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M-theory and the birth of the Universe

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Admittedly, a rather formidable title...

For now, no definitive answers, only explorative results.

Talk based on arXiv:2009.06525, further references on slides 13–14.

1. Standard FLRW cosmology

The Einstein gravitational field equation of general relativity (GR) reads [1]:

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = -8\pi G\, T^{\text{(SM)}}_{\mu\nu}\,, \tag{1}$$

with $R_{\mu\nu}$ the Ricci tensor, R the Ricci scalar, $T_{\mu\nu}$ the energy-momentum tensor of the matter (Standard Model), and G Newton's gravitational coupling constant. The spacetime indices μ, ν run over $\{0,\, 1,\, 2,\, 3\}$.

For cosmology, the spatially flat Robertson–Walker metric is

$$ds^2\,\Big|^{\text{(RW)}} \equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu\,\Big|^{\text{(RW)}} = -dt^2 + a^2(t)\; \delta_{ij}\, dx^i\, dx^j\,, \tag{2}$$

with $x^0 = c\, t$ and $c = 1$. The spatial indices i, j run over $\{1,\, 2,\, 3\}$.

1. Standard FLRW cosmology

For a homogeneous perfect fluid with energy density $\rho_M(t)$ and pressure $P_M(t)$, we get the spatially flat Friedmann equations [1]:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho_M\,, \tag{3a}$$

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G\,P_M\,, \tag{3b}$$

$$\dot{\rho}_M + 3\,\frac{\dot{a}}{a}\,\Big[\rho_M + P_M\Big] = 0\,, \tag{3c}$$

$$P_M = P_M\big(\rho_M\big)\,, \tag{3d}$$

where the overdot stands for differentiation with respect to t and (3d) corresponds to the equation-of-state (EOS) relation between pressure and energy density of the fluid.

1. Big bang in FLRW cosmology

For relativistic matter with constant EOS parameter $w_M \equiv P_M/\rho_M = 1/3$, the Friedmann–Lemaître–Robertson–Walker solution $a(t)$ is given by [1]

$$a(t)\Big|_{\text{FLRW}}^{(w_M=1/3)} = \sqrt{t/t_0}\,, \qquad \text{for} \quad t > 0\,, \tag{4a}$$

$$\rho_M(t)\Big|_{\text{FLRW}}^{(w_M=1/3)} = \rho_{M0}/a^4(t) \propto 1/t^2\,, \qquad \text{for} \quad t > 0\,, \tag{4b}$$

where the cosmic scale factor has normalization $a(t_0) = 1$ at $t_0 > 0$.

The FLRW solution displays the **big bang singularity** for $t \to 0^+$,

$$\lim_{t\to 0^+} a(t) = 0\,, \tag{5}$$

with diverging curvature and energy density. But, at $t = 0$, the theory (GR+SM) is no longer valid and we can ask what happens <u>really</u> at the big bang? Or, more precisely, <u>how to describe the birth of the Universe</u>?

2. Regularized big bang

First, let us try to control the divergences by considering a new *Ansatz* for a "regularized" big bang [2]:

$$
\begin{aligned}
ds^2 \Big|^{\text{(reg-bb)}} &\equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu \Big|^{\text{(reg-bb)}} \\
&= -\frac{t^2}{t^2+b^2}\, dt^2 + a^2(t)\, \delta_{ij}\, dx^i\, dx^j\,, \qquad \text{(6a)} \\
b^2 &> 0\,, \qquad \text{(6b)} \\
a^2(t) &> 0\,, \qquad \text{(6c)} \\
t &\in (-\infty,\, \infty)\,, \quad x^i \in (-\infty,\, \infty)\,, \qquad \text{(6d)}
\end{aligned}
$$

where we set $x^0 = c\,t$ and $c = 1$.

This metric $g_{\mu\nu}(x)$ is **degenerate**, with a vanishing determinant at $t = 0$.

