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M-theory and the birth of the Universe

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in memory of Martinus J. G. Veltman (1931-2021)

0. Introduction

Two preliminary remarks:

- The Future of Particle Physics may very well be tied to that of Gravitation and Cosmology.
- The present talk has no definitive answers, only a few suggestive results (the ermine on da Vinci's painting appears to know it all, but does not tell us...).

[Czartoryski Museum, Kraków, PL]

0. Introduction

<u>Outline of main talk</u>:

1. Standard Friedmann cosmology
2. Regularized big bang
3. New phase from M-theory
4. Conclusions
5. References

<u>Technical details</u>:

A. Extraction of the spacetime points
B. Extraction of the spacetime metric
C. Various emergent spacetimes
D. More on the Lorentzian signature

1. Standard Friedmann cosmology

The Einstein gravitational field equation of general relativity (GR) reads [1]:

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = -8\pi G\, T_{\mu\nu}^{\text{(SM)}}\,, \tag{1}$$

with $R_{\mu\nu}$ the Ricci tensor, R the Ricci scalar, $T_{\mu\nu}^{\text{(SM)}}$ the energy-momentum tensor of the matter described by the Standard Model (SM), and G the Newton gravitational coupling constant. The spacetime indices μ, ν run over $\{0,\, 1,\, 2,\, 3\}$.

For cosmology, the spatially flat Robertson–Walker (RW) metric is

$$ds^2\,\Big|^{\text{(RW)}} \equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu\,\Big|^{\text{(RW)}} = -dt^2 + a^2(t)\; \delta_{ij}\, dx^i\, dx^j\,, \tag{2}$$

with $x^0 = c\, t$ and $c = 1$. The spatial indices i, j run over $\{1,\, 2,\, 3\}$.

1. Standard Friedmann cosmology

Considering a homogeneous perfect fluid with energy density $\rho_M(t)$ and pressure $P_M(t)$, we get the spatially flat Friedmann equations [1]:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho_M\,, \tag{3a}$$

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G\,P_M\,, \tag{3b}$$

$$\dot{\rho}_M + 3\,\frac{\dot{a}}{a}\,\Big[\rho_M + P_M\Big] = 0\,, \tag{3c}$$

$$P_M = P_M\big(\rho_M\big)\,, \tag{3d}$$

where the overdot stands for differentiation with respect to t and (3d) corresponds to the equation-of-state (EOS) relation between pressure and energy density of the fluid.

1. Standard Friedmann cosmology

For relativistic matter with constant EOS parameter $w_M \equiv P_M/\rho_M = 1/3$, the Friedmann–Lemaître–Robertson–Walker (FLRW) solution is [1]

$$a(t)\Big|_{\text{FLRW}}^{(w_M=1/3)} = \sqrt{t/t_0}\,, \quad \text{for} \quad t>0\,, \tag{4a}$$

$$\rho_M(t)\Big|_{\text{FLRW}}^{(w_M=1/3)} = \rho_{M0}/a^4(t) \propto 1/t^2\,, \quad \text{for} \quad t>0\,, \tag{4b}$$

where the cosmic scale factor has normalization $a(t_0)=1$ at $t_0>0$.

This FLRW solution displays the **big bang singularity** for $t \to 0^+$,

$$\lim_{t\to 0^+} a(t) = 0\,, \tag{5}$$

with diverging curvature and energy density. But, at $t=0$, the theory (GR+SM) is no longer valid and we can ask what happens really?

Or, more precisely, how to describe the birth of the Universe?

2. Regularized big bang

First, we set out to **control the divergences** by using a new *Ansatz* for the "regularized" big bang [2]:

$$ds^2\Big|^{\text{(reg-bb)}} \equiv g_{\mu\nu}(x)\,dx^\mu\,dx^\nu\Big|^{\text{(reg-bb)}}$$

$$= -\frac{t^2}{t^2+b^2}\,dt^2 + a^2(t)\,\delta_{ij}\,dx^i\,dx^j\,, \qquad \text{(6a)}$$

$$b^2 > 0\,, \quad a^2(t) > 0\,, \qquad \text{(6b)}$$

$$t \in (-\infty,\,\infty)\,, \quad x^i \in (-\infty,\,\infty)\,, \qquad \text{(6c)}$$

with $x^0 = c\,t$ and $c = 1$. The length scale $b \neq 0$ acts as regulator.

This metric $g_{\mu\nu}(x)$ is **degenerate**, with a vanishing determinant at $t = 0$. Physically, the $t = 0$ slice corresponds to a **spacetime defect**.

