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Cosmological constant and q–theory

(new title, new talk*)

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* For the announced talk, see arXiv:1103.1569.

0.1 Introduction

The main **Cosmological Constant Problem** (CCP1) can be phrased as follows (Pauli, 1933; Bohr, 1948; Veltman, 1974; see [1, 2] for reviews):

why do the quantum fields in the vacuum not produce naturally a large cosmological constant Λ *in the Einstein field equations?*

The magnitude of the problem is enormous:

$$|\Lambda^{\text{theory}}|/|\Lambda^{\text{experiment}}| \geq 10^{42}\,.$$

0.1 Introduction

Indeed, it is known that QCD in the laboratory involves a vacuum energy density (e.g., gluon condensate) of <u>order</u>

$$\left|\epsilon_V^{\text{(QCD)}}\right| \sim \left(100\ \text{MeV}\right)^4 \sim 10^{32}\ \text{eV}^4\,.$$

Moreover, this energy density can be expected to <u>change</u> as the temperature T of the Universe drops,

$$\epsilon_V^{\text{(QCD)}} = \epsilon_V^{\text{(QCD)}}(T)\,.$$

How can the Universe then <u>end up</u> with a vacuum energy density

$$\left|\Lambda^{\text{(obs)}}\right| < 10^{-28}\ \text{g cm}^{-3} \sim 10^{-10}\ \text{eV}^4\ ?$$

Here, there are 42 orders of magnitude to explain:

$$\left|\Lambda^{\text{(obs)}}/\epsilon_V^{\text{(QCD)}}\right| \leq 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,001\,.$$

0.1 Introduction

Even more CCPs after the discovery of the “accelerating Universe”:

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{Planck}})^4$?

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}} \big|_{\text{present}} \sim +10^{-11}\ \text{eV}^4$?

Hundreds of papers have been published on CCP2. But, most likely:

CCP1 needs to be solved first before CCP2 can even be addressed.

0.1 Introduction

Here, a review of one particular approach to CCP1 by Volovik and the speaker, which goes under the name of ***q*–theory** [3, 4, 5].

Turning to CCP2, some brief remarks on three possible **mechanisms** for the creation of a positive remnant vacuum energy density $\rho_V(t_{\text{present}})$ [or effective cosmological constant Λ_{eff}] from:

- nonperturbative QCD [6, 7];
- new TeV–scale physics [8, 9];
- light massive neutrinos [10].

0.2 Outline

1. **Basics of q–theory** $\longleftarrow$ most important part of talk
2. **Two questions**
3. **Remnant $\rho_V(t_{\text{present}})$** $\longleftarrow$ predictions?
4. **Conclusions**
5. **References**

1. Basics of q–theory

Crucial insight [3]: there is vacuum energy and vacuum energy.

More specifically and introducing an appropriate notation:

the vacuum energy density $\boxed{\epsilon}$ appearing in the action
need not be the same as
the vacuum energy density $\boxed{\rho_V}$ in the Einstein field equations.

How can this happen concretely . . .

1. Basics of q–theory

Consider the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Study, then, the **macroscopic** equations of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy:

- Take the mass density ρ of a liquid, for example, liquid Argon.
- This ρ describes microscopic quantities ($\rho = m_{\text{Ar}}\, n_{\text{Ar}}$ with number density n_{Ar} and mass m_{Ar} of the atoms).
- Still, ρ obeys the macroscopic equations of hydrodynamics, because of particle-number and mass conservation.

However, is the quantum vacuum just like a normal liquid?

1. Basics of q–theory

No, as the quantum vacuum is known to be **Lorentz invariant** (cf. exp. limits at the 10^{-15} level in the photon sector [11, 12, 13]).

The Lorentz invariance of the vacuum rules out the standard type of charge density, which arises from the time component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material liquids.

1. Basics of q–theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{var}}(q)\,, \tag{1}$$

including a possible constant term Λ_{bare} from the zero-point energies of the fields of the Standard Model (SM).

From ① thermodynamics and ② Lorentz invariance follows that [3]

$$P_V \overset{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \overset{②}{=} -\rho_V\,, \tag{2}$$

where the first equality corresponds to an integrated form of the Gibbs–Duhem equation for chemical potential $\mu \equiv d\epsilon/dq$.

Recall GD eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

1. Basics of q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{Planck}})^4$, but they cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 = \text{const} \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\,\frac{d\,\epsilon(q)}{d\,q}\right]_{q=q_0} = 0\,, \tag{3}$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle . . .

But, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q (see later).

2. Two questions

Q1: How does the adjustment-type solution (3) of CCP1 circumvent Weinberg's no–go "theorem" [1]?

Answer: q is a non-fundamental scalar field; see Sec. 2.1.

Q2: How did the Universe get the right value q_0?

One possible answer is that q_0 (or the corresponding chemical potential μ_0) is fixed globally as an integration constant, being conserved throughout the history of the Universe [4].

Another possible answer uses a generalization of q–theory, for which the 'correct' value q_0 arises dynamically; see Sec. 2.2.

2.1 Four-form realization

Vacuum variable q may arise from a 3–form gauge field A [14, 15].

Start from the effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = \int_{\mathbb{R}^4} d^4x \sqrt{-\det g}\,\Big(K_N\, R[g] + \Lambda_{\text{SM}} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \tag{4}$$

with gravitational coupling constant $K_N \equiv 1/(16\pi G_N)$ and $\hbar = c = 1$.

Change this theory by the introduction of one field, A, to get [4, 5]:

$$\widetilde{S}^{\text{eff}}[A,g,\psi] \;=\; \int_{\mathbb{R}^4} d^4x \sqrt{-\det g}\,\Big(K(q)\, R[g] + \widetilde{\epsilon}(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \tag{5a}$$

$$q \;\equiv\; -\tfrac{1}{24}\, \epsilon^{\alpha\beta\gamma\delta}\, \nabla_\alpha A_{\beta\gamma\delta}\,/\,\sqrt{-g}\,, \tag{5b}$$

where q arises from the four-form field strength $F = d\,A$.

Variational principle gives generalized Einstein and Maxwell equations:

2.1 Four-form realization

$$2K(q)\left(R_{\alpha\beta} - g_{\alpha\beta}\,R/2\right) = -2\left(\nabla_\alpha\nabla_\beta - g_{\alpha\beta}\,\Box\right)K(q) + \rho_V(q)\,g_{\alpha\beta} - T^M_{\alpha\beta}\,, \tag{6a}$$

$$\frac{d\rho_V(q)}{dq} + R\,\frac{dK(q)}{dq} = 0\,, \tag{6b}$$

with a vacuum energy density,

$$\rho_V = \widetilde{\epsilon} - q\left(\frac{d\widetilde{\epsilon}}{dq} + R\,\frac{dK}{dq}\right) = \widetilde{\epsilon} - q\,\mu\,, \tag{7}$$

for integration constant (chemical potential) μ. Eq. (7) is precisely of the Gibbs–Duhem form (2) in Minkowski spacetime ($R = 0$). Technically, an extra $g_{\alpha\beta}$ term on the RHS of (6a) appears because $q = q(A,\,g)$.

Answer to Q1: (5b) shows that q is a non-fundamental scalar field, which invalidates Weinberg's argument (see [5] for details).

2.2 Vector-field realization

Realization of vacuum variable q by aether-type velocity field u_β [17, 18], setting $E_{\text{UV}} = E_{\text{Planck}}$. For a flat RW metric with cosmic time t, there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \quad\rightarrow\quad q_0\, t\,, \quad u_b(t) = 0\,, \quad H(t) \rightarrow 1/t\,, \tag{8a}$$

$$u_\alpha{}^\beta \equiv \nabla_\alpha\, u^\beta \quad\rightarrow\quad q_0\, \delta_\alpha{}^\beta\,. \tag{8b}$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t\, E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. Then, the field equations are [17]:

$$\ddot{v} + 3\,h\,\dot{v} - 3\,h^2\,v \quad=\quad 0\,, \tag{9a}$$

$$2\,\lambda - (\dot{v})^2 - 3\,(h\,v)^2 \quad=\quad 6\,h^2\,, \tag{9b}$$

with the overdot standing for differentiation with respect to τ. Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\widehat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\widehat{q}_0 = \sqrt{\lambda/2}$.

