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M-theory and the emergence of spacetime

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0. Preliminary remarks

- The present talk is really a “UFBSM” talk, where the extravagant acronym is explained as follows:

- BSM=Beyond the Standard Model of **elementary particle physics**
- FBSM=**Far**-BSM, also including **gravity** $\left(E_{\text{Planck}} \approx 10^{19}\ \text{GeV}\right)$
- UFBSM=**Ultra**-FBSM, also including the **emergence of spacetime**

Disclaimer:

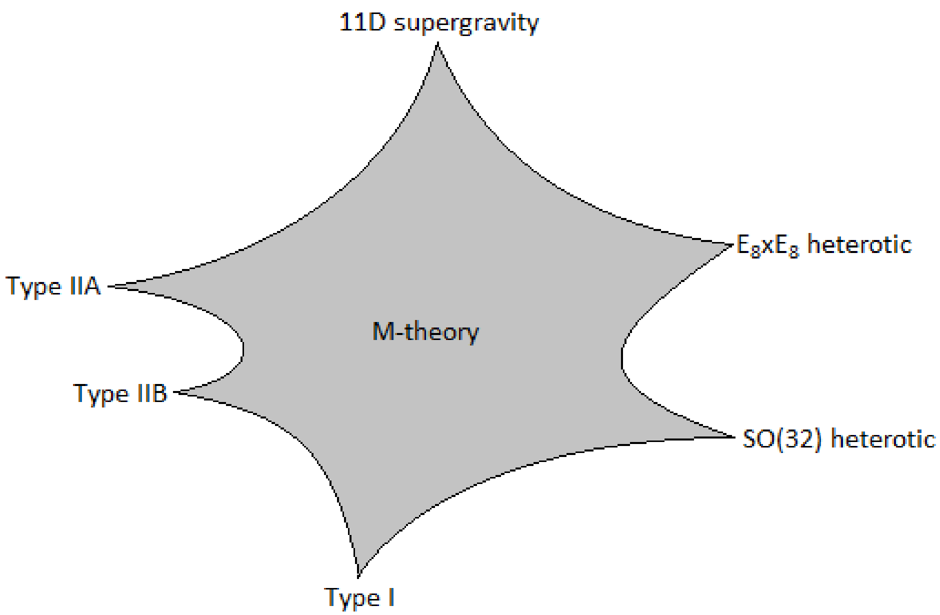
- The present talk, with a rather formidable title, provides no definitive answers (the Sphinx of Giza knows the answers but does not tell us...).

0. Outline

1. M-theory
   (a) Universality class
   (b) IIB matrix model
   (c) Conceptual question $\leftarrow$ focus of this talk
2. Suggested answer
   (a) Large-N factorization
   (b) Large-N master field
   (c) Emergent classical spacetime
3. Technical details – short version
4. Conclusions
5. References
6. Appendices (Technical details – long version)

1a. M-theory – Universality class

M-theory is a hypothetical theory that unifies all five consistent versions of 10D superstring theory (cf. [1, 2, 3]); see the well-known “nerve-cell” sketch.



The assumption is that all theories of the figure belong to the same **universality class**.

For an explicit description, we use the **IIB matrix model** of Kawai and collaborators [4, 5], which has been proposed as a nonperturbative formulation of type-IIB superstring theory and, thereby, of M-theory.

1b. IIB matrix model

The IIB matrix model has a **finite number** of $N \times N$ traceless Hermitian matrices: ten bosonic matrices A^μ and essentially eight fermionic (Majorana–Weyl) matrices Ψ_α.

The partition function Z of the IIB matrix model is defined by the following “path” integral [4, 5]:

$$Z = \int dA\, d\Psi\;\; \exp\left(-\, S/\ell^4\,\right) = \int dA\;\; \exp\left(-\, S_{\text{eff}}/\ell^4\,\right)\;, \tag{1a}$$

$$S = -\mathrm{Tr}\left(\frac{1}{4}\,\left[A^\mu,\, A^\nu\right]\,\left[A^\rho,\, A^\sigma\right]\,\widetilde{\delta}_{\mu\rho}\,\widetilde{\delta}_{\nu\sigma} + \frac{1}{2}\,\overline{\Psi}_\beta\,\widetilde{\Gamma}^\mu_{\beta\alpha}\,\widetilde{\delta}_{\mu\nu}\,\left[A^\nu,\, \Psi_\alpha\right]\right), \tag{1b}$$

$$\widetilde{\delta}_{\mu\nu} = \Big[\mathrm{diag}(1,\, 1,\, \ldots\, ,\, 1)\Big]_{\mu\nu}\,, \quad \text{for} \quad \mu, \nu \in \{1,\, 2,\, \ldots\, ,\, 10\}\,. \tag{1c}$$

Expectation values of further observables will be discussed later.