2. Regularized big bang

The standard Einstein equation (1) with the new metric *Ansatz* (6) and a homogeneous perfect fluid gives **modified** spatially flat Friedmann equations:

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\,\rho_M\,, \tag{7a}$$

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\ddot{a}}{a}+\frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2\right)-\frac{b^2}{t^3}\,\frac{\dot{a}}{a}=-4\pi G\,P_M\,, \tag{7b}$$

$$\dot{\rho}_M+3\,\frac{\dot{a}}{a}\left[\rho_M+P_M\right]=0\,, \tag{7c}$$

$$P_M=P_M\big(\rho_M\big)\,, \tag{7d}$$

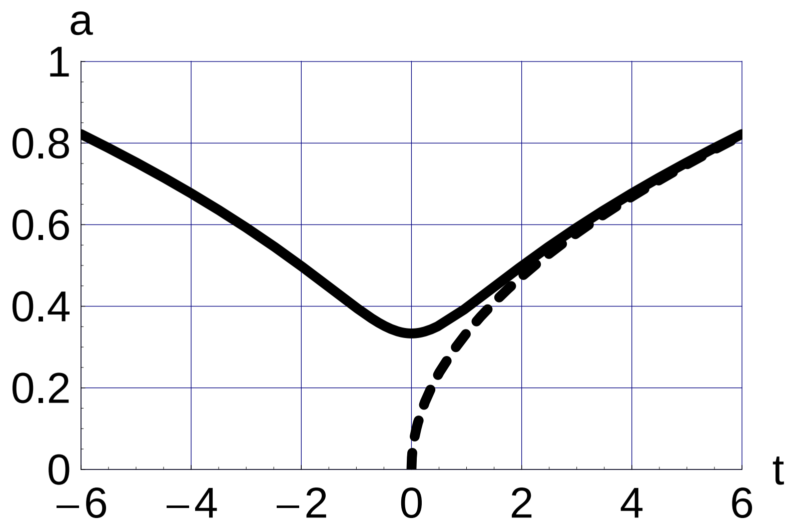
where the overdot stands again for differentiation with respect to t.

2. Regularized big bang

For constant EOS parameter $w_M = 1/3$, the solution $a(t)$ of (7) reads

$$a(t)\Big|^{(w_M=1/3)}_{\text{(reg-bb)}} = \sqrt[4]{\left(t^2+b^2\right)/\left(t_0^2+b^2\right)}\,, \tag{8}$$

which is **perfectly smooth** at $t = 0$ as long as $b \neq 0$. The figure below compares this regularized solution (full curve) with the singular FLRW solution (dashed curve).



2. Regularized big bang

Two possible scenarios:

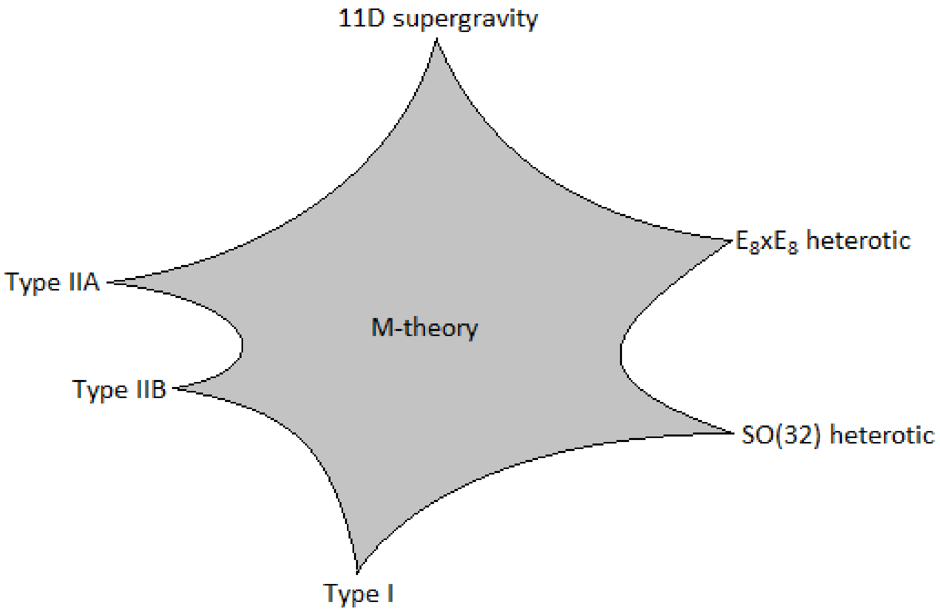
1. **nonsingular bouncing cosmology** [3, 4] from $t = -\infty$ to $t = \infty$ (valid for $b \gg l_{\text{Planck}}$?) [gravitational waves generated in the pre-bounce epoch keep on propagating into the postbounce epoch];
2. **new physics phase** at $t = 0$ pair-produces [5] a "universe" for $t > 0$ and an "antiuniverse" for $t < 0$ (valid for $b \sim l_{\text{Planck}}$?).