2. Regularized big bang

With the standard Einstein equation (1) and a homogeneous perfect fluid, get **modified** spatially flat Friedmann equations:

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\,\rho_M\,, \tag{7a}$$

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\ddot{a}}{a}+\frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2\right)-\frac{b^2}{t^3}\,\frac{\dot{a}}{a}=-4\pi G\,P_M\,, \tag{7b}$$

$$\dot{\rho}_M+3\,\frac{\dot{a}}{a}\,\Big[\rho_M+P_M\Big]=0\,, \tag{7c}$$

$$P_M=P_M\big(\rho_M\big)\,, \tag{7d}$$

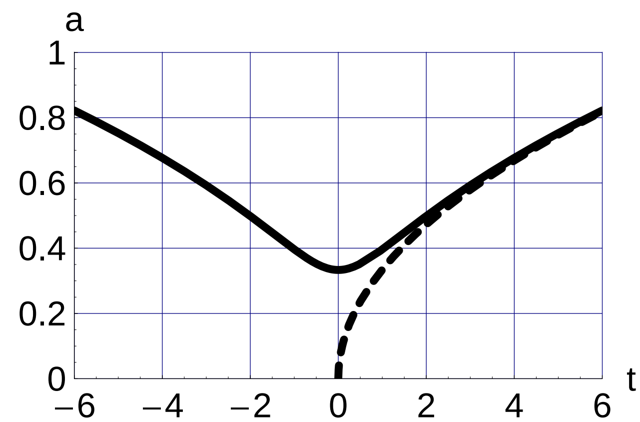
where the overdot stands again for differentiation with respect to t.

2. Bounce or new phase?

For constant EOS parameter $w_M = 1/3$, the new solution $a(t)$ is

$$a(t)\Big|^{(w_M=1/3)}_{\text{mod. FLRW}} = \sqrt[4]{\left(t^2+b^2\right)/\left(t_0^2+b^2\right)}\,, \tag{8}$$

which is **perfectly smooth** at $t = 0$ as long as $b \neq 0$. Figure compares with the singular FLRW solution, as shown by the dashed curve.



Two possibilities:

1. **nonsingular bouncing cosmology** [3, 4] from $t = -\infty$ to $t = \infty$ (valid for $b \gg l_{\text{Planck}}$?) [gravitational waves generated in the pre-bounce epoch keep on propagating into the postbounce epoch];

2. **new phase** at $t = 0$ pair-produces [5] a "universe" for $t > 0$ and an "antiuniverse" for $t < 0$ (valid for $b \sim l_{\text{Planck}}$?). $\Leftarrow$ THIS TALK

3. IIB matrix model

For an explicit description of such a new phase, we can use the IIB matrix model [6, 7], which has been suggested as a nonperturbative definition of superstring theory (M–theory).

The model has $N \times N$ traceless Hermitian matrices, ten bosonic matrices A^μ and essentially eight fermionic (Majorana–Weyl) matrices Ψ_α.

The partition function Z is defined by a “path” integral [6, 7, 8]:

$$Z = \int dA\, d\Psi\;\; \exp\left(i\, S/\ell^4\right) = \int dA\;\; \exp\left(i\, S_{\text{eff}}/\ell^4\right)\;, \tag{9a}$$

$$S = -\text{Tr}\left(\frac{1}{4}\left[A^\mu,\, A^\nu\right]\left[A^\rho,\, A^\sigma\right]\widetilde{\eta}_{\mu\rho}\,\widetilde{\eta}_{\nu\sigma} + \frac{1}{2}\,\overline{\Psi}_\beta\,\widetilde{\Gamma}^\mu_{\beta\alpha}\,\widetilde{\eta}_{\mu\nu}\left[A^\nu,\, \Psi_\alpha\right]\right), \tag{9b}$$

$$\widetilde{\eta}_{\mu\nu} = \Big[\text{diag}\big(-1,\, 1,\, \ldots\,,\, 1\big)\Big]_{\mu\nu}\,. \tag{9c}$$

The model length scale “ℓ” has been introduced, so that A^μ has the dimension of length and Ψ_α the dimension of $(\text{length})^{3/2}$.

3. Emergence of a classical spacetime?

Well, the matrices A^μ and Ψ_α in (9a) are merely integration variables and there is no obvious small dimensionless parameter to motivate a saddle-point approximation, so the question is:

$\boxed{\text{where is the classical spacetime?}}$

Recently, we have suggested to revisit an old idea, the large-N master field of Witten [9], for a possible origin of classical spacetime in the context of IIB matrix model [10].

In this short talk, we have only time to remind you of this mysterious master field (name coined by Coleman) and to give you the final result.

3. Large-N factorization

The gauge-invariant bosonic observable

$$w^{\mu_1 \cdots \mu_m} = \mathrm{Tr}\left(A^{\mu_1} \ldots A^{\mu_m}\right) \tag{10}$$

has expectation values

$$\langle w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n}\ \cdots \rangle = \frac{1}{Z} \int dA \left(w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n}\ \cdots\right) e^{i\, S_{\text{eff}}/\ell^4}. \tag{11}$$

Now, the following factorization property holds to leading order in N:

$$\left\langle\, w^{\mu_1 \cdots \mu_m}\, w^{\mu_1 \cdots \mu_m} \,\right\rangle \stackrel{N}{=} \left\langle w^{\mu_1 \cdots \mu_m}\right\rangle \left\langle w^{\mu_1 \cdots \mu_m}\right\rangle, \tag{12}$$

without sums over repeated indices.