1b. IIB matrix model

Two technical remarks:

1. The model shown in (1) is the original model with "Euclidean" coupling constants $\widetilde{\delta}_{\mu\nu}$, but it is also possible to consider a "Lorentzian" version [6, 7] with a complex Feynman phase factor $\exp\left(i\, S/\ell^4\,\right)$ in the path integral and coupling constants $\widetilde{\eta}_{\mu\nu} = \Big[\text{diag}\big(-1,\,1,\,\ldots\,,\,1\big)\Big]_{\mu\nu}$ for $\mu, \nu \in \{0,\,1,\,\ldots\,,\,9\}$.

2. A model length scale "ℓ" has been introduced, so that A^μ has the dimension of length and Ψ_α the dimension of $(\text{length})^{3/2}$.

1c. Conceptual question

Now, the IIB matrix model (1) just gives **numbers**, Z and further expectation values (see later), while the matrices A^{μ} and Ψ_{α} in (1a) are merely integration variables.

Moreover, there is no obvious small dimensionless parameter to motivate a saddle-point approximation.

Hence, the **conceptual** question: $\boxed{\text{where is the classical spacetime?}}$

Recently, we have suggested to revisit an old idea, the large-N master field of Witten [8], for a possible origin of classical spacetime in the context of the IIB matrix model [9].

In this relatively short talk, we have only time to remind you of this mysterious master field (a name coined by Coleman) and to give some preliminary results.

2a. Suggested answer – Large-N factorization

Consider the gauge-invariant bosonic observable

$$w^{\mu_1 \cdots \mu_m} = \mathrm{Tr}\left(A^{\mu_1} \cdots A^{\mu_m}\right). \tag{2}$$

Arbitrary strings of these w observables have **expectation values**

$$\langle w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n}\, \cdots \rangle = \frac{1}{Z}\int dA\left(w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n}\, \cdots\right) e^{-S_{\text{eff}}/\ell^4}, \tag{3}$$

with normalization $\langle\, 1\, \rangle = 1$.

For a string of two identical w observables, the following **factorization property** holds to leading order in N:

$$\langle\, w^{\mu_1 \cdots \mu_m}\, w^{\mu_1 \cdots \mu_m}\, \rangle \stackrel{N}{=} \langle w^{\mu_1 \cdots \mu_m}\rangle\, \langle w^{\mu_1 \cdots \mu_m}\rangle, \tag{4}$$

without sums over repeated indices. Similar large-N factorization properties hold for <u>all</u> expectation values (3).

The leading-order equality (4) is a truly remarkable result for a statistical (quantum) theory.

2b. Large-N master field

Indeed, according to Witten [8], the factorization (4) implies that the path integrals (3) are saturated by a single configuration, the so-called **master field** $\widehat{\boldsymbol{A}}^{\boldsymbol{\mu}}$.

Considering one w observable for simplicity, we then have for its expectation value:

$$\langle w^{\mu_1 \,\cdots\, \mu_m} \rangle \stackrel{N}{=} \text{Tr}\left(\widehat{A}^{\mu_1} \cdots \widehat{A}^{\mu_m} \right) \equiv \widehat{w}^{\mu_1 \,\cdots\, \mu_m} \,, \tag{5}$$

and similarly for the other expectation values,

$$\langle w^{\mu_1 \,\cdots\, \mu_m} \, w^{\nu_1 \,\cdots\, \nu_n} \, \cdots \, w^{\omega_1 \,\cdots\, \omega_z} \rangle \stackrel{N}{=} \widehat{w}^{\mu_1 \,\cdots\, \mu_m} \, \widehat{w}^{\nu_1 \,\cdots\, \nu_n} \, \cdots \, \widehat{w}^{\omega_1 \,\cdots\, \omega_z} \,. \tag{6}$$

Hence, we do not have to perform the path integrals on the right-hand side of (3): we "only" need ten traceless Hermitian matrices $\widehat{A}^{\mu}$ to get all these expectation values from the simple procedure of replacing each A^{μ} in the observables by $\widehat{A}^{\mu}$.