For both scenarios, the $t = 0$ slice corresponds to a spacetime defect, which manifests itself as a discontinuity of the extrinsic curvature K on constant-t hypersurfaces. Also, there is a discontinuity at $t = 0$ of the expansion function θ for a bunch of timelike geodesics [Wang, PhD thesis, KIT, 2020].

It is not clear, for the first scenario, what physical mechanism determines the relatively large value of b. For the second scenario, the hope is that the new physics sets the value of b. $\Leftarrow$ THIS TALK

3. New phase from M-theory

M-theory is a hypothetical theory that unifies all five consistent versions of 10D superstring theory (cf. [6, 7]).



For an explicit description of the new phase replacing the big bang, we use the **IIB matrix model** of Kawai and collaborators [8, 9], which has been proposed as a nonperturbative definition of type-IIB superstring theory (and, thereby, of M-theory).

3. New phase from M-theory

This IIB matrix model has $N \times N$ traceless Hermitian matrices, ten bosonic matrices A^μ and essentially eight fermionic (Majorana–Weyl) matrices Ψ_α.

The partition function Z of the Lorentzian IIB matrix model is defined by the following "path" integral [8, 9, 10, 11]:

$$Z = \int dA\, d\Psi\;\; \exp\big(i\, S/\ell^4\big) = \int dA\;\; \exp\big(i\, S_{\text{eff}}/\ell^4\big)\;, \tag{9a}$$

$$S = -\text{Tr}\left(\frac{1}{4}\,\big[A^\mu,\, A^\nu\big]\,\big[A^\rho,\, A^\sigma\big]\,\widetilde{\eta}_{\mu\rho}\,\widetilde{\eta}_{\nu\sigma} + \frac{1}{2}\,\overline{\Psi}_\beta\,\widetilde{\Gamma}^\mu_{\beta\alpha}\,\widetilde{\eta}_{\mu\nu}\,\big[A^\nu,\, \Psi_\alpha\big]\right), \tag{9b}$$

$$\widetilde{\eta}_{\mu\nu} = \Big[\text{diag}\big(-1,\, 1,\, \ldots\,,\, 1\big)\Big]_{\mu\nu}\,, \quad \text{for} \quad \mu, \nu \in \{0,\, 1,\, \ldots\,,\, 9\}\,. \tag{9c}$$

A model length scale "ℓ" has been introduced, so that A^μ has the dimension of length and Ψ_α the dimension of $(\text{length})^{3/2}$.

Expectation values of further observables will be discussed later.

3. New phase from M-theory

Now, the matrices A^μ and Ψ_α in (9a) are merely integration variables.

Moreover, there is no obvious small dimensionless parameter to motivate a saddle-point approximation.

Hence, the **conceptual** question: where is the classical spacetime?

Recently, I have suggested to revisit an old idea, the large-N master field of Witten [12], for a possible origin of classical spacetime in the context of IIB matrix model [13].

First, I will remind you of this mysterious master field (name coined by Coleman) and give you the final result.
Then, time permitting, I will highlight a few of the technical details collected in the Appendices.

3. New phase from M-theory

Consider the gauge-invariant bosonic observable

$$w^{\mu_1 \cdots \mu_m} = \text{Tr}\left(A^{\mu_1} \ldots A^{\mu_m}\right) . \tag{10}$$

Strings of these observables have expectation values

$$\langle w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n} \; \cdots \rangle = \frac{1}{Z} \int dA \left(w^{\mu_1 \cdots \mu_m}\, w^{\nu_1 \cdots \nu_n} \; \cdots \right) e^{i\, S_{\text{eff}}/\ell^4} . \tag{11}$$

The following **factorization property** holds to leading order in N:

$$\left\langle\, w^{\mu_1 \cdots \mu_m}\, w^{\mu_1 \cdots \mu_m} \,\right\rangle \stackrel{N}{=} \left\langle w^{\mu_1 \cdots \mu_m} \right\rangle \left\langle w^{\mu_1 \cdots \mu_m} \right\rangle , \tag{12}$$

without sums over repeated indices. Similar large-N factorization properties hold for all expectation values (11).

The leading-order equality (12) is a truly remarkable result for a statistical (quantum) theory.

3. New phase from M-theory

Indeed, according to Witten [12], the factorization (12) implies that the path integrals (11) are saturated by a single configuration, namely by the so-called **master field** $\widehat{A}^{\,\mu}$.

Considering one w observable for simplicity, we then have for its expectation value ("Wilson loop"):

$$\langle w^{\mu_1 \,\cdots\, \mu_m} \rangle \stackrel{N}{=} \text{Tr}\left(\widehat{A}^{\,\mu_1} \ldots \widehat{A}^{\,\mu_m}\right), \tag{13}$$

and similarly for the other expectation values (11).

Hence, we do not have to perform the path integrals on the right-hand side of (11): we "only" need ten traceless Hermitian matrices $\widehat{A}^{\,\mu}$ to get all these expectation values from the simple recipe of replacing each $A^{\,\mu}$ in the observables by $\widehat{A}^{\,\mu}$, just as was done in (13).