2.2 Vector-field realization

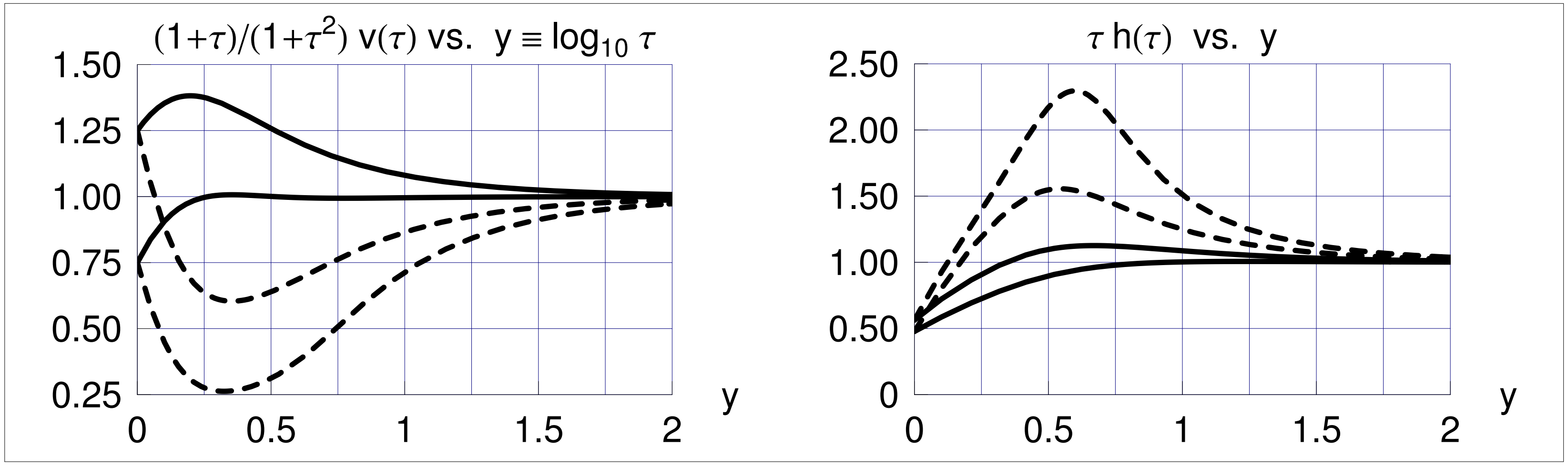

Fig. 1: Four numerical solutions of ODEs (9ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25$.

$\Rightarrow$ Minkowski value $\widehat{q}_0 = \sqrt{\lambda/2} = 1$ arises dynamically [see left panel].

$\Rightarrow$ Minkowski spacetime is an attractor in this aether-type theory [5].

(Incidentally, this theory may be relevant to the early Universe but not the present one [19].)

2.3 Recap

To summarize, the q–theory approach to the main Cosmological Constant Problem (CCP1) provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics harbors an appropriate q–type variable.

Still, better to have one possible solution than none.

3. Remnant ρ_V

Now, the remaining problems (or puzzles, rather):

CCP2a – why $\Lambda_{\text{eff}} \neq 0$?

CCP2b – why $\Lambda_{\text{eff}} \sim \rho_{\text{matter}}\big|_{\text{now}} \sim 10^{-29}$ g cm^{-3} $\sim 10^{-11}$ eV^4 ?

Last one also goes under the name of ‘cosmic coincidence puzzle’ (ccp).

Here, discuss <u>three</u> possible mechanisms in the framework of q–theory.

(If time is short, fast forward.)

3.1 Remnant ρ_V – QCD

Gluon condensate [20] from quantum chromodynamics (QCD):

$$\widetilde{q} \equiv \langle \frac{1}{4\pi^2} G^{a\,\mu\nu}\, G^a{}_{\mu\nu} \rangle = \langle \frac{1}{4\pi^2} G_{a\,\kappa\lambda}\, g^{\kappa\mu} g^{\lambda\nu}\, G^a{}_{\mu\nu} \rangle \,, \tag{10}$$

with Yang–Mills field strength $G^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + f^{abc}\, A^b{}_\mu A^c{}_\nu$
for $su(3)$ structure constants f^{abc}.

Particle physics experiments: $\widetilde{q} \sim (300\,\text{MeV})^4$.

Observational cosmology: $\rho_V \sim (2\,\text{meV})^4$.

$\Rightarrow$ How to reconcile the typical QCD vacuum energy density $\epsilon_{\text{QCD}} \sim 10^{34}$ eV^4 with the observed value $\rho_V \sim 10^{-11}$ eV^4 ?

General q–theory argument (Sec. 1):

in equilibrium, $\widetilde{q}$ has self-adjusted to the value $\widetilde{q}_0$ with $\rho_V(\widetilde{q}_0) = 0$.

3.1 Remnant ρ_V – QCD

Effective action for the gluon condensate q from (10) [dropping tilde]:

$$S_{\text{eff}} = S_{\text{grav}} + S_{\text{vac}} = \int d^4x \sqrt{-\text{det}(g)} \left(\frac{1}{16\pi G_N} R[g] + \epsilon_{\text{vac}}(q) \right). \quad (11)$$

Energy-momentum tensor for the gravitational field equations:

$$T^{\text{vac}}_{\alpha\beta} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{vac}}}{\delta g^{\alpha\beta}} = \epsilon_{\text{vac}}(q)\, g_{\alpha\beta} - 2\, \frac{\text{d}\epsilon_{\text{vac}}(q)}{\text{d}q} \frac{\delta q}{\delta g^{\alpha\beta}}$$

$$= \left(\epsilon_{\text{vac}}(q) - q\, \frac{\text{d}\epsilon_{\text{vac}}(q)}{\text{d}q} \right) g_{\alpha\beta} \equiv \rho_V(q)\, g_{\alpha\beta}\,, \quad (12)$$

which is, again, of the Gibbs–Duhem form (2).

$\Rightarrow$ equilibrium state: $q = q_0$ with $\rho_V(q_0) = 0$ and $g_{\alpha\beta}(x) = \eta^{\text{Minkowski}}_{\alpha\beta}$.

3.1 Remnant ρ_V – QCD

In a nonequilibrium state such as the expanding Universe [with Hubble parameter $H(t) \neq 0$], there is a perturbation of the vacuum:

$$q = q_0 + \delta q(H) \neq q_0 \quad \Rightarrow \quad \rho_V(q) \sim \frac{\mathrm{d}\rho_V}{\mathrm{d}q}\,\delta q(H) \neq 0\,. \tag{13}$$

For QCD, this is a difficult IR problem (cf. [21, 22]). *A priori*, can have

$$\begin{aligned} \rho_V(H) \quad &\sim \quad 0 + H^2\,\Lambda_{\text{QCD}}^2 + H^4 + \cdots \\ &\qquad + |H|\,\Lambda_{\text{QCD}}^3 + |H|^3\,\Lambda_{\text{QCD}} + \cdots \end{aligned} \tag{14}$$

Linear term in H gives correct order of magnitude for asymptotic ρ_V [6].

As a spatially-flat RW universe has Ricci scalar $R = 6\big(2\,H^2 + \dot{H}\big)$, the $|H|\,\Lambda_{\text{QCD}}^3$ term suggests a modified-gravity action with term $|R|^{1/2}\,|q|^{3/4}$.

This QCD-scale modified-gravity universe fits astronomical data well [7].

3.2 Remnant ρ_V – Electroweak kick

Reconsider the four-form realization of q, taken to be operative at an UV (Planckian) energy scale.

In the very early Universe, the vacuum energy density $\rho_V(t)$ rapidly drops to zero and stays there, but small effects may occur at cosmic temperatures T of the order of the TeV scale . . .

Simple picture:

Take a glass of water and shake the glass $\Rightarrow$ water responds.