2c. Emergent classical spacetime

Now, the meaning of the suggestion on slide 7 is clear:

classical spacetime may reside in the master-field matrices $\widehat{A}^{\,\mu}$.

<u>Heuristics</u>:

- The expectation values $\langle w^{\mu_1\,\cdots\,\mu_m}\; w^{\nu_1\,\cdots\,\nu_n}\;\ldots\; w^{\omega_1\,\cdots\,\omega_z}\rangle$ from (3) correspond to a large part of the **information content** of the IIB matrix model (but, of course, not all the information).
- That **same** information is contained in the master-field matrices $\widehat{A}^{\,\mu}$, which give the same numbers $(\widehat{w}^{\,\mu_1\,\cdots\,\mu_m}\; \widehat{w}^{\nu_1\,\cdots\,\nu_n}\;\ldots\; \widehat{w}^{\,\omega_1\,\cdots\,\omega_z})$, at least to leading order in N and where $\widehat{w}$ is the observable w evaluated for $\widehat{A}$.
- From these master-field matrices $\widehat{A}^{\,\mu}$, it appears possible to **extract** the points and metric of an emergent classical spacetime (recall that the original matrices $A^{\,\mu}$ were merely integration variables).

2c. Emergent classical spacetime

Next, **assume** that the matrices $\widehat{A}^{\mu}$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal.

Then, it is possible [9] to extract a discrete set of spacetime points $\{\widehat{x}_k^{\mu}\}$ with an index $k \in \{1, \ldots, K\}$ for integer $K = N/n$.

These discrete spacetime points sample a smooth manifold with continuous spacetime coordinates x^{μ} and an emergent inverse metric $g^{\mu\nu}(x)$, for which we have an explicit expression [9].

The metric $g_{\mu\nu}(x)$ is obtained as matrix inverse of $g^{\mu\nu}(x)$.

It has been established [10, 11, 12] that, in principle, it is possible to get, from appropriate distributions of the extracted spacetime points $\{\widehat{x}_k^{\mu}\}$, the metrics of the Minkowski and the spatially flat Robertson–Walker spacetimes.

For discussion relating to a "tamed" big bang, see the recent review [13].

3a. Some details – Extracted spacetime points

Assume that the matrices $\widehat{A}^{\mu}$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by numerical results [6, 7]).

Then, skipping over a few subtleties, the **spacetime points** are obtained as follows:

Let n be a divisor of N,

$$N = K\,n\,, \tag{7}$$

where both K and n are positive integers. Now, the extracted spacetime points $\widehat{x}^{\,\mu}_{k}$, for $k \in \{1,\,\ldots\,,\,K\}$, are obtained as **averaged eigenvalues of the $\mathbf{n} \times \mathbf{n}$ blocks** along the diagonals of the master-field matrices $\widehat{A}^{\mu}$.

Further details on the extracted spacetime points are given in App. 6A.

3b. Extracted spacetime metric

By considering the effective action of a low-energy scalar degree of freedom σ "propagating" over the discrete spacetime points $\widehat{x}_k^{\,\mu}$, the following expression for the **emergent inverse metric** is obtained [5, 9]:

$$g^{\mu\nu}(x) \;\;\sim\;\; \int_{\mathbb{R}^D} d^D y\; \rho_{\text{av}}(y)\; (x-y)^\mu\, (x-y)^\nu\; f(x-y)\; r(x,\,y)\,, \qquad (8)$$

with continuous spacetime coordinates x^μ having the dimension of length and spacetime dimension $D = 9+1 = 10$ for the original model.

The multiple integral (8) contains several functions which follow from the **distributions** of the extracted discrete spacetime points. As such, these functions are determined by the master-field matrices $\widehat{A}^{\,\mu}$.

The emergent metric $g_{\mu\nu}(x)$ is obtained as matrix inverse of $g^{\mu\nu}(x)$.

Further details on the emergent inverse metric are given in App. 6B.

3c. Emerging Lorentzian signature

The matrix model (1) has been defined with “Euclidean” coupling constants $\widetilde{\delta}_{\mu\nu}$, but the emergent spacetime metric from (8) need not necessarily have a Euclidean signature.

It can be shown that an appropriate behavior of the functions in the integrand of (8) can give rise to a Lorentzian signature.