3. New phase from M-theory

Now, the meaning of the suggestion on slide 13 is clear:

classical spacetime resides in the model master-field matrices $\widehat{A}^{\,\mu}$.

Assume that the matrices $\widehat{A}^{\,\mu}$ of the Lorentzian-IIB-matrix-model master field are known and that they are approximately band-diagonal.

Then, it is possible [13] to extract a discrete set of spacetime points $\{\widehat{x}_k^{\,\mu}\}$ and the emergent inverse metric $g^{\mu\nu}(x)$, with the metric $g_{\mu\nu}(x)$ obtained as matrix inverse.

It is even possible [14] that the large-N master field of the Lorentzian IIB matrix model gives rise to the regularized-big-bang metric (6) of GR.

Final result: effective length parameter b of the regularized-big-bang metric (6) is calculated in terms of the IIB-matrix-model length scale ℓ,

$$b_{\text{eff}} \sim \ell \stackrel{?}{\sim} l_{\text{Planck}} \equiv \sqrt{\hbar\, G/c^3} \approx 1.62 \times 10^{-35}\ \text{m}\,. \tag{14}$$

Technical details are collected in the Appendices.

4. Conclusions

It is conceivable that a **new physics phase** replaces the big bang singularity suggested by our current theories (GR&SM).

For an explicit calculation, we have considered the **IIB matrix model**, which has been proposed as a nonperturbative definition of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the **large-N master field** $\widehat{A}^{\mu}$ of the model.

In principle, the IIB-matrix-model master field $\widehat{A}^{\mu}$ can give rise to the regularized-big-bang metric with length parameter $b \sim \ell$, where ℓ is the length scale of the matrix model.

At this moment, the outstanding task is to **calculate** the exact IIB-matrix-model master field $\widehat{A}^{\mu}$ or, at least, to get a reliable approximation of it...

....

Time permitting, we can mention a few technical details. ($\rightarrow$ slide 3)

5. References

[1] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (Princeton University Press, Princeton, NJ, 2017).

[2] F.R. Klinkhamer, “Regularized big bang singularity,” Phys. Rev. D **100**, 023536 (2019), arXiv:1903.10450.

[3] F.R. Klinkhamer and Z.L. Wang, “Nonsingular bouncing cosmology from general relativity,” Phys. Rev. D **100**, 083534 (2019), arXiv:1904.09961.

[4] F.R. Klinkhamer and Z.L. Wang, “Nonsingular bouncing cosmology from general relativity: Scalar metric perturbations,” Phys. Rev. D **101**, 064061 (2020), arXiv:1911.06173.

[5] L. Boyle, K. Finn, and N. Turok, “CPT-symmetric Universe,” Phys. Rev. Lett. **121**, 251301 (2018), arXiv:1803.08928.

[6] E. Witten, “String theory dynamics in various dimensions,” Nucl. Phys. B **443**, 85 (1995), arXiv:hep-th/9503124.

[7] P. Horava and E. Witten, “Heterotic and type I string dynamics from eleven dimensions,” Nucl. Phys. B **460**, 506 (1996), arXiv:hep-th/9510209.

5. References

[8] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, “A large-N reduced model as superstring,” Nucl. Phys. B **498**, 467 (1997), arXiv:hep-th/9612115.

[9] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya, and T. Tada, “IIB matrix model,” Prog. Theor. Phys. Suppl. **134**, 47 (1999), arXiv:hep-th/9908038.

[10] S.W. Kim, J. Nishimura, and A. Tsuchiya, “Expanding (3+1)-dimensional universe from a Lorentzian matrix model for superstring theory in (9+1)-dimensions,” Phys. Rev. Lett. **108**, 011601 (2012), arXiv:1108.1540.

[11] J. Nishimura and A. Tsuchiya, “Complex Langevin analysis of the space-time structure in the Lorentzian type IIB matrix model,” JHEP **1906**, 077 (2019), arXiv:1904.05919.

[12] E. Witten, “The $1/N$ expansion in atomic and particle physics,” in G. ’t Hooft et. al (eds.), *Recent Developments in Gauge Theories*, Cargese 1979 (Plenum Press, New York, 1980).

[13] F.R. Klinkhamer, “IIB matrix model: Emergent spacetime from the master field,” to appear in Prog. Theor. Exp. Phys., arXiv:2007.08485.

[14] F.R. Klinkhamer, “Regularized big bang and IIB matrix model,” arXiv:2009.06525.

A. Extraction of the spacetime points

Aoki et al. [9] have argued that the **eigenvalues** of the matrices A^{μ} of model (9) can be interpreted as **spacetime coordinates**, so that the model has a ten-dimensional $\mathcal{N}=2$ spacetime supersymmetry.