If vacuum energy density is really like a liquid, then it can be 'shaken.' Here, the 'shaking' is done by massive particles.

3.2 Remnant ρ_V – Electroweak kick

Key steps of **frozen-electroweak-kick mechanism** [8, 9]:

- Presence of massive particles with electroweak interactions [average mass $< M > = E_{\text{ew}} \sim$ TeV] changes the expansion rate $H(t)$ of the Universe compared to the radiation-dominated case.
- Change of the expansion rate kicks $\rho_V(t)$ away from zero.
- Quantum-dissipative effects operating at cosmic time t_{ew} set by E_{ew} may result in finite remnant value of ρ_V.
- Phenomenological description of this process with a simple field-theoretic model.
- Required E_{ew} value ranges from 2 to 20 TeV, depending on the number of new particles and details of the model.

3.3 Remnant ρ_V – Massive neutrinos

In the very early Universe, vacuum energy density $\rho_V(t)$ drops fast.

Consider the effects from massive SM fermions (larger in number than the SM bosons). These fermions get a mass as the temperature drops below a critical value and decrease $\rho_V(t)$ in a stepwise manner [10].

In fact, the mass effects on the zero-point energies of the SM quark and charged-lepton fields are:

$$\rho_V^{\text{(charged fermion)}} \sim \int^{(E_{\text{cutoff}})} \frac{d^3\mathbf{p}}{(2\pi)^3} \left(-\sqrt{|\mathbf{p}|^2 + M^2} + |\mathbf{p}| \right)$$
$$\sim -M^2 \left(E_{\text{cutoff}}\right)^2 \sim -M^2 \left(E_{\text{ew}}\right)^2 , \qquad (15)$$

with $E_{\text{cutoff}} \sim E_{\text{ew}}$ in the last step (cf. symmetry restoration at $T_c \sim E_{\text{ew}}$). But a massive neutrino is different and may have $E_{\text{cutoff}} \sim M_\nu$, so that:

$$\rho_V^{\text{(neutrino)}} \sim -(M_\nu)^4 . \qquad (16)$$

3.3 Remnant ρ_V – Massive neutrinos

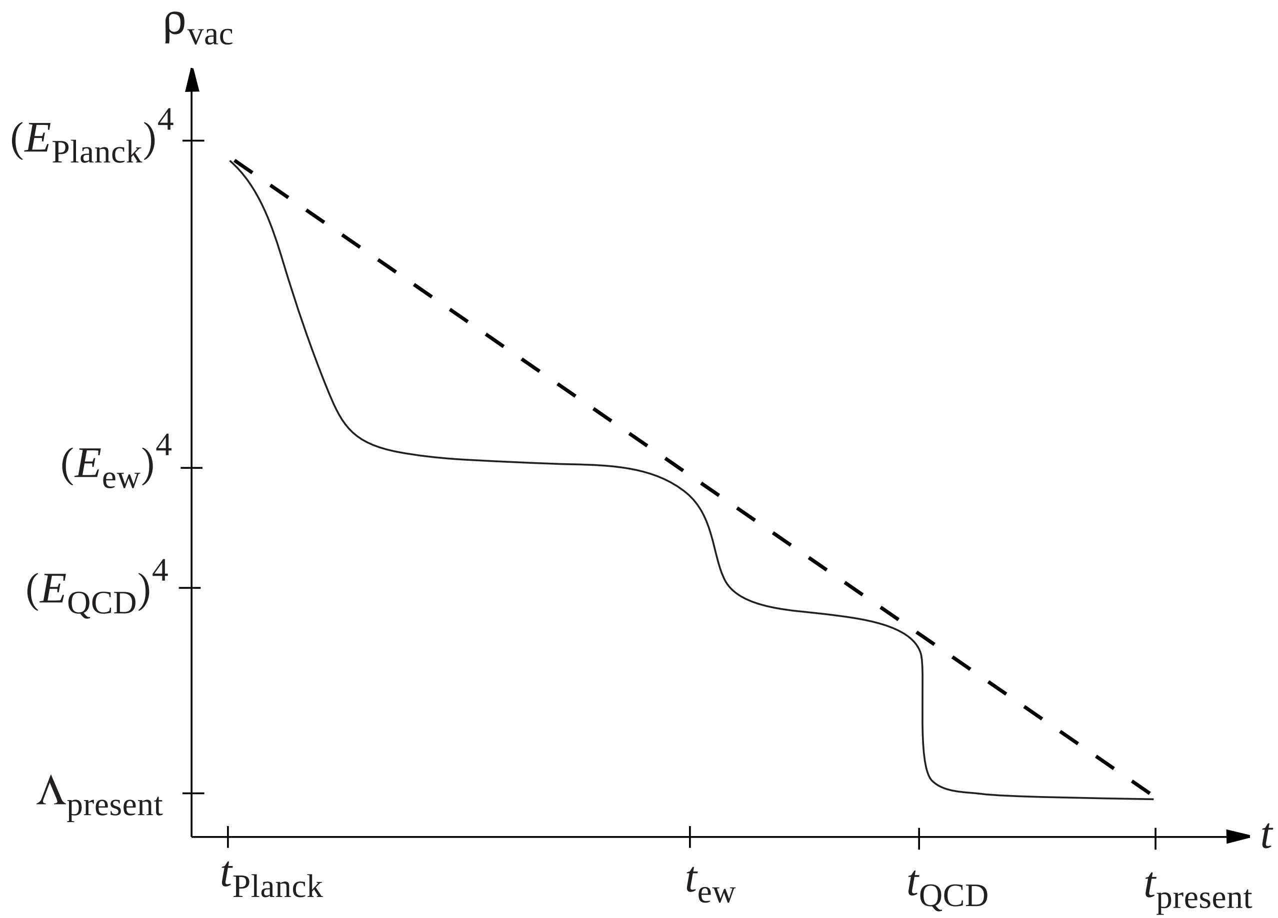

Fig. 2: Approximate double-log plot of the relaxation of the vacuum energy density. Dashed curve: relaxation according to $< \rho_{\text{vac}}(t) > \sim (E_{\text{Planck}})^2/t^2$ from Ref. [4]. Full curve: dissipative processes and cosmological phase transitions included [10].

3.3 Remnant ρ_V – Massive neutrinos

Key steps of **non-equilibrated-neutrino mechanism** [10]:

- The last fermions to get massive are the neutrinos, consider the heaviest one.
- This neutrino gives, in principle, the change $\Delta\rho_V \sim -(M_\nu)^4$, aiming for $\rho_{V,\,\infty} = 0$ of the self-sustained equilibrium state from q–theory.
- But perhaps interactions of virtual neutrinos in the quantum vacuum are too weak to make the transition.
- Lack of negative contribution corresponds to a positive ρ_V value.

Final formula for a single light massive neutrino:

$$\rho_V(t_{\text{present}}) \;\sim\; 0 + (M_\nu)^4 \,. \tag{17}$$

For three neutrino flavors with near-maximal mixing, neutrino mass spectrum is close to the minimal one: $0 \lesssim m_{\nu n} \lesssim 0.05$ eV for $n = 1, 2, 3$.

4. Conclusions

CCP1: Self-adjustment of a special type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0 =$ const.

CCP2: Within the q–theory framework, a finite remnant value of $\rho_V(t)$ may result from

(i) nonperturbative “finite-size effects” of QCD
$\Rightarrow$ verified by lattice-gauge-theory simulations?

or

(ii) a “kick” by massive particles with $M \sim E_{\text{ew}}$
$\Rightarrow$ new TeV–scale physics beyond the SM?

or

(iii) non-equilibrated neutrinos in the quantum vacuum
$\Rightarrow$ small neutrino masses, $m_{\nu n} \lesssim 0.05$ eV?