This leading-order equality (12) states that the expectation value of the square of w equals the square of the expectation value of w, which is a truly remarkable result for a statistical (quantum) theory.

3. Large-N master field

According to Witten [9], the factorization (12) implies that the path integrals (11) are saturated by a single configuration, namely by the so-called **master field** $\widehat{A}^{\mu}$.

Considering one observable w, for simplicity, we then have the following expectation value:

$$\langle w^{\mu_1 \cdots \mu_m} \rangle \stackrel{N}{=} \text{Tr}\left(\widehat{A}^{\mu_1} \ldots \widehat{A}^{\mu_m} \right), \tag{13}$$

and similarly for the other expectation values (11). Hence, we "only" need ten complex-number matrices to get all expectation values (11).

3. Emergent classical spacetime

Now, the meaning of the previous suggestion [10] is clear:

$$\boxed{\text{classical spacetime resides in the master-field matrices } \widehat{A}^{\mu}.}$$

In fact, it is possible to extract the spacetime points $\widehat{x}_k^{\mu}$ and the emergent inverse metric $g^{\mu\nu}(x)$ [the metric $g_{\mu\nu}(x)$ is obtained as matrix inverse].

It is even possible [11] that the large-N master field of the Lorentzian IIB matrix model gives rise to the regularized-big-bang metric (6) of GR.

<u>Final result</u>: effective length parameter b of the regularized-big-bang metric (6) calculated in terms of the IIB-matrix-model length scale ℓ,

$$b_{\text{eff}} \sim \ell \stackrel{?}{\sim} l_{\text{Planck}} \equiv \sqrt{\hbar\, G/c^3} \approx 1.62 \times 10^{-35}\,\text{m}\,. \tag{14}$$

<u>Details</u> skipped in this short talk ($\rightarrow$ slides after the References).

Outstanding task: get the exact IIB-matrix-model master field $\widehat{A}^{\mu}$ or, at least, a reliable approximation...

4. References

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[10] F.R. Klinkhamer, "IIB matrix model: Emergent spacetime from the master field," arXiv:2007.08485.

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A. Extraction of spacetime points

Make a particular global gauge transformation [8] on the matrices $\widehat{A}^{\mu}$ of the IIB-matrix-model master field,

$$\underline{\widehat{A}}^{\mu} = \underline{\Omega}\, \widehat{A}^{\mu}\, \underline{\Omega}^{\dagger}\,, \quad \underline{\Omega} \in SU(N)\,, \tag{15}$$

so that the transformed 0-component is diagonal and has ordered eigenvalues $\widehat{\alpha}_i \in \mathbb{R}$,

$$\underline{\widehat{A}}^{0} = \text{diag}\big(\widehat{\alpha}_1, \widehat{\alpha}_2, \ldots, \widehat{\alpha}_{N-1}, \widehat{\alpha}_N\big)\,, \tag{16a}$$

$$\widehat{\alpha}_1 \leq \widehat{\alpha}_2 \leq \ldots \leq \widehat{\alpha}_{N-1} \leq \widehat{\alpha}_N\,, \tag{16b}$$

$$\sum_{i=1}^{N} \widehat{\alpha}_i = 0\,. \tag{16c}$$

The ordering (16b) will turn out to be crucial for the time coordinate $\widehat{t}$ obtained later.

A. Extraction of spacetime points

A relatively simple procedure [10] approximates the eigenvalues of the spatial matrices $\underline{\widehat{A}}^{\,m}$ but still manages to order them along the diagonal.

This procedure corresponds, in fact, to a type of **coarse graining** of some of the information contained in the master field.

We start from the following trivial observation:
if M is an $N \times N$ Hermitian matrix, then any $n \times n$ block centered on the diagonal of M is also Hermitian, which holds for $n \geq 1$ and $n \leq N$.

Let K be an odd divisor of N, so that

$$N \;=\; K\,n\,, \quad K = 2\,L + 1\,, \tag{17}$$

where both L and n are positive integers.

A. Extraction of spacetime points

Consider, in each of the ten matrices $\widehat{\underline{A}}^{\mu}$, the K blocks of size $n \times n$ centered on the diagonal.