Here, we will turn to the eigenvalues of the **master-field** matrices $\widehat{A}^{\mu}$. Assume that the matrices $\widehat{A}^{\mu}$ of the Lorentzian-IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by numerical results [10, 11]). Then, make a particular global gauge transformation [10],

$$\underline{\widehat{A}}^{\mu} = \underline{\Omega}\, \widehat{A}^{\mu}\, \underline{\Omega}^{\dagger}\,, \quad \underline{\Omega} \in SU(N)\,, \tag{15}$$

so that the transformed 0-component matrix is diagonal and has ordered eigenvalues $\widehat{\alpha}_i \in \mathbb{R}$,

$$\underline{\widehat{A}}^{0} = \mathrm{diag}\big(\widehat{\alpha}_1,\, \widehat{\alpha}_2,\, \ldots\,,\, \widehat{\alpha}_{N-1},\, \widehat{\alpha}_N\big)\,, \tag{16a}$$

$$\widehat{\alpha}_1 \leq \widehat{\alpha}_2 \leq \ldots \leq \widehat{\alpha}_{N-1} \leq \widehat{\alpha}_N\,, \qquad \sum_{i=1}^{N} \widehat{\alpha}_i = 0\,. \tag{16b}$$

A. Extraction of the spacetime points

The ordering (16b) will turn out to be crucial for the time coordinate $\widehat{t}$ to be obtained later.

A relatively simple procedure [13] **approximates** the eigenvalues of the spatial matrices $\underline{\widehat{A}}^{m}$ but still manages to **order them along the diagonal**, matching the temporal eigenvalues $\widehat{\alpha}_i$ from (16).

We start from the following trivial observation:

if M is an $N \times N$ Hermitian matrix, then any $n \times n$ block centered on the diagonal of M is also Hermitian, which holds for $1 \leq n \leq N$.

Now, let K be an odd divisor of N, so that

$$N \;=\; K\,n\,, \quad K = 2\,L + 1\,, \tag{17}$$

where both L and n are positive integers.

A. Extraction of the spacetime points

Consider, in each of the ten matrices $\underline{\widehat{A}}^{\mu}$, the K blocks of size $n \times n$ centered on the diagonal.

We already know the diagonalized blocks of $\underline{\widehat{A}}^{0}$ from (16a), which allows us to define the following time coordinate $\widehat{t}\,(\sigma)$ for $\sigma \in (0,\, 1]$:

$$\widehat{x}^{\,0}\left(k/K\right) \equiv \widetilde{c}\,\widehat{t}\left(k/K\right) \equiv \frac{1}{n}\sum_{j=1}^{n}\widehat{\alpha}_{(k-1)\,n+j}\,, \tag{18}$$

with $k \in \{1,\, \ldots,\, K\}$ and a velocity $\widetilde{c}$ to be set to unity later. The time coordinates from (18) are ordered,

$$\widehat{t}\left(1/K\right) \,\leq\, \widehat{t}\left(2/K\right) \,\leq\, \ldots \,\leq\, \widehat{t}\left(1-1/K\right) \,\leq\, \widehat{t}\left(1\right), \tag{19}$$

because the $\widehat{\alpha}_i$ are, according to (16b).

A. Extraction of the spacetime points

Next, obtain the eigenvalues of the $n \times n$ blocks of the nine spatial matrices $\underline{\widehat{A}}^{\,m}$ and denote these real eigenvalues by $\big(\widehat{\beta}^{\,m}\big)_i$, with $i \in \{1,\, \ldots,\, N\}$.

Define, just as for the time coordinate in (18), the following nine spatial coordinates $\widehat{x}^{\,m}(\sigma)$ for $\sigma \in \{(0,\, 1]$:

$$\widehat{x}^{\,m}\big(k/K\big) \equiv \frac{1}{n} \sum_{j=1}^{n} \Big[\, \widehat{\beta}^{\,m} \,\Big]_{(k-1)\,n+j} \,, \tag{20}$$

with $k \in \{1,\, \ldots,\, K\}$.

A. Extraction of the spacetime points

If the master-field matrices $\underline{\widehat{A}}^{\mu}$ are approximately **band-diagonal** and if the eigenvalues of the spatial $n \times n$ blocks show significant **scattering**, then the expressions (18) and (20) may provide suitable spacetime points, which, in a somewhat different notation, are denoted

$$\widehat{x}_k^{\mu} = \left(\widehat{x}_k^{0},\, \widehat{x}_k^{m}\right) \equiv \Big(\widehat{x}^{\,0}\big(k/K\big),\, \widehat{x}^{\,m}\big(k/K\big)\Big)\,, \tag{21}$$

where k runs over $\{1,\, \ldots,\, K\}$.

Each of these coordinates $\widehat{x}_k^{\mu}$ has the dimension of length, which traces back to the dimension of the bosonic matrix variable A^{μ} as mentioned below (9c).

