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Revisiting the cosmological constant problem

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0.1 Introduction

The main **Cosmological Constant Problem** (CCP1) can be phrased as follows (Pauli, 1933; Bohr, 1948; Veltman, 1974; see [1, 2] for two reviews):

why do the quantum fields in the vacuum not produce naturally a large cosmological constant Λ in the Einstein field equations?

The magnitude of the problem is enormous:

$$|\Lambda^{\text{theory}}|/|\Lambda^{\text{experiment}}| \geq 10^{54}\,.$$

The large number on the RHS arises as follows.

0.1 Introduction

With the ATLAS and CMS results [3, 4] in support of the Higgs mechanism, it is clear that the EWSM in the laboratory involves a vacuum energy density of order

$$\left|\epsilon_V^{\text{(EWSM)}}\right| \sim \big(100\ \text{GeV}\big)^4 \sim 10^{44}\ \text{eV}^4\,.$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_V^{\text{(EWSM)}} = \epsilon_V^{\text{(EWSM)}}(T)\,.$$

How can the Universe then end up with a vacuum energy density

$$\left|\Lambda^{\text{(obs)}}\right| < 10^{-28}\ \text{g cm}^{-3} \sim 10^{-10}\ \text{eV}^4\,?$$

Here, there are 54 orders of magnitude to explain:

$$\left|\Lambda^{\text{(obs)}}/\epsilon_V^{\text{(EWSM)}}\right| \leq 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,001\,.$$

0.1 Introduction

Still more CCPs after the discovery of the “accelerating Universe”:

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{Planck}})^4$?

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{present}} \sim +10^{-11}\ \text{eV}^4$?

Hundreds of papers have been published on CCP2. But, most likely:

$\boxed{\text{CCP1 needs to be solved first before CCP2 can even be addressed.}}$

0.1 Introduction

Here, a discussion of one particular approach to CCP1 by Volovik and the speaker, which goes under the name of $\boldsymbol{q}$**–theory** [5, 6, 7] (a brief review appears in [8]).

It is instructive to consider two explicit realizations of q–theory:

1. with a three-form gauge field [9, 10, 11, 12],
2. with a massless vector-field [13, 14].

The vector-field realization, in particular, is found to give **Minkowski spacetime** as an **attractor** of the field equations.

But a new problem arises: the danger of ruining the standard **Newtonian physics** of small self-gravitating systems [15].

This disaster can, however, be avoided by a special model with two vector fields [16, 17].

0.2 Outline

1. **Basics of q–theory** $\longleftarrow$ most important part of talk
2. **Two realizations**
3 **Newtonian gravity recovered** $\longleftarrow$ no collateral damage
4. **Conclusion**
5. **References**

1. Basics of q–theory

Crucial insight [5]: $\boxed{\text{there is vacuum energy and vacuum energy.}}$

More specifically and introducing an appropriate notation:

the vacuum energy density $\boxed{\epsilon}$ appearing in the action
need not be the same as
the vacuum energy density $\boxed{\rho_V}$ in the Einstein field equations.

How can this happen concretely . . .

1. Basics of q–theory

Consider the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Study, then, the **macroscopic** equations of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy:

- Take the mass density ρ of a liquid, for example, liquid Argon.
- This ρ describes microscopic quantities ($\rho = m_{\text{Ar}}\, n_{\text{Ar}}$ with number density n_{Ar} and mass m_{Ar} of the atoms).
- Still, ρ obeys the macroscopic equations of hydrodynamics, because of particle-number and mass conservation.

However, is the quantum vacuum just like a normal liquid?

1. Basics of q–theory

No, as the quantum vacuum is known to be **Lorentz invariant** (cf. experimental limits at the 10^{-15} level in the photon sector [18]).

The Lorentz invariance of the vacuum rules out the standard type of charge density, which arises from the time component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material liquids.

1. Basics of q–theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{var}}(q)\,, \tag{1}$$

including a possible constant term Λ_{bare} from the zero-point energies of the fields of the Standard Model (SM).

From ① thermodynamics and ② Lorentz invariance follows that [5]

$$P_V \overset{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \overset{②}{=} -\rho_V\,, \tag{2}$$

where the first equality corresponds to an integrated form of the Gibbs–Duhem equation for chemical potential $\mu \equiv d\epsilon/dq$.

Recall GD eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

1. Basics of q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{Planck}})^4$, but they cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 = \text{const} \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\, \frac{d\,\epsilon(q)}{d\,q} \right]_{q=q_0} = 0\,, \tag{3}$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle . . .

But, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q (see Sec. 2).

2. Two realizations

Start with two obvious questions:

Q1: How does the adjustment-type solution (3) of CCP1 circumvent Weinberg's no–go "theorem" [2]?

Answer: q is a non-fundamental scalar field; see Sec. 2.1.

Q2: How did the Universe get the right value q_0?

One possible answer is that q_0 (or the corresponding chemical potential μ_0) is fixed globally as an integration constant, being conserved throughout the history of the Universe [6].

Another possible answer uses a generalization of q–theory, for which the 'correct' value q_0 arises dynamically; see Sec. 2.2.

2.1 Four-form realization

Vacuum variable q may arise from a 3–form gauge field A [9, 10].

Start from the effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = \int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K_N\,R[g] + \Lambda_{\text{SM}} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \tag{4}$$

with gravitational coupling constant $K_N \equiv 1/(16\pi G_N)$ and $\hbar = c = 1$.

Change this theory by the introduction of one field, A, to get [6, 7]:

$$\widetilde{S}^{\text{eff}}[A,g,\psi] \;=\; \int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K(q)\,R[g] + \epsilon(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \tag{5a}$$

$$q \;\equiv\; \boxed{-\frac{1}{24}\,\epsilon^{\alpha\beta\gamma\delta}\,\nabla_\alpha A_{\beta\gamma\delta}\,/\,\sqrt{-g}}\,, \tag{5b}$$

where q arises from the four-form field strength $F = d\,A$.

Variational principle gives generalized Einstein and Maxwell equations:

2.1 Four-form realization

$$
\begin{aligned}
2K(q)\left(R_{\alpha\beta} - g_{\alpha\beta}\,R/2\right) &= -2\left(\nabla_\alpha\nabla_\beta - g_{\alpha\beta}\,\Box\right)K(q) \\
&\quad +\rho_V(q)\,g_{\alpha\beta} - T^M_{\alpha\beta}\,, \qquad \text{(6a)} \\
\frac{d\rho_V(q)}{dq} + R\,\frac{dK(q)}{dq} &= 0\,, \qquad \text{(6b)}
\end{aligned}
$$

with a vacuum energy density,

$$
\rho_V = \epsilon - q\left(\frac{d\epsilon}{dq} + R\,\frac{dK}{dq}\right) = \epsilon - q\,\mu\,, \qquad \text{(7)}
$$

for integration constant (chemical potential) μ. Eq. (7) is precisely of the Gibbs–Duhem form (2) in Minkowski spacetime ($R = 0$). Technically, the extra $g_{\alpha\beta}$ term on the RHS of (6a) appears because $q = q(A,\, g)$.

Answer to Q1: (5b) shows that q is a non-fundamental scalar field, which invalidates Weinberg's argument (see [7] for details).

2.2 Vector-field realization

Vacuum variable q comes from an aether-type velocity field u_β [13, 14], setting $E_{\text{UV}} = E_{\text{Planck}}$. For a flat RW metric with cosmic time t, there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \quad \to \quad q_0\, t\,, \quad u_b(t) = 0\,, \quad H(t) \to 1/t\,, \tag{8a}$$

$$u_\alpha{}^\beta \equiv \nabla_\alpha\, u^\beta \quad \to \quad \boxed{q_0\ \delta_\alpha{}^\beta}\,. \tag{8b}$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t\, E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. With an action quadratic in the variable $u_\alpha{}^\beta$, the field equations are [13]:

$$\ddot{v} + 3\, h\, \dot{v} - 3\, h^2\, v \quad = \quad 0\,, \tag{9a}$$

$$2\, \lambda - (\dot{v})^2 - 3\, (h\, v)^2 \quad = \quad 6\, h^2\,, \tag{9b}$$

with the overdot standing for differentiation with respect to τ. Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\widehat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\widehat{q}_0 = \sqrt{\lambda/2}$.