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App. A1: Relaxation of $\rho_V(t)$

Spatially-flat (F)RW universe with two types of matter, massive ('type 1') and massless ('type 2') particles. From fields eqs. (6), get ODEs [4]:

$$6\left(H\,\frac{dK}{dq}\,\frac{dq}{dt}+K\,H^2\right) = \rho_V+\rho_{M1}+\rho_{M2}\,, \qquad \text{(A.1a)}$$

$$6\,\frac{dK}{dq}\left(\frac{dH}{dt}+2H^2\right) = \frac{d\rho_V}{dq}\,, \qquad \text{(A.1b)}$$

$$\frac{d\rho_{M1}}{dt}+3\left[1+w_{M1}\right]H\,\rho_{M1} = 0\,, \qquad \text{(A.1c)}$$

$$\frac{d\rho_{M2}}{dt}+4\,H\,\rho_{M2} = 0\,, \qquad \text{(A.1d)}$$

with equation-of-state (EOS) function $w_{M1}(t)$.

App. A1: Relaxation of $\rho_V(t)$

If Universe starts out with $\rho_V \sim (E_{\text{Planck}})^4$ at $t \sim t_{\text{Planck}}$, then $\rho_V \to 0$ by oscillations of $q(t)$ and coupling to the gravitational field.

Indeed, with simple *Ansätze*

$$K(q) = \tfrac{1}{2}\, q\,, \tag{A.2a}$$

$$\rho_V(q) = \tfrac{1}{2}\,(q-q_0)^2 + \mathsf{O}\big((q-q_0)^3\big)\,, \tag{A.2b}$$

the following behavior is found [4]:

$$q(\tau)/q_0 - 1 \sim \tau^{-1}\,\sin\tau\,, \tag{A.3a}$$

$$r_V(\tau) \sim \tau^{-2}\,\sin^2\tau\,, \tag{A.3b}$$

in terms of the dimensionless cosmic time τ and the dimensionless vacuum energy density r_V obtained by scaling with $q_0 = (E_{\text{Planck}})^2$.

App. A1: Relaxation of $\rho_V(t)$

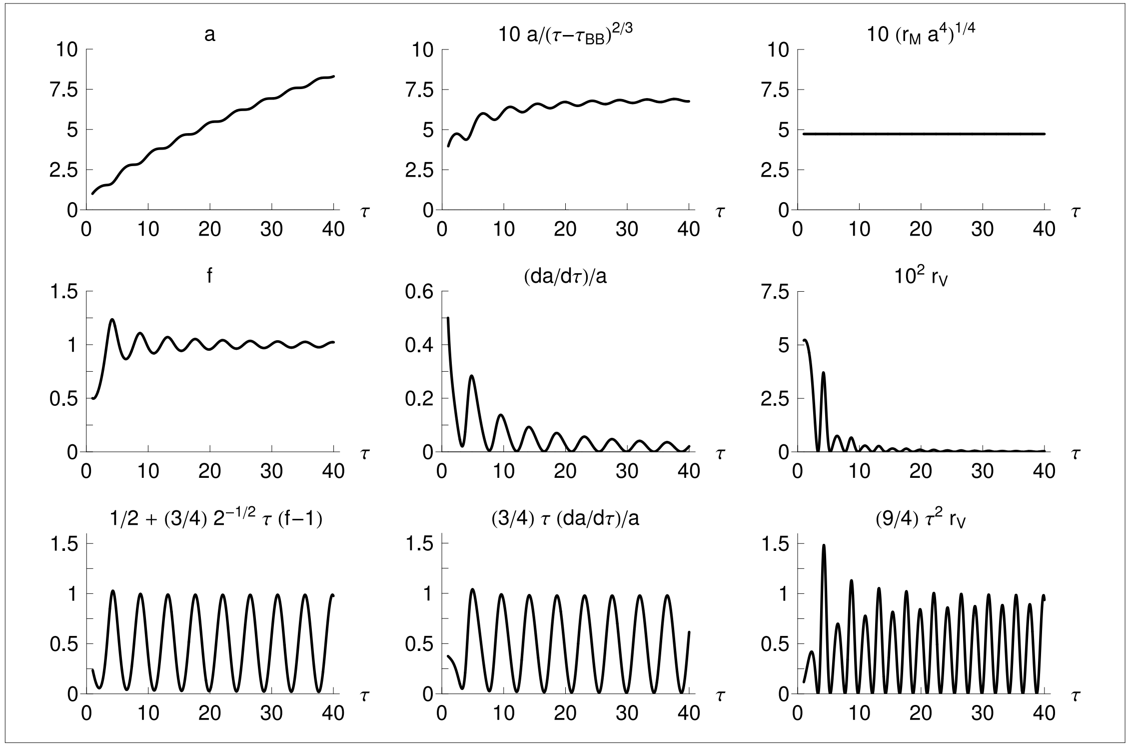

Fig. A1: Flat (F)RW model universe [4] with ultrarelativistic matter ($w_M \equiv P_M/\rho_M = 1/3$) and dynamic vacuum energy density ($w_V \equiv P_V/\rho_V = -1$). The dimensionless q–type variable is denoted by f. Asymptotic behavior on bottom-row panels: $|f-1| \propto 1/\tau$, $h \propto 1/\tau$, and $r_V \propto 1/\tau^2$.

App. A2: Remnant ρ_V from EW kick

Theoretical value of the effective cosmological constant given by

$$\Lambda^{\text{theory}} \equiv \lim_{t\to\infty} \rho_V^{\text{theory}}(t) = r_V^{\text{num}} \, (E_{\text{ew}})^8 \,/\, (E_{\text{Planck}})^4 \,, \tag{A.4}$$

with number $r_V^{\text{num}} \equiv r_V(\tau_{\text{freeze}})$ from solution ODEs (cf. Fig. A2).
Eq. (A.4) already suggested in [16], but first calculated in [8, 9].

Equating (A.4) to the experimental value $\Lambda^{\text{exp}} \approx (2\ \text{meV})^4$ gives

$$E_{\text{ew}} = \left(\frac{\Lambda^{\text{exp}}}{r_V^{\text{num}}}\right)^{1/8} (E_{\text{Planck}})^{1/2} \approx 3.8\ \text{TeV} \left(\frac{0.013}{r_V^{\text{num}}}\right)^{1/8} . \tag{A.5}$$

Analytic bound: $r_V^{\text{num}} \lesssim 1 \Rightarrow E_{\text{ew}} \gtrsim 2$ TeV.
Numerical results for r_V^{num} give E_{ew} estimates of Table A1.

App. A2: Remnant ρ_V from EW kick

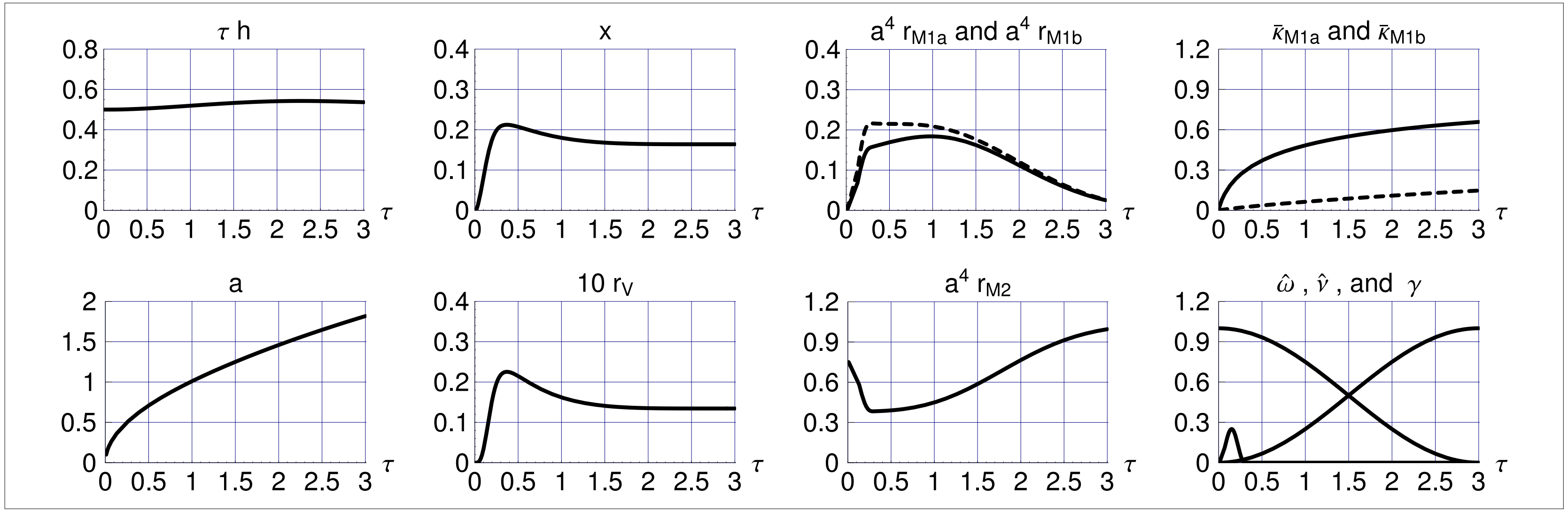

Fig. A2 (same as Fig. B4 in App. B2): Sudden presence of massive (type–1) particles kicks vacuum energy density $r_V(t)$ away from zero. Quantum-dissipative effects freeze $r_V(t)$ to a nonzero value as $t \to \infty$.