See App. B in Ref. [9] and App. D in Ref. [13] for some toy-model calculations.

Here, further details on the emerging Lorentzian signature are given in App. 6C.

4. Conclusions

It is conceivable that a **new physics phase** gives rise to classical spacetime, gravity, and matter, as described by our current theories (General Relativity and the Standard Model).

For an explicit calculation, we have considered the **IIB matrix model**, which has been proposed as a nonperturbative formulation of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the **large-N master field** $\widehat{\boldsymbol{A}}^{\boldsymbol{\mu}}$ of the IIB matrix model.

In principle, the IIB-matrix-model master field $\widehat{A}^{\mu}$ can produce the metrics of the Minkowski and the spatially flat Robertson–Walker spacetimes.

At this moment, the outstanding task is to **calculate** the exact IIB-matrix-model master field $\widehat{A}^{\mu}$ or, at least, to get a reliable approximation of it ...

5. References

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6A. Extraction of the spacetime points

Aoki et al. [5] have argued that the **eigenvalues** of the matrices A^μ of model (1) can be interpreted as **spacetime coordinates**, so that the model has a ten-dimensional $\mathcal{N}=2$ spacetime supersymmetry.

Here, we will turn to the eigenvalues of the **master-field** matrices $\widehat{A}^\mu$.

Assume that the matrices $\widehat{A}^\mu$ of the Lorentzian-IIB-matrix-model master field are known and that they are approximately band-diagonal with width $\Delta N < N$ (as suggested by numerical results [6, 7]). Then, make a particular global gauge transformation [6],

$$\underline{\widehat{A}}^{\mu} = \underline{\Omega}\,\widehat{A}^{\mu}\,\underline{\Omega}^{\dagger}\,, \quad \underline{\Omega} \in SU(N)\,, \tag{9}$$

so that the transformed 0-component matrix is diagonal and has ordered eigenvalues $\widehat{\alpha}_i \in \mathbb{R}$,

$$\underline{\widehat{A}}^{0} = \mathrm{diag}\big(\widehat{\alpha}_1,\,\widehat{\alpha}_2,\,\ldots\,,\,\widehat{\alpha}_{N-1},\,\widehat{\alpha}_N\big)\,, \tag{10a}$$

$$\widehat{\alpha}_1 \leq \widehat{\alpha}_2 \leq \ldots \leq \widehat{\alpha}_{N-1} \leq \widehat{\alpha}_N\,, \qquad \sum_{i=1}^{N}\widehat{\alpha}_i = 0\,. \tag{10b}$$

6A. Extraction of the spacetime points

A relatively simple procedure [9] **approximates** the eigenvalues of the spatial matrices $\underline{\widehat{A}}^{m}$ but still manages to **order them along the diagonal**, matching the temporal eigenvalues $\widehat{\alpha}_i$ from (10).

We start from two trivial observations:

- If M is an $N \times N$ Hermitian matrix, then any $n \times n$ block centered on the diagonal of M is also Hermitian, which holds for $1 \leq n \leq N$.
- If the matrix M is, moreover, band-diagonal with width $\Delta N < N$, then the eigenvalues of the $n \times n$ blocks on the diagonal approximate the original eigenvalues of M, provided $n \gtrsim \Delta N$.

Now, let K be an odd divisor of N, so that

$$N \;=\; K\, n\,, \quad K = 2\,L + 1\,, \tag{11}$$

where both L and n are positive integers.

6A. Extraction of the spacetime points

Consider, in each of the ten matrices $\underline{\widehat{A}}^{\mu}$, the K adjacent blocks of size $n \times n$ centered on the diagonal.

We already know the diagonalized blocks of $\underline{\widehat{A}}^{0}$ from (10a), which allows us to define the following time coordinate $\widehat{t}\,(\sigma)$ for $\sigma \in (0,\, 1]$:

$$\widehat{x}^{\,0}\,(k/K) \equiv \widetilde{c}\,\widehat{t}\,(k/K) \equiv \frac{1}{n}\sum_{j=1}^{n} \widehat{\alpha}_{(k-1)\,n+j}\,, \tag{12}$$

with $k \in \{1,\, \ldots,\, K\}$ and a velocity $\widetilde{c}$ to be set to unity later. The time coordinates from (12) are <u>ordered</u>,

$$\widehat{t}\,(1/K)\;\leq\;\widehat{t}\,(2/K)\;\leq\;\ldots\;\leq\;\widehat{t}\,(1-1/K)\;\leq\;\widehat{t}\,(1)\,, \tag{13}$$

precisely because the $\widehat{\alpha}_i$ are, according to (10b).