We already know the diagonalized blocks of $\widehat{\underline{A}}^{0}$ from (16a), which allows us to define the following time coordinate $\widehat{t}\,(\sigma)$ for $\sigma \in (0,\, 1]$:

$$\widehat{x}^{\,0}\,(k/K) \equiv \widetilde{c}\,\widehat{t}\,(k/K) \equiv \left(\frac{1}{n} \sum_{j=1}^{n} \widehat{\alpha}_{(k-1)\,n+j} \right) , \qquad (18)$$

with $k \in \{1,\, \ldots,\, K\}$ and a velocity $\widetilde{c}$ to be set to unity later. The time coordinates from (18) are <u>ordered</u>,

$$\widehat{t}\,(1/K) \,\leq\, \widehat{t}\,(2/K) \,\leq\, \ldots \,\leq \widehat{t}\,(1-1/K) \,\leq \widehat{t}\,(1)\,, \qquad (19)$$

because the $\widehat{\alpha}_i$ are, according to (16b).

A. Extraction of spacetime points

Next, obtain the eigenvalues of the $n \times n$ blocks of the nine spatial matrices $\underline{\widehat{A}}^{\,m}$ and denote these real eigenvalues by $\left(\widehat{\beta}^{\,m}\right)_i$, with $i \in \{1, \ldots, N\}$.

Define, just as for the time coordinate in (18), the following nine spatial coordinates $\widehat{x}^{\,m}(\sigma)$ for $\sigma \in \{(0,\, 1]$:

$$\widehat{x}^{\,m}\big(k/K\big) \equiv \frac{1}{n} \sum_{j=1}^{n} \left[\,\widehat{\beta}^{\,m}\,\right]_{(k-1)\,n+j}\,, \qquad (20)$$

with $k \in \{1, \ldots, K\}$.

A. Extraction of spacetime points

The expressions (18) and (20) may provide suitable spacetime points, which, in a somewhat different notation, are denoted

$$\widehat{x}_k^{\,\mu} = \left(\widehat{x}_k^{\,0},\, \widehat{x}_k^{\,m}\right) \equiv \Big(\widehat{x}^{\,0}\big(k/K\big),\, \widehat{x}^{\,m}\big(k/K\big)\Big)\,, \tag{21}$$

where k runs over $\{1,\, \ldots,\, K\}$.

Each of these ten coordinates has the dimension of length, which traces back to the dimension of the bosonic matrix variable A^μ as mentioned below (9c).

To summarize, the extracted spacetime points $\widehat{x}_k^{\,\mu}$ are obtained as **averaged eigenvalues** of the $n \times n$ blocks along the diagonals of the gauge-transformed master-field matrices $\underline{\widehat{A}}^{\,\mu}$ from (15)–(16).

B. Extraction of spacetime metric

The points $\widehat{x}_k^\mu$ effectively build a spacetime manifold with continuous (interpolating) coordinates x^μ if there is also an emerging metric $g_{\mu\nu}(x)$.

By considering the effective action of a low-energy scalar degree of freedom ϕ “propagating” over the discrete spacetime points $\widehat{x}_k^\mu$, the following expression for the emergent <u>inverse</u> metric is obtained [7, 10]:

$$g^{\mu\nu}(x) \sim \int_{\mathbb{R}^D} d^D y\; \rho_{\text{av}}(y)\; (x-y)^\mu\, (x-y)^\nu\; f(x-y)\; r(x,\,y)\,, \tag{22a}$$

$$\rho_{\text{av}}(y) \equiv \langle\langle\, \rho(y)\, \rangle\rangle\,, \tag{22b}$$

with continuous spacetime coordinates x^μ having the dimension of length and spacetime dimension $D = 9+1 = 10$ for the original model.

The average $\langle\langle\, \rho(y)\, \rangle\rangle$ corresponds, for the extraction procedure described earlier, to averaging over different block sizes n and block positions along the diagonal in the master-field matrices $\underline{\widehat{A}}^\mu$.

B. Extraction of spacetime metric

The quantities that enter the integral (22) are the density function

$$\rho(x) \;\equiv\; \sum_{k=1}^{K} \delta^{(D)}\big(x - \widehat{x}_k\big)\,, \tag{23}$$

the density correlation function $r(x,\, y)$ defined by

$$\langle\langle\, \rho(x)\, \rho(y)\, \rangle\rangle \;\equiv\; \langle\langle\, \rho(x)\, \rangle\rangle\; \langle\langle\, \rho(y)\, \rangle\rangle\; r(x,\, y)\,, \tag{24}$$

and a sufficiently localized function $f(x)$ from the scalar effective action.