App. A2: Remnant ρ_V from EW kick

Table A1: Preliminary estimates [9, (a)] of the energy scale E_{ew} for hierarchy parameter $\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4 \gg 1$. Both massive type–1 and massless type–2 particles are assumed to have been in thermal equilibrium before the "kick" and the number of type–2 particles is taken as $N_{\text{eff},2} = 10^2$. See App. B2 for details.
Left: Prescribed kick with type–1 particles of equal mass $M = E_{\text{ew}}$ and, for dissipative coupling constant $\zeta = 2$, E_{ew} shown as a function of the effective number of d.o.f. $N_{\text{eff},1}$.
Right: Dynamic kick with case–A type–1 mass spectrum $(N_{1a},\, M_{1a}\,;\, N_{1b},\, M_{1b}) = (40,\, 2\times E_{\text{ew}};\, 60,\, 1/3\times E_{\text{ew}})$ and $E_{\text{ew}} =< M_{1i} >$ shown as a function of ζ.

ζ	$N_{\text{eff},1}$	E_{ew} [TeV]
2	1	8.5
2	10^1	4.9
2	10^2	$\boxed{3.2}$
2	10^3	2.8
2	10^4	2.7

ζ	$N_{\text{eff},1}$	E_{ew} [TeV]
0.2	10^2	14.8
2	10^2	$\boxed{3.8}$
20	10^2	5.6

App. A3: Remnant ρ_V from m_ν

For three neutrino flavors with near-maximal mixing, a heuristic argument suggests [10]:

$$\rho_V(t_{\text{present}}) \stackrel{?}{=} c_\nu \, \Big((m_{\nu 1})^2 + (m_{\nu 2})^2 + (m_{\nu 3})^2 \Big) \, \Big| m_{\nu 1} \, m_{\nu 2} \, m_{\nu 3} \Big|^{2/3} , \quad \text{(A.6)}$$

where c_ν is a positive coefficient assumed to be of order unity.

With neutrino-oscillation data, then have 3 equations for 3 unknowns. Taking $c_\nu = 1$, the solutions for the two possible hierarchies are:

$$\Big(m_{\nu 1},\, m_{\nu 2},\, m_{\nu 3} \Big) \Big|^{(c_\nu = 1)} \stackrel{?}{=} \begin{cases} (2.793 \times 10^{-6},\, 8.775,\, 48.99) \times \text{meV}, \\ (48.99,\, 49.77,\, 1.783 \times 10^{-7}) \times \text{meV}. \end{cases} \quad \text{(A.7)}$$

These neutrino masses cannot be detected by the KATRIN tritium beta-decay detector (0.2–eV design sensitivity).

Table A2 shows that the same conclusion holds for $c_\nu \geq 10^{-8}$.

App. A3: Remnant ρ_V from m_ν

Table A2: Neutrino masses [in units of meV] from (A.6) and neutrino-oscillation data $(m_{\nu 3})^2 - (m_{\nu 1})^2 = \pm\, 2400$ and $(m_{\nu 2})^2 - (m_{\nu 1})^2 = 77$.

c_ν	$m_{\nu 1}$	$m_{\nu 2}$	$m_{\nu 3}$	$m_{\nu 1}$	$m_{\nu 2}$	$m_{\nu 3}$
1	2.793×10^{-6}	8.775	48.99	48.99	49.77	1.783×10^{-7}
10^{-2}	2.793×10^{-3}	8.775	48.99	48.99	49.77	1.783×10^{-4}
10^{-4}	2.638	9.163	49.06	48.99	49.77	0.1783
10^{-6}	48.13	48.93	68.68	62.95	63.56	39.54
10^{-8}	172.4	172.6	179.2	177.0	177.2	170.1
10^{-10}	551.9	552.0	554.1	553.3	553.4	551.2

$\Rightarrow$ neutrino masses from $c_\nu \gtrsim 10^{-4}$ are close to the minimal values needed to explain the neutrino-oscillation data.

App. B1: Electroweak kick

Analytic solution [8] of the ODEs (A.1) which

- starts from a standard radiation-dominated FRW universe with $\rho_V = 0$,
- is perturbed around $t = t_{\text{ew}} \sim E_{\text{Planck}}/(E_{\text{ew}})^2$ with $\rho_V \neq 0$,
- resumes the standard radiation-dominated expansion with $\rho_V = 0$.

Specifically, the vacuum energy density for $t \sim t_{\text{ew}}$ is given by

$$\rho_V(t) \sim (1 - 3\, w_{M1})^2(t)\, H(t)^4\,, \tag{B.1}$$

which has a peak value of order $(t_{\text{ew}})^{-4} \sim \left((E_{\text{ew}})^2 / E_{\text{Planck}}\right)^4$ but vanishes as $t \to \infty$.

$\Rightarrow$ standard (nondissipative) dynamic equations of q–theory do not produce a constant $\rho_{V,\,\text{remnant}} > 0$ from the electroweak kick.

App. B1: Electroweak kick

As argued in [8], quantum-dissipative effects of the vacuum energy density may lead to a finite remnant value of order

$$\Lambda \equiv \rho_{V,\,\text{remnant}} \sim \left((E_{\text{ew}})^2 / E_{\text{Planck}} \right)^4 \sim (10^{-3}\ \text{eV})^4 \,, \tag{B.2}$$

for $E_{\text{ew}} \sim 1$ TeV and $E_{\text{Planck}} \sim 10^{15}$ TeV. In fact, expression (B.2) was already suggested by Arkani-Hamed, Hall, Kolda, and Murayama [16].

It is possible [9, (a)] to modify the “classical” q–theory equations (A.1) in such a way as to recover (B.2).
Even better, a simple field-theory model has been presented in [9, (b)].

Details for modified ODEs in App. B2 and for simple model in App. C.

App. B2: Model universe

Model universe with three components (see App. A of [9, (a)]):

0. Vacuum variable q entering the gravitational coupling $K(q)$.
1. Massive 'type 1' particles (subspecies $i = a, b, c, \ldots$) with masses M_i of order $E_{\text{ew}} \sim 1$ TeV and electroweak interactions.
2. Massless 'type 2' particles with electroweak interactions.

Now, proceed as follows:

- Consider a flat RW universe with Hubble parameter $H(t)$.
- Allow for energy exchange between the two matter components, so that total type–1 energy density peaks around $t_{\text{ew}} \equiv E_{\text{Planck}}/(E_{\text{ew}})^2$.
- Get EOS function $\overline{\kappa}_{M1i}(t) \equiv 1 - 3\, w_{M1i}(t)$ with $\overline{\kappa}_{M1i}(t) \sim 0$ for $t \ll t_{\text{ew}}$ in the ultrarelativistic regime.
- Introduce a dissipative coupling constant $\zeta = \text{O}(1)$ and a function $\gamma(t)$ which equals 1 for $t \ll t_{\text{ew}}$ and drops to zero for $t > t_{\text{ew}}$.