2.2 Vector-field realization

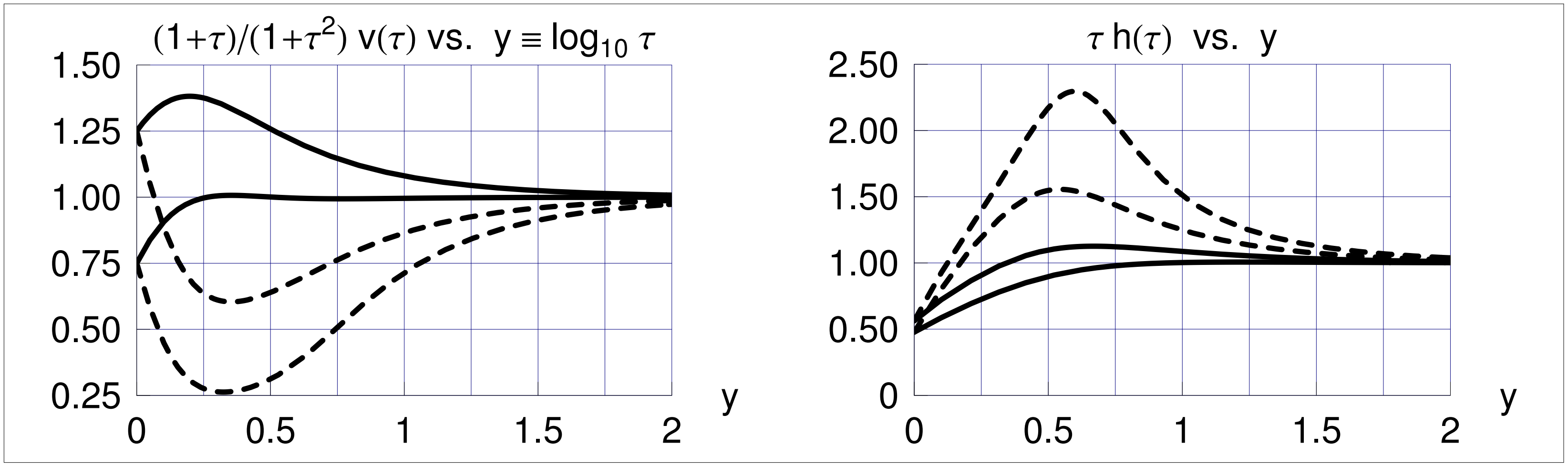

Fig. 1: Four numerical solutions of ODEs (9ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25$.

$\Rightarrow$ Minkowski value $\widehat{q}_0 = \sqrt{\lambda/2} = 1$ arises dynamically [see left panel].

$\Rightarrow$ Minkowski spacetime is an attractor in this aether-type theory [7].

But. as mentioned above, there is serious collateral damage which needs to be avoided ($\rightarrow$ Sec. 3)

2.3 Recap

To summarize, the q–theory approach to the main Cosmological Constant Problem (CCP1) provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the "beyond-the-Standard-Model" physics contains such a q–type variable.

GENERAL REMARK: it is clear that the SM harbors huge vacuum energy densities, which somehow need to be cancelled by new d.o.f., possibly related to the fundamental theory of spacetime and gravity.

Bad news: nothing is known about these fundamental d.o.f.

Good news: even though the detailed (high-energy) microphysics is unknown, it may be possible to describe the macroscopic (low-energy) effects along the lines of q–theory, just as for the hydrodynamics of water.

3. Newtonian gravity

As mentioned in the Introduction, the original Dolgov model [13] leads to an unacceptable modification of the standard Newtonian physics of small self-gravitating systems (first noted by Rubakov and Tinyakov [15]).

SPECIAL MODEL [16]:

Two massless vector fields $A_\alpha(x)$ and $B_\alpha(x)$ with effective action:

$$S_{\text{eff}} = -\int d^4x\,\sqrt{-g}\left(\frac{1}{2}\,(E_{\text{Planck}})^2\,R + \epsilon(Q_A,\,Q_B) + \Lambda\right), \quad \text{(10a)}$$

$$Q_A \equiv \sqrt{A_{\alpha;\beta}\;A^{\alpha;\beta}}\,, \quad Q_B \equiv \sqrt{B_{\alpha;\beta}\;B^{\alpha;\beta}}\,, \quad \text{(10b)}$$

$$E_{\text{Planck}} \equiv (8\pi\,G_N)^{-1/2} \approx 2.44 \times 10^{18}\ \text{GeV}\,. \quad \text{(10c)}$$

3. Newtonian gravity

The vacuum energy density ϵ is taken to have the following structure:

$$\epsilon \quad = \quad \frac{Q_A^4 - Q_B^4}{Q_A^2\, Q_B^2}\,, \tag{11a}$$

For later use, we give the corresponding results for the gravitating vacuum energy density $\widetilde{\epsilon}$ and inverse vacuum compressibility X^{-1} :

$$\widetilde{\epsilon} \quad \equiv \quad \epsilon - Q_A\,\frac{d\epsilon}{dQ_A} - Q_B\,\frac{d\epsilon}{dQ_B} = \frac{Q_A^4 - Q_B^4}{Q_A^2\, Q_B^2} = \epsilon\,, \tag{11b}$$

$$X^{-1} \quad \equiv \quad Q_A^2\,\frac{d^2\epsilon}{dQ_A\, dQ_A} + Q_B^2\,\frac{d^2\epsilon}{dQ_B\, dQ_B} + 2\,Q_A\,Q_B\,\frac{d^2\epsilon}{dQ_A\, dQ_B} = 0\,. \tag{11c}$$