6A. Extraction of the spacetime points

Next, obtain the eigenvalues of the $n \times n$ blocks of the nine spatial matrices $\underline{\widehat{A}}^{\,m}$ and denote these real eigenvalues by $\left(\widehat{\beta}^{\,m}\right)_i$, with a label $i \in \{1,\, \ldots,\, N\}$ respecting the order of the n-dimensional blocks.

Define, just as for the time coordinate in (12), the following nine spatial coordinates $\widehat{x}^{\,m}(\sigma)$ for $\sigma \in \{(0,\, 1]$:

$$\widehat{x}^{\,m}\big(k/K\big) \equiv \frac{1}{n}\,\sum_{j=1}^{n}\,\left[\,\widehat{\beta}^{\,m}\,\right]_{(k-1)\,n+j}\,, \tag{14}$$

with $k \in \{1,\, \ldots,\, K\}$.

6A. Extraction of the spacetime points

If the master-field matrices $\widehat{\underline{A}}^{\mu}$ are approximately **band-diagonal** (width $\Delta N < N$) and if the eigenvalues of the spatial $n \times n$ blocks (with $n \gtrsim \Delta N$) show significant **scattering**, then the expressions (12) and (14) may provide suitable spacetime points. In a somewhat different notation, these points are denoted

$$\widehat{x}_k^{\mu} = \left(\widehat{x}_k^{0},\, \widehat{x}_k^{m}\right) \equiv \Big(\widehat{x}^{\,0}(k/K),\, \widehat{x}^{\,m}(k/K)\Big)\,, \tag{15}$$

where k runs over $\{1,\, \ldots,\, K\}$.

Each of these coordinates $\widehat{x}_k^{\mu}$ has the dimension of length, which traces back to the dimension of the bosonic matrix variable A^{μ}.

To summarize, with $N = K\,n$ and $n \gtrsim \Delta N$, the extracted spacetime points $\widehat{x}_k^{\mu}$, for $k \in \{1,\, \ldots,\, K\}$, are obtained as **averaged eigenvalues** of the $n \times n$ blocks along the diagonals of the gauge-transformed master-field matrices $\widehat{\underline{A}}^{\mu}$ from (9)–(10).

6B. Extraction of the spacetime metric

The points $\widehat{x}_k^{\mu}$ effectively build a spacetime manifold with continuous (interpolating) coordinates x^{μ} if there is also an emerging metric $g_{\mu\nu}(x)$.

By considering the effective action of a low-energy scalar degree of freedom σ "propagating" over the discrete spacetime points $\widehat{x}_k^{\mu}$, the following expression for the **emergent inverse metric** is obtained [5, 9]:

$$g^{\mu\nu}(x) \sim \int_{\mathbb{R}^D} d^D y\, \rho_{\text{av}}(y)\, (x-y)^{\mu}\, (x-y)^{\nu}\, f(x-y)\, r(x,\, y)\,, \tag{16a}$$

$$\rho_{\text{av}}(y) \equiv \langle\langle\, \rho(y)\, \rangle\rangle\,, \tag{16b}$$

with continuous spacetime coordinates x^{μ} having the dimension of length and spacetime dimension $D = 9+1 = 10$ for the original model.

The average $\langle\langle\, \rho(y)\, \rangle\rangle$ corresponds, for the extraction procedure of App. 6A, to averaging over different block sizes n and block positions along the diagonal in the master-field matrices $\underline{\widehat{A}}^{\mu}$.

6B. Extraction of the spacetime metric

The quantities that enter the integral (16) are the density function

$$\rho(x) \;\equiv\; \sum_{k=1}^{K} \delta^{(D)}\big(x - \widehat{x}_k\big)\,, \tag{17}$$

the density correlation function $r(x,\,y)$ defined by

$$\langle\langle\, \rho(x)\, \rho(y)\, \rangle\rangle \;\equiv\; \langle\langle\, \rho(x)\, \rangle\rangle\; \langle\langle\, \rho(y)\, \rangle\rangle\; r(x,\,y)\,, \tag{18}$$

and a localized real function $f(x)$ from the scalar effective action,

$$S_{\text{eff}}[\sigma] \sim \sum_{k,\,l}\; \frac{1}{2}\, f\big(\widehat{x}_k - \widehat{x}_l\big)\, \big(\sigma_k - \sigma_l\big)^2\,, \tag{19}$$

where σ_k is the field value at the point $\widehat{x}_k$ (the scalar degree of freedom σ arises from a perturbation of the master field $\underline{\widehat{A}}^{\mu}$; see App. A in Ref. [9]).