App. B2: Model universe

Modified q–theory ODEs (standard ODEs recovered for $\zeta = 0$ and $\gamma = 1$):

$$6\,\left(H\,K'\,\dot{q} + K\,H^2\right) = \rho_V + \sum_{i=a,b,c,\ldots} \rho_{M1i} + \rho_{M2}\,, \tag{B.3a}$$

$$6\,K'\left(\dot{H} + 2H^2\right) = \gamma\,\rho'_V + (1-\gamma)\,\frac{K'}{K}\left[2\rho_V + \sum_i \tfrac{1}{2}\,\overline{\kappa}_{M1i}\,\rho_{M1i}\right], \tag{B.3b}$$

$$\dot{\rho}_{M1i} + (4 - \overline{\kappa}_{M1i})H\rho_{M1i} = \frac{N_{1i}}{N_1}\left[\frac{\lambda_{21}}{t_{\text{ew}}}\widehat{\omega}\,\rho_{M2} - \frac{\zeta}{\gamma}q\,\dot{\rho}'_V\right] - \frac{\lambda_{12}}{t_{\text{ew}}}\widehat{\nu}\,\rho_{M1i}, \tag{B.3c}$$

$$\dot{\rho}_{M2} + 4\,H\,\rho_{M2} = -\frac{\lambda_{21}}{t_{\text{ew}}}\,\widehat{\omega}\,\rho_{M2} + \frac{\lambda_{12}}{t_{\text{ew}}}\,\widehat{\nu}\,\sum_i \rho_{M1i}\,, \tag{B.3d}$$

where the overdot [prime] stands for differentiation with respect to t [q]. Functions γ, $\widehat{\omega}$, and $\widehat{\nu}$ shown in Figs. B1–B4 below.

App. B2: Model universe

Use simple *Ansätze*: $\rho_V(q) \propto (q - q_0)^2$ and $K(q) \propto q$.

With t_{ew} and $\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4 \gg 1$, define dimensionless variables:

$$\tau \equiv (t_{\text{ew}})^{-1}\, t\,, \qquad h \equiv t_{\text{ew}}\, H\,, \tag{B.4a}$$

$$r_V \equiv (t_{\text{ew}})^4\, \rho_V\,, \qquad r_{Mn} \equiv \xi^{-1}\, (t_{\text{ew}})^4\, \rho_{Mn}\,, \tag{B.4b}$$

$$x \equiv \xi\, \big(q/q_0 - 1\big)\,. \tag{B.4c}$$

Figures B1–B3 and B4 show numerical results for $\xi = 10^2$ and $\xi = \infty$.

App. B2: Model universe

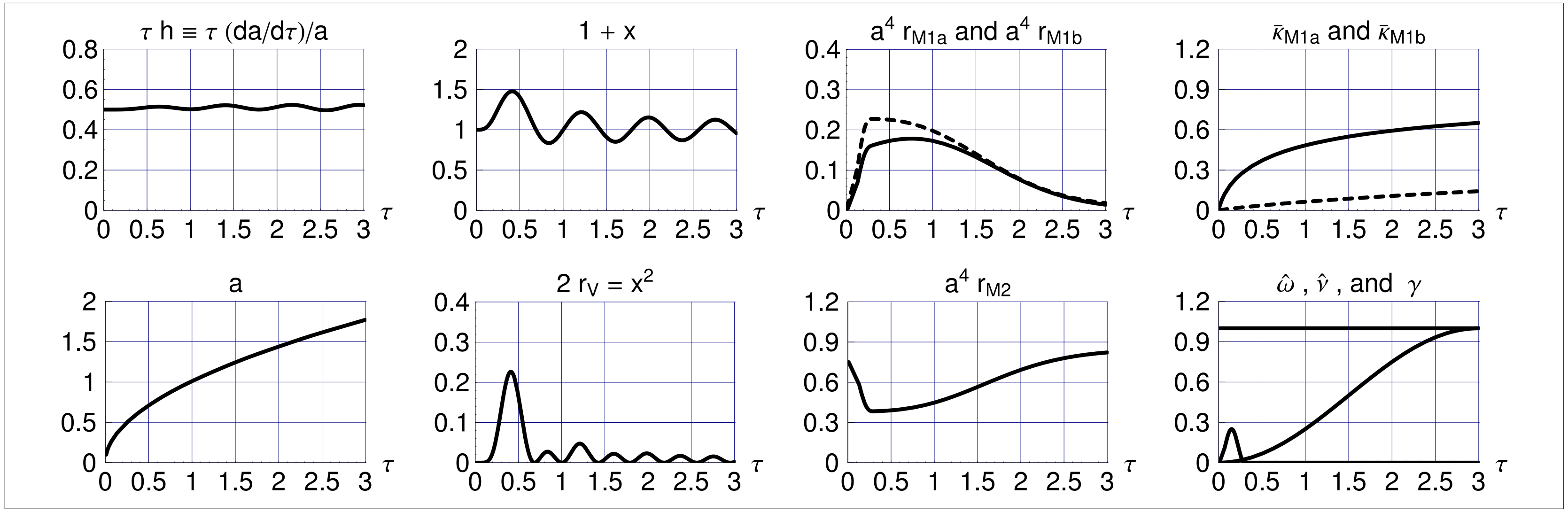

Fig. B1: Numerical solution [9, (a)] of standard (nondissipative) q–theory ODEs (B.3) for $\zeta = 0$ and $\gamma = 1$. The hierarchy parameter is $\xi = 10^2$ [oscillatory effects suppressed for larger values of ξ, recovering the smooth behavior of (B.1)]. Further coupling constants $\{\lambda_{21},\, \lambda_{12}\} = \{18,\, 2\}$ and case–A type–1 mass spectrum $(N_{1a},\, M_{1a}\,;\, N_{1b},\, M_{1b}) = (40,\, 2\, E_{\text{ew}};\, 60,\, 1/3\, E_{\text{ew}})$.

App. B2: Model universe

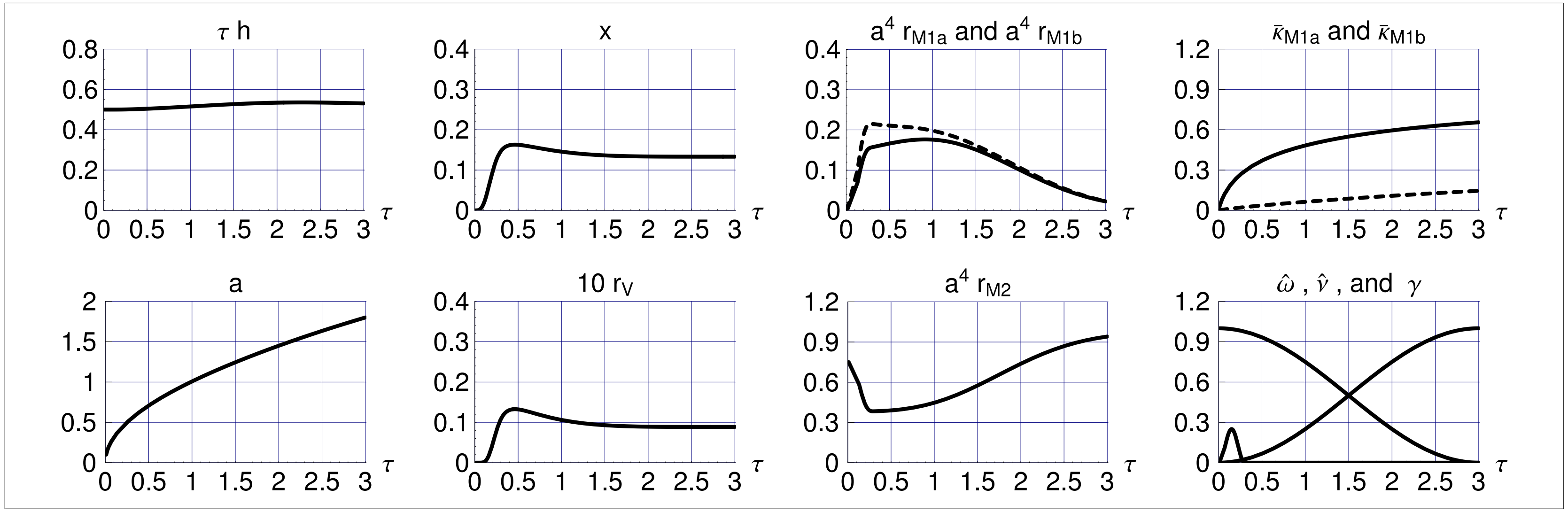

Fig. B2: Same as Fig. B1, but now for the <u>modified</u> q–theory ODEs (B.3) with dissipative coupling constant $\zeta = 2$ and $\gamma(\tau) = 0$ for $\tau \geq \tau_{\text{freeze}} = 3$.

