)$ and $\widetilde{r}^{\,2}(\xi)$, we get energy-momentum components without singular behavior at $\xi = 0$.

Work in progress . . .

5.2 New wormhole – Recap vacuum solution

Awaiting the general analysis, we recall that we already have an exact wormhole-type solution of the vacuum Einstein gravitational field equation, as discussed on slide 14 for the special-case metric.

In terms of the general *Ansatz* (15), the solution reads

$$\left\{\widetilde{\phi}(\xi),\, \widetilde{r}^{\,2}(\xi)\right\}\Big|^{\text{(K-worm-gen)}}_{\text{vacuum sol}} = \left\{0,\, \lambda^2+\xi^2\right\}, \tag{16a}$$

$$T^{\mu}{}_{\nu}(\xi)\Big|^{\text{(K-worm-gen)}}_{\text{vacuum sol}} = 0\,. \tag{16b}$$

Unlike Minkowski spacetime, this flat vacuum-wormhole spacetime has asymptotically two flat 3-spaces with different orientations.

6. New wormhole – Two final remarks

First, the vacuum-wormhole solution (16) has the length scale λ as a free parameter and, if there is a preferred value $\overline{\lambda}$ in Nature, then that value can only come from a theory beyond GR.

An example of such a theory would be nonperturbative superstring theory in the formulation of the IKKT matrix model [Ishibashi–Kawai–Kitazawa–Tsuchiya, 1997], also known as IIB matrix model.

That matrix model could give rise to an emergent spacetime with or without spacetime defects [Klinkhamer, 2021].

If defects do appear, then the typical length scale $\overline{\lambda}$ of a remnant vacuum-wormhole defect would be related to the IIB-matrix-model length scale ℓ (the Planck length $l_P \propto G^{1/2}$ might also be related to ℓ).

6. New wormhole – Two final remarks

Second, the main objective of the present talk has been to reduce the hurdles to overcome in the quest of traversable wormholes. Specifically, we have removed the requirement of exotic matter.

But there remains, at least, one important hurdle, namely to construct a suitable spacetime defect or to harvest one, if already present as a remnant from an early phase.

7. References

[1] M.S. Morris and K.S. Thorne, “Wormholes in space-time and their use for interstellar travel: A tool for teaching general relativity,” Am. J. Phys. **56**, 395 (1988).

[2] H.G. Ellis, “Ether flow through a drainhole: A particle model in general relativity,” J. Math. Phys. **14**, 104 (1973).

[3] K.A. Bronnikov, “Scalar-tensor theory and scalar charge,” Acta Phys. Polon. B **4**, 251 (1973).

[4] F.R. Klinkhamer, Traversable wormhole without exotic matter, arXiv:2301.00724.

[5] F.R. Klinkhamer, “Regularized big bang singularity,” Phys. Rev. D **100**, 023536 (2019), arXiv:1903.10450.

[6] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, “A large-N reduced model as superstring,” Nucl. Phys. B **498**, 467 (1997), arXiv:hep-th/9612115.

[7] F.R. Klinkhamer, “IIB matrix model: Emergent spacetime from the master field,” Prog. Theor. Exp. Phys. **2021**, 013B04 (2021), arXiv:2007.08485.