As $r(x,\, y)$ is dimensionless and $f(x)$ has dimension $1/(\text{length})^2$, the inverse metric $g^{\mu\nu}(x)$ from (22) is seen to be dimensionless.

The metric $g_{\mu\nu}$ is simply obtained as the matrix inverse of $g^{\mu\nu}$.

To summarize, the emergent metric is obtained from **correlations** of the extracted spacetime points.

C. What emergent spacetime?

The obvious question, now, is what spacetime and metric do we get?

We don’t know, as we do not know the IIB-matrix-model master field.

But, awaiting the final result on the master field, we can already investigate what properties the master field would need to have in order to be able to produce certain desired emerging metrics.

The results presented here are, therefore, solely exploratory.

C. Emergent Minkowski and RW metrics

We restrict ourselves to four “large” spacetime dimensions [8], setting

$$D = 3 + 1 = 4\,, \tag{25}$$

and use length units that normalize the IIB-matrix-model length scale,

$$\ell = 1\,. \tag{26}$$

Then, it is possible to choose appropriate functions $\rho_{\text{av}}(y)$, $f(x-y)$, and $r(x,\, y)$, so that the Minkowski metric is obtained [formally given by (6a) for $b^2 = 0$ and $a^2(t) = 1$].

Similarly, it is possible to choose appropriate functions $\rho_{\text{av}}(y)$, $f(x-y)$, and $r(x,\, y)$, so that the spatially flat Robertson–Walker metric is obtained [formally given by (6a) for $b^2 = 0$].

C. Emergent regularized-big-bang metric

In order to get an inverse metric whose component g^{00} diverges at $t = 0$, it is necessary to relax the convergence properties of the y^0 integral in (22a) by adapting the functions $\rho_{\text{av}}(y)$, $f(x - y)$, and $r(x,\, y)$.

In this way, it is possible to obtain the following inverse metric [11]:

$$g^{\mu\nu}_{\text{(eff)}} \sim \begin{cases} -\dfrac{t^2 + c_{-2}}{t^2}\,, & \text{for } \mu = \nu = 0\,, \\ 1 + c_2\, t^2 + c_4\, t^4 + \ldots\,, & \text{for } \mu = \nu = m \in \{1,\, 2,\, 3\}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{27}$$

with real dimensionless coefficients c_n that result from the requirement that the t^n terms, for $n > 0$, vanish in g^{00}.

C. Emergent regularized-big-bang metric

The matrix inverse of (27) gives the following metric:

$$g_{\mu\nu}^{(\text{eff})} \sim \begin{cases} -\dfrac{t^2}{t^2 + c_{-2}}\,, & \text{for } \mu = \nu = 0\,, \\ \dfrac{1}{1 + c_2\, t^2 + c_4\, t^4 + \ldots}\,, & \text{for } \mu = \nu = m \in \{1,\, 2,\, 3\}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{28}$$

which has, for $c_{-2} > 0$, a vanishing determinant at $t = 0$ and is, therefore, degenerate.

C. Emergent regularized-big-bang metric

The emergent metric (28) has indeed the structure of the regularized-big-bang metric (6a), with the following effective parameters:

$$b_{\text{eff}}^2 \sim c_{-2}\,\ell^2\,, \tag{29a}$$

$$a_{\text{eff}}^2(t) \sim 1 - c_2\,\left(t/\ell\right)^2 + \ldots\,, \tag{29b}$$

where the IIB-matrix-model length scale ℓ has been restored and where the leading coefficients c_{-2} and c_2 have been calculated.

By choosing the *Ansatz* parameters appropriately, we get $c_2 < 0$ in (29b), so that the emerged classical spacetime corresponds to the spacetime of a nonsingular cosmic bounce at $t = 0$, as obtained in (8) from Einstein's gravitational field equation with a $w_M = 1/3$ perfect fluid.

C. Cosmological interpretation

The proper cosmological interpretation of the emerged classical spacetime is perhaps as follows.

The new physics phase is assumed to be described by the IIB matrix model and the corresponding large-N master field gives rise to the points and metric of a classical spacetime.

If the master field has an appropriate structure, the emerged metric has a tamed big bang, with a metric similar to the regularized-big-bang metric of GR [2] but now having an effective length parameter b_{eff} proportional to the IIB-matrix-model length scale ℓ .

In fact, one possible interpretation is that the new phase has produced a universe-antiuniverse pair [5], that is, a “universe” for $t > 0$ and an “antiuniverse” for $t < 0$.