App. B2: Model universe

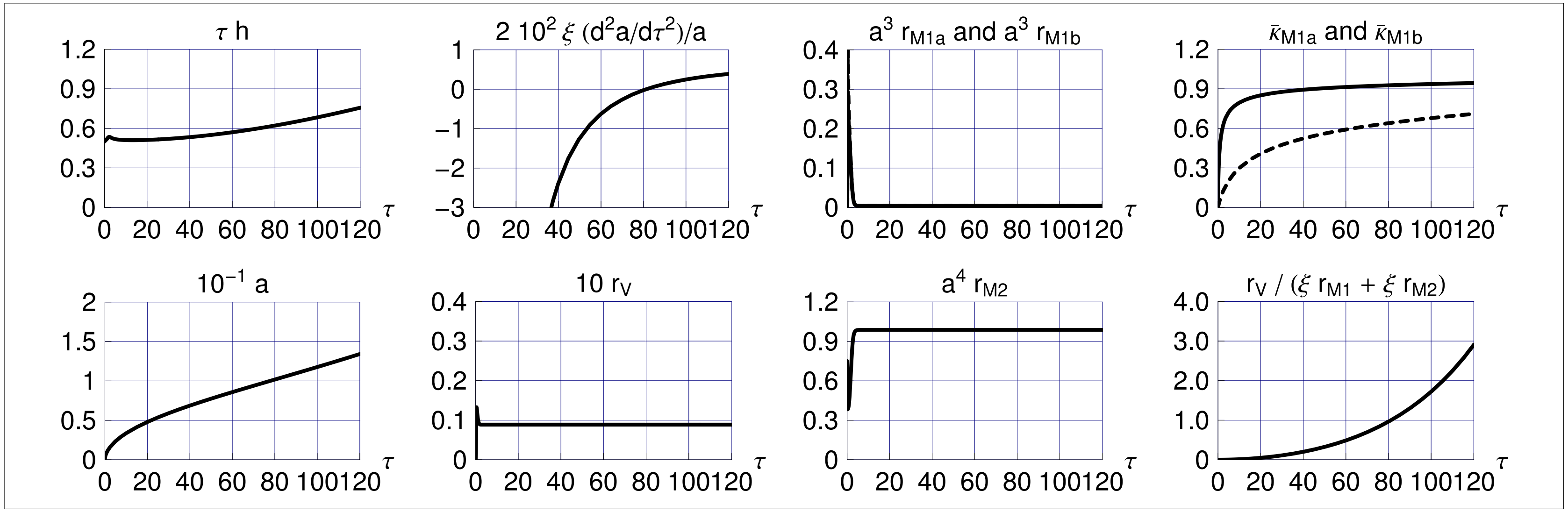

Fig. B3: Same as Fig. B2, but evolved further.

App. B2: Model universe

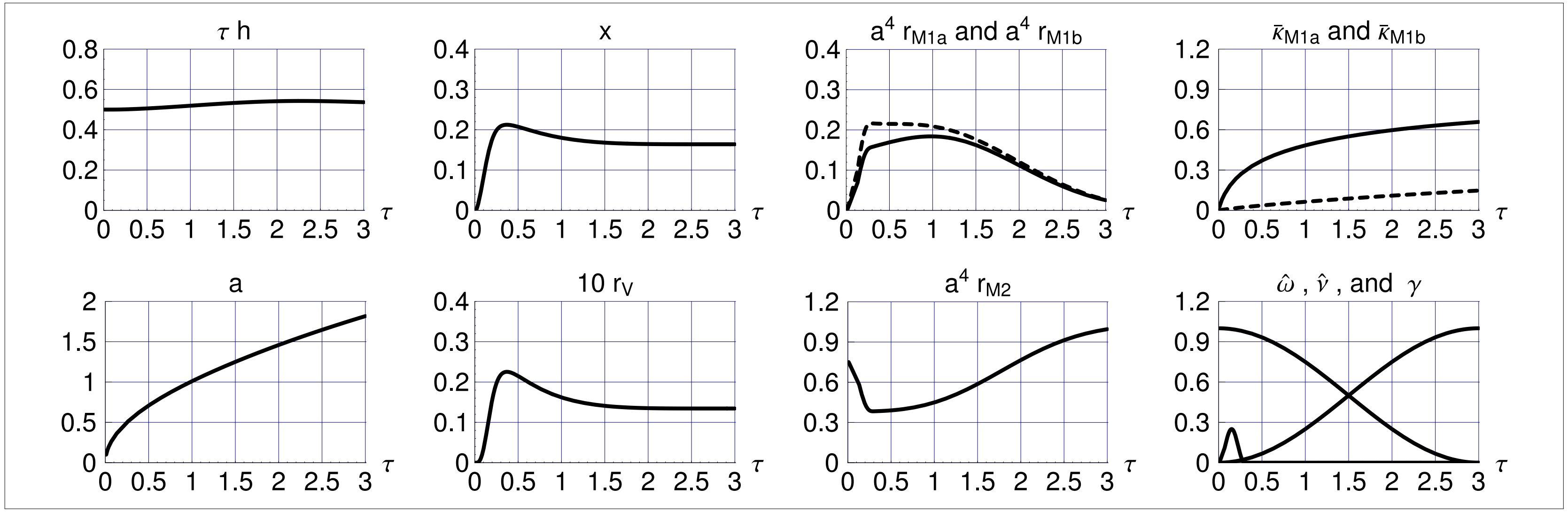

Fig. B4: Same as Fig. B2, but now for $\xi = \infty$.

App. C: Field-theoretic model

Simple field-theoretic model [9, (b)] generates an effective cosmological constant (remnant vacuum energy density) of order $\Lambda_{\text{eff}} \sim (\text{meV})^4$ from **TeV–scale ultramassive particles with electroweak interactions**.

The model is **simple** in the sense that it involves only a few types of fields and two energy scales, E_{Planck} and E_{ew}.

Specifically, two types of scalars:

- ultramassive (type–1) fields ϕ_a for $a = 1, \ldots, N_1$;
- massless (type–2) fields ψ_b for $b = 1, \ldots, N_2$;
- take $N_1 \overset{①}{=} N_2 \overset{②}{=} 10^2$ from ② SM and ① SUSY?.

Basic model equations are ($\hbar = c = k = 1$; signature $-, +, +, +$):

App. C: Field-theoretic model

$$S_{\text{eff},\,T} = \int_{\mathbb{R}^4} d^4x\,\sqrt{-g}\,\Big(K_T(q)\,R[g] + \epsilon_V(q) + \mathcal{L}^{M}_{\text{eff},\,T}[\phi,\psi,g]\Big)\,, \tag{C.1a}$$

$$q \equiv -\frac{1}{24}\,\epsilon^{\alpha\beta\gamma\delta}\,\nabla_{[\alpha}A_{\beta\gamma\delta]}\,/\,\sqrt{-g}\,, \tag{C.1b}$$

$$\rho_V(q) \equiv \epsilon_V(q) - \mu_0\,q = \frac{1}{2}\,(q-q_0)^2\,, \tag{C.1c}$$

$$K_T(q) = \begin{cases} q/2 & \text{for} \quad T > T^{(+)}_{c,\,K}\,, \\ q_0/2 & \text{for} \quad T \leq T^{(+)}_{c,\,K}\,, \end{cases} \tag{C.1d}$$

$$q_0 = 1/(8\pi\,G_N) \equiv (E_{\text{Planck}})^2 \approx (2.44\times 10^{18}\ \text{GeV})^2\,. \tag{C.1e}$$