3. Newtonian gravity

The Dolgov-type *Ansatz* for the vector fields $A_\alpha(x)$ and $B_\beta(x)$ and for the metric $g_{\alpha\beta}(x)$ is:

$$\begin{aligned} A_0 &= A_0(t) \equiv V(t)\,, \quad A_1 = A_2 = A_3 = 0\,, && \text{(12a)} \\ B_0 &= B_0(t) \equiv W(t)\,, \quad B_1 = B_2 = B_3 = 0\,, && \text{(12b)} \\ (g_{\alpha\beta}) &= \text{diag}\big(1,\, -a(t),\, -a(t),\, -a(t)\big)\,, && \text{(12c)} \end{aligned}$$

where $a(t)$ is the cosmic scale factor of the spatially flat Friedmann–Robertson–Walker (FRW) universe considered.

Solving the field equations from (10a) for the *Ansatz* fields (12) gives the explicit functions $\overline{V}(t) \propto t$, $\overline{W}(t) \propto t$, and $\overline{a}(t) \propto t$.

3. Newtonian gravity

MAIN ARGUMENT:

Small-scale perturbations around the background solution from (10a) and (12) give the following equation for the metric perturbation:

$$
\begin{aligned}
(8\pi\, G_N)^{-1} \left\{\text{“}\,\partial^2\,\widehat{h}\,\text{”}\right\}^{\text{(GR)}} &+ \left[\Lambda+\widetilde{\epsilon}\,\right]_{\text{asymp}} \left\{\text{“}\,\widehat{h}\,\text{”}\right\} \\
&+ \left[X^{-1}\right]_{\text{asymp}} \left\{t^2\,\text{“}\,\partial^2\,\widehat{h}\,\text{”} + t\,\text{“}\,\partial\,\widehat{h}\,\text{”} + \text{“}\,\widehat{h}\,\text{”}\right\} \\
&+ \left[\epsilon-\widetilde{\epsilon}\,\right]_{\text{asymp}} \left\{t^2\,\text{“}\,\partial^2\,\widehat{h}\,\text{”} + t\,\text{“}\,\partial\,\widehat{h}\,\text{”} + \text{“}\,\widehat{h}\,\text{”}\right\} = T_{\text{ext}}\,. \qquad \text{(13a)}
\end{aligned}
$$

The Minkowski-attractor solution of the special model with (11) gives

$$
\begin{aligned}
\left[\Lambda+\widetilde{\epsilon}\,\right]_{\text{asymp}} &= 0\,, \qquad \text{(13b)} \\
\left[X^{-1}\right]_{\text{asymp}} &= \left[\epsilon-\widetilde{\epsilon}\,\right]_{\text{asymp}} = 0\,. \qquad \text{(13c)}
\end{aligned}
$$

3. Newtonian gravity

Hence, the linear equation (13a) from the special model is the same as the linear equation from standard GR, which reduces to the standard Poisson equation of standard Newtonian gravity.

Newtonian gravity is indeed restored, but the Hubble expansion is too fast: $H(t) \equiv \dot{a}/a = t^{-1}$.

Possible to have another model [17] with non-minimal gravitational couplings, which has the standard FRW expansion [$H(t) = (1/2)\, t^{-1}$] and the standard local Newtonian dynamics [$G = G_N$].

This last paper also gives a mathematical discussion of the attractor behavior.

4. Conclusion

Self-adjustment of a special type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0 =$ const. In principle, this solves the main cosmological constant problem (CCP1).

A generic problem of adjustment-type solutions of CCP1 is the catastrophic modification of the Newtonian dynamics of small self-gravitating systems [e.g., $G = G(t) \neq G_N$'].

For a very special model with two massless vector-fields, it is possible to have asymptotically both a standard FRW universe on large scales and standard Newtonian dynamics on small scales.

The physical interpretation of this particular type of model is, however, unclear. Somehow, the two vector fields conspire to give a self-adjusting fluid with infinite compressibility (i.e., perfectly soft and flexible).

Such a fluid may have applications not only to cosmology but even to cosmetics . . .

5. References

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