To summarize, with $N = K\,n$, the extracted spacetime points $\widehat{x}_k^{\mu}$, for $k \in \{1,\, \ldots,\, K\}$, are obtained as **averaged eigenvalues** of the $n \times n$ blocks along the diagonals of the gauge-transformed master-field matrices $\underline{\widehat{A}}^{\mu}$ from (15)–(16).

B. Extraction of the spacetime metric

The points $\widehat{x}_k^{\mu}$ effectively build a spacetime manifold with continuous (interpolating) coordinates x^{μ} if there is also an emerging metric $g_{\mu\nu}(x)$.

By considering the effective action of a low-energy scalar degree of freedom σ "propagating" over the discrete spacetime points $\widehat{x}_k^{\mu}$, the following expression for the **emergent inverse metric** is obtained [9, 13]:

$$g^{\mu\nu}(x) \sim \int_{\mathbb{R}^D} d^D y \; \rho_{\text{av}}(y) \; (x-y)^{\mu} \, (x-y)^{\nu} \; f(x-y) \; r(x,\, y)\,, \tag{22a}$$

$$\rho_{\text{av}}(y) \equiv \langle\langle\, \rho(y) \,\rangle\rangle\,, \tag{22b}$$

with continuous spacetime coordinates x^{μ} having the dimension of length and spacetime dimension $D = 9 + 1 = 10$ for the original model.

The average $\langle\langle\, \rho(y) \,\rangle\rangle$ corresponds, for the extraction procedure of App. A, to averaging over different block sizes n and block positions along the diagonal in the master-field matrices $\underline{\widehat{A}}^{\mu}$.

B. Extraction of the spacetime metric

The quantities that enter the integral (22) are the density function

$$\rho(x) \equiv \sum_{k=1}^{K} \delta^{(D)}\big(x - \widehat{x}_k\big)\,, \tag{23}$$

the density correlation function $r(x,\,y)$ defined by

$$\langle\langle\, \rho(x)\,\rho(y)\,\rangle\rangle \equiv \langle\langle\, \rho(x)\,\rangle\rangle\,\langle\langle\, \rho(y)\,\rangle\rangle\, r(x,\,y)\,, \tag{24}$$

and a localized real function $f(x)$ from the scalar effective action,

$$S_{\text{eff}}[\sigma] \sim \sum_{k,\,l} \frac{1}{2}\, f\left(\widehat{x}_k - \widehat{x}_l\right)\, \left(\sigma_k - \sigma_l\right)^2\,, \tag{25}$$

where σ_k is the field value at the point $\widehat{x}_k$ (the scalar degree of freedom σ arises from a perturbation of the master field $\underline{\widehat{A}}^{\mu}$; see App. A in Ref. [13]).
As $r(x,\,y)$ is dimensionless and $f(x)$ has dimension $1/(\text{length})^2$, the inverse metric $g^{\mu\nu}(x)$ from (22) is seen to be dimensionless.
The metric $g_{\mu\nu}$ is simply obtained as the matrix inverse of $g^{\mu\nu}$.

B. Extraction of the spacetime metric

A few **heuristic remarks** [14] may help to clarify expression (22a).

In the standard continuum theory [i.e., a scalar field $\sigma(x)$ propagating over a given continuous spacetime manifold with metric $g_{\mu\nu}(x)$], two nearby points x' and x'' have approximately equal field values, $\sigma(x') \sim \sigma(x'')$, and two distant points x' and x''' generically have very different field values, $|\sigma(x') - \sigma(x''')|/|\sigma(x') + \sigma(x''')| \gtrsim 1$.

The logic is inverted for our discussion. Two approximately equal field values, $\sigma_1 \sim \sigma_2$, may still have a relatively small action (25) if $f(\widehat{x}_1 - \widehat{x}_2) \sim 1$ and inserting $f \sim 1$ in (22a) gives a "large" value for the inverse metric $g^{\mu\nu}$ and, hence, a "small" value for the metric $g_{\mu\nu}$, meaning that the spacetime points $\widehat{x}_1$ and $\widehat{x}_2$ are close (in units of ℓ).

Two different field values σ_1 and σ_3 have a small action (25) if $f(\widehat{x}_1 - \widehat{x}_3) \sim 0$ and inserting $f \sim 0$ in (22a) gives a "small" value for the inverse metric $g^{\mu\nu}$ and, hence, a "large" value for the metric $g_{\mu\nu}$, meaning that the spacetime points $\widehat{x}_1$ and $\widehat{x}_3$ are separated by a large distance (in units of ℓ).

B. Extraction of the spacetime metric

To summarize, the emergent metric, in the context of the IIB matrix model, is obtained from **correlations** of the extracted spacetime points and the master-field perturbations.

The obvious question, now, is which spacetime and metric do we get?

We don't know, as we do not have the IIB-matrix-model master field.