App. C: Field-theoretic model

$$\mathcal{L}^{M}_{\text{eff},\,T} = \frac{1}{2}\,\partial_\alpha\psi\cdot\partial^\alpha\psi + \frac{1}{2}\,\partial_\alpha\phi\cdot\partial^\alpha\phi + \frac{1}{2}\,M^2\,(\phi\cdot\phi) + g_T\,(\psi\cdot\psi)\,(\phi\cdot\phi)\,, \tag{C.2a}$$

$$g_T = \begin{cases} g_0\left(1-(T/T_{c,\,g})^2\right) & \text{for} \quad T \leq T_{c,\,g}\,, \\ 0 & \text{for} \quad T > T_{c,\,g}\,, \end{cases} \tag{C.2b}$$

$$M = E_{\text{ew}}\,, \tag{C.2c}$$

$$T_{c,\,g} = \mathsf{O}(E_{\text{ew}})\,. \tag{C.2d}$$

$$T_{c,\,g} > T^{(+)}_{c,\,K} = \mathsf{O}(E_{\text{ew}})\,. \tag{C.2e}$$

$$\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4\,. \tag{C.3}$$

App. C: Field-theoretic model

Spatially flat, homogeneous, and isotropic (F)RW universe.

Timescale set by

$$t_{\text{ew}} \equiv E_{\text{Planck}}/(E_{\text{ew}})^2 = \big(1/\text{meV}\big)\,\big(\text{TeV}/E_{\text{ew}}\big)^2 \,. \tag{C.4}$$

Dimensionless variables:

$$\tau \equiv (t_{\text{ew}})^{-1}\, t\,, \qquad h \equiv t_{\text{ew}}\, H\,, \tag{C.5a}$$

$$r_{Mn} \equiv \xi^{-1}\,(t_{\text{ew}})^4\,\rho_{Mn}\,, \qquad r_V \equiv (t_{\text{ew}})^4\,\rho_V = x^2/2\,, \tag{C.5b}$$

$$x \equiv \xi\,\big(q/q_0 - 1\big)\,. \tag{C.5c}$$

App. C: Field-theoretic model

Dimensionless ODEs:

$$\left(\dot{h} + 2h^2\right)\left(x^2/2 + \xi\left(r_{M1} + r_{M2} - 3\,h^2\right)\right) - h\,x\,\dot{x} = 0\,, \tag{C.6a}$$

$$\dot{r}_{M1} + \left(4 - \overline{\kappa}_{M1}\right) h\, r_{M1} - \lambda_{21}\, r_{M2} + \lambda_{12}\, r_{M1} = 0\,, \tag{C.6b}$$

$$\dot{r}_{M2} + 4\, h\, r_{M2} + \lambda_{21}\, r_{M2} - \lambda_{12}\, r_{M1} = 0\,, \tag{C.6c}$$

$$3\,h\,\dot{x}\,\theta\left[r_{M2}(\tau) - r_{c,\,K}\right] - \left(x^2/2 + \xi\left(r_{M1} + r_{M2} - 3\,h^2\right)\right) = 0\,, \tag{C.6d}$$

with EOS function $\overline{\kappa}_{M1}$ from [9, (a)] and coupling parameters $\left[\lambda \propto (g_0)^2\right]$:

$$\lambda_{12}(\tau) = \lambda\,\theta[r_{c,\,g} - r_{M2}]\left(1 - \sqrt{r_{M2}/r_{c,\,g}}\right)^2\,, \tag{C.6e}$$

$$\lambda_{21}(\tau) = \lambda_{12}(\tau)\ \exp\left[-\left(\frac{\pi N_2}{30\, r_{M2}(\tau_{\text{min}})}\right)^{1/4} \frac{a(\tau)}{a(\tau_{\text{min}})}\,\frac{M}{E_{\text{ew}}}\right]. \tag{C.6f}$$

App. C: Field-theoretic model

Model universe has early phase given by a standard radiation-dominated FRW universe $\Rightarrow$ fully determined boundary conditions of ODEs.

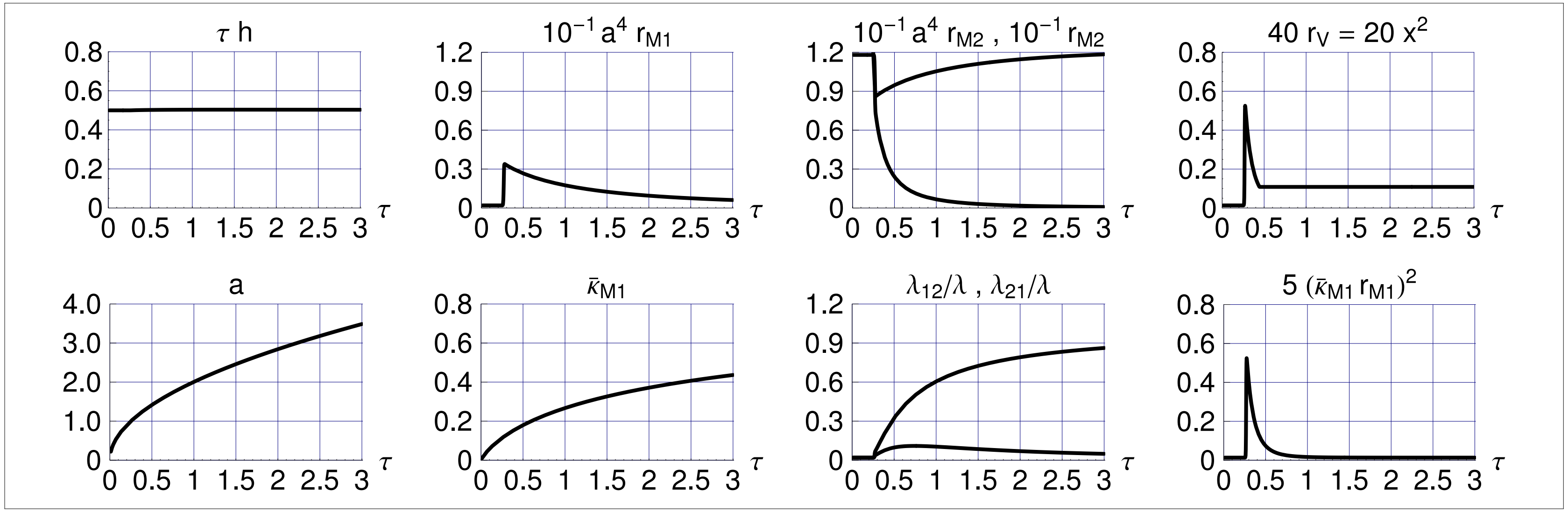

Fig. C1: Numerical solution [9, (b)] of the dimensionless ODEs (C.6). Model parameters are $\{\xi,\,\lambda,\,r_{c,\,g},\,r_{c,\,K}\} = \{10^7,\,10^4,\,12,\,3\}$. The ODEs are solved over the interval $[\tau_{\text{min}},\,\tau_{\text{max}}] = [0.01,\,3]$ with the boundary conditions at $\tau = \tau_{\text{bcs}} = 0.25$: $\{x,\,h,\,a,\,r_{M1},\,r_{M2}\} = \{0,\,2,\,1,\,0,\,12\}$. Essentially the same results for $\xi = 10^{60}$.

App. C: Field-theoretic model

The calculated value $r_{V,\,\text{remnant}} \approx 2.4 \times 10^{-3}$ gives $E_{\text{ew}} \approx 4.7$ TeV, according to (A.5).

But, here, main focus on the physical content of a theory capable of generating the observed cosmological "constant" of our Universe.

Hence, analytic result of interest:

$$\lim_{\tau\to\infty} r_V(\tau)\Big|^{\xi=\infty} = \frac{1}{8}\Big(\overline{\kappa}_{M1}(\tau_{\text{freeze}})\; r_{M1}(\tau_{\text{freeze}})\Big)^2\Big|_{r_{M2}(\tau_{\text{freeze}})=r_{c,\,K}} \,. \qquad \text{(C.7)}$$