As $r(x,\,y)$ is dimensionless and $f(x)$ has dimension $1/(\text{length})^2$, the inverse metric $g^{\mu\nu}(x)$ from (16) is seen to be dimensionless.
The metric $g_{\mu\nu}$ is simply obtained as the matrix inverse of $g^{\mu\nu}$.

6B. Extraction of the spacetime metric

A few **heuristic remarks** [12] may help to clarify expression (16a).

In the standard continuum theory [i.e., a scalar field $\sigma(x)$ propagating over a given continuous spacetime manifold with metric $g_{\mu\nu}(x)$], two nearby points x' and x'' have approximately equal field values, $\sigma(x') \sim \sigma(x'')$, and two distant points x' and x''' generically have very different field values, $|\sigma(x') - \sigma(x''')|/|\sigma(x') + \sigma(x''')| \gtrsim 1$.

The logic is inverted for our discussion. Two approximately equal field values, $\sigma_1 \sim \sigma_2$, may still have a relatively small action (19) if $f(\widehat{x}_1 - \widehat{x}_2) \sim 1$ and inserting $f \sim 1$ in (16a) gives a "large" value for the inverse metric $g^{\mu\nu}$ and, hence, a "small" value for the metric $g_{\mu\nu}$, meaning that the spacetime points $\widehat{x}_1$ and $\widehat{x}_2$ are close (in units of ℓ).

Two different field values σ_1 and σ_3 have a small action (19) if $f(\widehat{x}_1 - \widehat{x}_3) \sim 0$ and inserting $f \sim 0$ in (16a) gives a "small" value for the inverse metric $g^{\mu\nu}$ and, hence, a "large" value for the metric $g_{\mu\nu}$, meaning that the spacetime points $\widehat{x}_1$ and $\widehat{x}_3$ are separated by a large distance (in units of ℓ).

6B. Extraction of the spacetime metric

To summarize, the emergent metric, in the context of the IIB matrix model, is obtained from **correlations** of the extracted spacetime points and the master-field perturbations.

The obvious question, now, is which spacetime and metric <u>do</u> we get?

We don't know, as we do <u>not</u> have the IIB-matrix-model master field.

But, awaiting the final result on the master field, we can already investigate what properties the master field <u>would</u> need to have in order to be able to produce certain desired emerging metrics. Some exploratory results were presented in Refs. [11, 12].

[Note that, in principle, the origin of the expression (16) need not be the IIB matrix model but can be an entirely different theory, as long as the emerging inverse metric is given by a multiple integral with the same basic structure.]

6C. More on the Lorentzian signature

In Refs. [9, 11, 12], we have considered the Lorentzian IIB matrix model, which has two characteristics:

1. the "Lorentzian" coupling constants $\widetilde{\eta}_{\mu\nu}$ in the action:
2. the Feynman phase factor $e^{i\,S/\ell^4}$ in the "path" integral.

From the master field of this Lorentzian matrix model, we obtained the spacetime points from expressions (12) and (14) in App. 6A and the inverse spacetime metric from expression (16) in App. 6B.

Several Lorentzian inverse metrics were found in Refs. [11, 12], where the used *Ansätze* relied on having "Lorentzian" coupling constants $\widetilde{\eta}_{\mu\nu}$.

But there is another way [9] to obtain Lorentzian inverse metrics, namely by making an appropriately <u>odd</u> *Ansatz* for the correlations functions entering (16), so that the resulting matrix is <u>off-diagonal</u>.

6C. More on the Lorentzian signature

With this appropriately odd *Ansatz*, it is, in principle, also possible to get a Lorentzian inverse metric from the Euclidean matrix model, which has nonnegative coupling constants $\widetilde{\delta}_{\mu\nu}$ in the action and a weight factor $e^{-\,S/\ell^4}$ in the path integral. The spacetime points are extracted from the Euclidean master field (no gauge transformation needed) by the expression (14), where m now runs over $\{1,\,\ldots\,,\,D\}$.