But, awaiting the final result on the master field, we can already investigate what properties the master field would need to have in order to be able to produce certain desired emerging metrics.
Some exploratory results are presented in App. C.

[Note that, in principle, the origin of the expression (22) need not be the IIB matrix model but can be an entirely different theory, as long as the emerging inverse metric is given by a multiple integral with the same basic structure.]

C. Various emergent spacetimes

We restrict ourselves to four “large” spacetime dimensions [10, 11], setting

$$D = 3 + 1 = 4\,, \tag{26}$$

and use length units that normalize the Lorentzian-IIB-matrix-model length scale,

$$\ell = 1\,. \tag{27}$$

Then, it is possible to choose appropriate functions $\rho_{\text{av}}(y)$, $f(x-y)$, and $r(x,\,y)$ in (22), so that the Minkowski metric is obtained [as given by (2) for $a^2(t)=1$].

Similarly, it is possible to choose appropriate functions $\rho_{\text{av}}(y)$, $f(x-y)$, and $r(x,\,y)$ in (22), so that the spatially flat Robertson–Walker metric (2) is obtained.

C. Various emergent spacetimes

In order to get an inverse metric whose component g^{00} diverges at $t = 0$, it is necessary to relax the convergence properties of the y^0 integral in (22a) by adapting the functions $\rho_{\text{av}}(y)$, $f(x - y)$, and $r(x, y)$.

In this way, it is possible to obtain the following inverse metric [14]:

$$g^{\mu\nu}_{\text{(eff)}} \sim \begin{cases} -\dfrac{t^2 + c_{-2}}{t^2}\,, & \text{for } \mu = \nu = 0\,, \\ 1 + c_2\,t^2 + c_4\,t^4 + \ldots\,, & \text{for } \mu = \nu = m \in \{1,\,2,\,3\}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{28}$$

with real dimensionless coefficients c_n that result from the requirement that t^n terms, for $n > 0$, vanish in $g^{00}_{\text{(eff)}}$.

C. Various emergent spacetimes

The matrix inverse of (28) gives the following Lorentzian metric:

$$g_{\mu\nu}^{(\text{eff})} \sim \begin{cases} -\dfrac{t^2}{t^2 + c_{-2}}\,, & \text{for } \mu = \nu = 0\,, \\ \dfrac{1}{1 + c_2\,t^2 + c_4\,t^4 + \ldots}\,, & \text{for } \mu = \nu = m \in \{1,\,2,\,3\}\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{29}$$

which has, for $c_{-2} > 0$, a vanishing determinant at $t = 0$ and is, therefore, degenerate.

C. Various emergent spacetimes

The emergent metric (29) has indeed the structure of the regularized-big-bang metric (6a), with the following effective parameters:

$$b^2_{\text{eff}} \sim c_{-2}\,\ell^2\,, \tag{30a}$$

$$a^2_{\text{eff}}(t) \sim 1 - c_2\,\left(t/\ell\right)^2 + \ldots\,, \tag{30b}$$

where the IIB-matrix-model length scale ℓ has been restored and where the leading coefficients c_{-2} and c_2 have been calculated [14].

By choosing the *Ansatz* parameters appropriately, we can get $c_2 < 0$ in (30b), so that the emergent classical spacetime corresponds to the spacetime of a nonsingular cosmic bounce at $t = 0$, as obtained in (8) from Einstein's gravitational field equation with a $w_M = 1/3$ perfect fluid.

C. Various emergent spacetimes

The proper **cosmological interpretation** of the emergent classical spacetime is perhaps as follows.

The new physics phase (replacing the big bang singularity of GR&SM) is assumed to be described by the IIB matrix model and the corresponding large-N master field gives rise to the points and metric of a classical spacetime.

If the master field has an appropriate structure, the emergent metric has a tamed big bang, with a metric similar to the regularized-big-bang metric of GR [2] but now having an effective length parameter b_{eff} proportional to the IIB-matrix-model length scale ℓ , as given by (30a).

In fact, one possible interpretation is that the new phase has produced a universe-antiuniverse pair [5], that is, a “universe” for $t > 0$ and an “antiuniverse” for $t < 0$.

D. More on the Lorentzian signature

Up till now, we have considered the Lorentzian IIB matrix model, which has two characteristics:

1. the "Lorentzian" coupling constants $\widetilde{\eta}_{\mu\nu}$ from (9c);
2. the Feynman phase factor $e^{i\,S/\ell^4}$ in the "path" integral (9a).

From the master field of this Lorentzian matrix model, we obtained the spacetime points from expressions (18) and (20) in App. A and the inverse metric from expression (22) in App. B.

Several Lorentzian inverse metrics were found in App. C, where the *Ansätze* used [14] relied on having "Lorentzian" coupling constants $\widetilde{\eta}_{\mu\nu}$.