The details of a toy-model calculation are as follows (expanding on a parenthetical remark in the last paragraph of App. B in Ref. [9]).

The calculation starts from the multiple integral (16) for spacetime dimension $D = 4$ by writing in the integrand

$$f(x-y)\,r(x,\,y) = f(x-y)\,\widetilde{r}(y-x)\,\overline{r}(x,\,y) = h(y-x)\,\overline{r}(x,\,y)\,, \quad (20)$$

where the new function $\overline{r}(x,\,y)$ has a more complicated dependence on x and y than the combination $x-y$.

6C. More on the Lorentzian signature

The $D = 4$ multiple integral (16), with y^0 replaced by y^4, is then evaluated at the spacetime point

$$x^\mu = 0\,, \tag{21a}$$

with the replacement (20) in the integrand, two further simplifications,

$$\langle\langle\, \rho(y)\,\rangle\rangle = 1\,, \qquad \overline{r}(x,\, y) = 1\,, \tag{21b}$$

and symmetric cutoffs on the integrals,

$$\int_{-1}^{1} dy^1 \,\ldots \int_{-1}^{1} dy^4\,. \tag{21c}$$

The only nontrivial contribution to the integrand of (16) now comes from the correlation function h as defined by (20).

6C. More on the Lorentzian signature

From (16) and (21), we then get the emergent inverse metric at $x^\mu = 0$

$$g^{\mu\nu}_{\text{test,E4}}(0) \sim \int_{-1}^{1} dy^1 \int_{-1}^{1} dy^2 \int_{-1}^{1} dy^3 \int_{-1}^{1} dy^4 \;\; y^\mu \, y^\nu \; h(y)\,. \tag{22}$$

Next, take an appropriate *Ansatz* for the correlation function h ,

$$h(y) = 1 - \gamma \left(y^1\, y^2 + y^1\, y^3 + y^1\, y^4 + y^2\, y^3 + y^2\, y^4 + y^3\, y^4 \right), \tag{23}$$

where γ multiplies monomials that are odd in two coordinates and even in the two others.

Note that the Ansatz (23) treats <u>all</u> coordinates y^1, y^2, y^3, and y^4 <u>equally</u>, in line with the coupling constants $\widetilde{\delta}_{\mu\nu}$ of the Euclidean matrix model.

6C. More on the Lorentzian signature

The integrals of (22) with *Ansatz* function (23) are trivial and we obtain

$$g_\gamma^{\mu\nu}(0) \sim \frac{16}{9} \begin{pmatrix} 3 & -\gamma & -\gamma & -\gamma \\ -\gamma & 3 & -\gamma & -\gamma \\ -\gamma & -\gamma & 3 & -\gamma \\ -\gamma & -\gamma & -\gamma & 3 \end{pmatrix}, \tag{24a}$$

where the matrix on the right-hand side has the following eigenvalues and corresponding (normalized) eigenvectors:

$$\mathcal{E}_\gamma = \frac{16}{9} \Big\{ (3-3\,\gamma)\,,\,(3+\gamma)\,,\,(3+\gamma)\,,\,(3+\gamma) \Big\}, \tag{24b}$$

$$\mathcal{V}_\gamma = \left\{ \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\}. \tag{24c}$$

6C. More on the Lorentzian signature

From (24b), we have the following signatures:

$$(+---) \quad \text{for} \quad \gamma \in (-\infty,\, -3)\,, \tag{25a}$$

$$(++++) \quad \text{for} \quad \gamma \in (-3,\, 1)\,, \tag{25b}$$

$$(-+++) \quad \text{for} \quad \gamma \in (1,\, \infty)\,. \tag{25c}$$

Hence, we obtain Lorentzian signatures for parameter values γ sufficiently far away from zero, $\gamma > 1$ or $\gamma < -3$.

The conclusion is that it is, in principle, possible to get a Lorentzian emergent inverse metric from the Euclidean IIB matrix model, provided the correlation functions have the appropriate structure.

This observation, if applicable, would remove the need for working with the (possibly more difficult) Lorentzian IIB matrix model.

Figure credits: https://www.egypttoday.com/siteimages/Larg/72515.jpg (Sphinx picture); https://commons.wikimedia.org (M-theory sketch)