But there is another way [13] to obtain Lorentzian inverse metrics, namely by making an appropriately <u>odd</u> *Ansatz* for the correlations functions entering (22), so that the resulting matrix is <u>off-diagonal</u>.

D. More on the Lorentzian signature

With this appropriately odd *Ansatz*, it is, in principle, also possible to get a Lorentzian inverse metric from the Euclidean matrix model, which has nonnegative coupling constants $\widetilde{\delta}_{\mu\nu}$ in the action and a weight factor $e^{-\,S/\ell^4}$ in the path integral. The spacetime points are extracted from the Euclidean master field (no gauge transformation needed) by the expression (20), where m now runs over $\{1,\,\ldots,\,D\}$.

The details of a toy-model calculation are as follows (expanding on a parenthetical remark in the last paragraph of App. B in Ref. [13]).

The calculation starts from the multiple integral (22) for spacetime dimension $D=4$ by writing in the integrand

$$f(x-y)\; r(x,\,y) = f(x-y)\; \widetilde{r}(y-x)\; \overline{r}(x,\,y) = h(y-x)\; \overline{r}(x,\,y)\,, \qquad (31)$$

where the new function $\overline{r}(x,\,y)$ has a more complicated dependence on x and y than the combination $x-y$.

D. More on the Lorentzian signature

The $D = 4$ multiple integral (22), with y^0 replaced by y^4, is then evaluated at the spacetime point

$$x^\mu = 0\,, \tag{32a}$$

with the replacement (31) in the integrand and two further simplifications:

$$\langle\langle\, \rho(y)\,\rangle\rangle = 1\,, \qquad \overline{r}(x,\, y) = 1\,, \tag{32b}$$

and symmetric cutoffs on the integrals,

$$\int_{-1}^{1} dy^1 \,\ldots \int_{-1}^{1} dy^4\,. \tag{32c}$$

The only nontrivial contribution to the integrand of (22) now comes from the correlation function h as defined by (31).

D. More on the Lorentzian signature

From (22) and (32), we then get the emergent inverse metric

$$g^{\mu\nu}_{\text{test,E4}}(0) \sim \int_{-1}^{1} dy^1 \int_{-1}^{1} dy^2 \int_{-1}^{1} dy^3 \int_{-1}^{1} dy^4 \;\; y^\mu \, y^\nu \; h_{\text{test,E4}}(y)\,, \qquad (33)$$

with the following *Ansatz* for the correlation function h :

$$h_{\text{test,E4}}(y) = 1 - \gamma \left(y^1\, y^2 + y^1\, y^3 + y^1\, y^4 + y^2\, y^3 + y^2\, y^4 + y^3\, y^4 \right), \qquad (34)$$

where γ multiplies monomials that are odd in two coordinates and even in the two others.

Note that the Ansatz (34) treats <u>all</u> coordinates y^1, y^2, y^3, and y^4 <u>equally</u>, in line with the coupling constants $\widetilde{\delta}_{\mu\nu}$ of the Euclidean matrix model.

D. More on the Lorentzian signature

The integrals of (33) with *Ansatz* function (34) are trivial and we obtain

$$g_\gamma^{\mu\nu}(0) \sim \frac{16}{9} \begin{pmatrix} 3 & -\gamma & -\gamma & -\gamma \\ -\gamma & 3 & -\gamma & -\gamma \\ -\gamma & -\gamma & 3 & -\gamma \\ -\gamma & -\gamma & -\gamma & 3 \end{pmatrix}, \qquad \text{(35a)}$$

where the matrix on the right-hand side has the following eigenvalues and normalized eigenvectors:

$$\mathcal{E}_\gamma = \frac{16}{9} \Big\{ (3-3\,\gamma)\,,\,(3+\gamma)\,,\,(3+\gamma)\,,\,(3+\gamma) \Big\}, \qquad \text{(35b)}$$

$$\mathcal{V}_\gamma = \left\{ \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \right\}. \qquad \text{(35c)}$$

D. More on the Lorentzian signature

From (35b), we have the following signatures:

$$(+---) \quad \text{for} \quad \gamma \in (-\infty,\, -3)\,, \tag{36a}$$

$$(++++) \quad \text{for} \quad \gamma \in (-3,\, 1)\,, \tag{36b}$$

$$(-+++) \quad \text{for} \quad \gamma \in (1,\, \infty)\,. \tag{36c}$$

Hence, we obtain Lorentzian signatures for parameter values γ sufficiently far away from zero, $\gamma > 1$ or $\gamma < -3$.

The conclusion is that it is, in principle, possible to get a Lorentzian emergent inverse metric from the Euclidean IIB matrix model, provided the correlation functions have the appropriate structure.

This observation, if applicable, would remove the need for working with the (possibly more difficult) Lorentzian IIB matrix model.

Figure credits: en.wikipedia.org (Lady with an ermine); commons.wikimedia.org (M-theory sketch).