Ninth Internat. Workshop DICE2018 Castiglioncello, Sept. 17–21, 2018

Spacetime–Matter–Quantum Mechanics

Spacetime defects

Frans R. Klinkhamer

Institute for Theoretical Physics,

Karlsruhe Institute of Technology (KIT),

76128 Karlsruhe, Germany

Email: frans.klinkhamer@kit.edu

1. Introduction

ANALOGY

quantum phase??? liquid

$\downarrow ??$ $\downarrow$

classical spacetime and gravity (General Theory of Relativity) crystal

$+$ $+$

possible “spacetime defects”? $\boxed{\overset{?}{\longleftarrow}}$ crystal defects

1. Introduction

LITTLE IS KNOWN ABOUT THE QUANTUM PHASE OF SPACETIME.

Loop Quantum Gravity does have something like “atoms of space,” but the emergence of a classical spacetime is not fully understood.

Here, we will stay at the classical level and use the framework of Einstein’s General Relativity (GR).

Specifically, we will

- obtain **soliton-like solutions** to describe certain spacetime defects (imperfections in the fabric of spacetime),
- investigate **novel effects**.

1. Introduction

OUTLINE TALK:

1. Introduction
2. Skyrmion spacetime defect [K, 2014a]
3. Antigravity [K&Queiruga, 2018a]
4. Stealth defect [K&Queiruga, 2018b]
5. Lensing [K&Wang, 2018]
6. Discussion

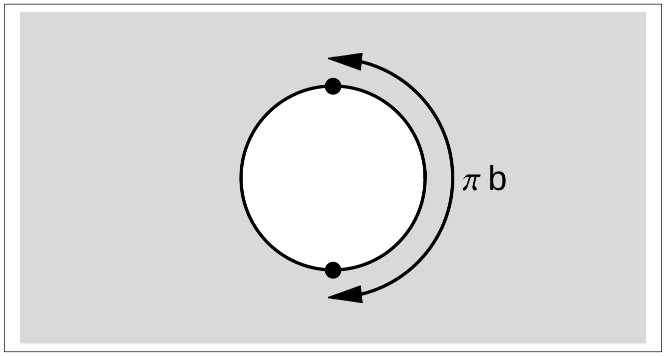
REMARKS:

- Solution of GR, allowing for degenerate metrics.
- A few slides can be skipped over, initially.
- References on the last slide.

2.1 Skyrmion spacetime defect – Manifold

BASIC IDEA:

Obtain a nonsingular defect solution of the Einstein field equation, with parameter $b > 0$ and topology as suggested by the sketch below:



Crux is the use of appropriate coordinates. The standard Cartesian coordinates of Euclidean 3-space are unsatisfactory, as a single point may have different coordinates. For example, $(x^1,\, x^2,\, x^3) = (0,\, b,\, 0)$ and $(x^1,\, x^2,\, x^3) = (0,\, -b,\, 0)$ correspond to the same point (dot in the figure).

It is possible to use three overlapping charts, each one centered on one of the three Cartesian coordinate axes [Schwarz, 2010, Guenther, 2017].

2.1 Skyrmion spacetime defect – Manifold

DETAILS:

Four-dimensional spacetime manifold:

$$\widetilde{M}_4 = \mathbb{R} \times \widetilde{M}_3 \,, \tag{1}$$

where $\widetilde{M}_3$ is a noncompact, orientable, nonsimply-connected manifold without boundary.

Up to a point, $\widetilde{M}_3$ is homeomorphic to the 3-dimensional real-projective plane,

$$\widetilde{M}_3 \simeq \mathbb{R}P^3 - p_\infty \,, \tag{2}$$

where p_∞ corresponds to the "point at spatial infinity."

2.1 Skyrmion spacetime defect – Manifold

For the explicit construction of $\widetilde{M}_3$, we perform local surgery on the 3-dimensional Euclidean space $E_3 = \big(\mathbb{R}^3,\, \delta_{mn}\big)$. We use the standard Cartesian and spherical coordinates on $\mathbb{R}^3$,

$$\vec{x} \quad \equiv \quad |\vec{x}|\,\widehat{x} = (x^1,\, x^2,\, x^3) = (r\sin\theta\cos\phi,\; r\sin\theta\sin\phi,\; r\cos\theta)\,, \tag{3a}$$

with ranges

$$x^m \quad \in \quad (-\infty,\, +\infty)\,, \quad r \geq 0\,, \quad \theta \in [0,\, \pi]\,, \quad \phi \in [0,\, 2\pi)\,. \tag{3b}$$

Now, $\widetilde{M}_3$ is obtained from $\mathbb{R}^3$ by removing the interior of the ball B_b with radius b and identifying antipodal points on the boundary $S_b \equiv \partial B_b$. With point reflection denoted $P(\vec{x}) = -\vec{x}$, the 3-space $\widetilde{M}_3$ is given by

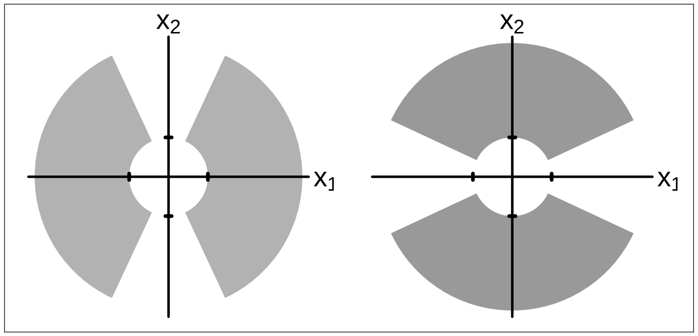
$$\widetilde{M}_3 = \Big\{\vec{x} \in \mathbb{R}^3 \;:\; \Big(|\vec{x}| \geq b > 0\Big) \wedge \Big(P(\vec{x}) \,\widehat{=}\, \vec{x} \;\text{ for }\; |\vec{x}| = b\Big)\Big\}\,, \tag{4}$$

where $\widehat{=}$ stands for point-wise identification.

2.1 Skyrmion spacetime defect – Manifold

As mentioned before, a relatively simple covering of $\widetilde{M}_3$ uses three charts of coordinates, labeled by $n = 1, 2, 3$.

Each chart surrounds one of the three Cartesian coordinate axes:



These coordinate charts are denoted

$$(X_n,\, Y_n,\, Z_n)\,, \quad \text{for} \quad n = 1,\, 2,\, 3\,. \tag{5}$$

CAREFUL: the triples (5) are non-Cartesian coordinates.

2.1 Skyrmion spacetime defect – Manifold

Specifically, the set of coordinates surrounding the x^2-axis segments with $|\vec{x}| \geq b$ is given by

$$X_2 = \begin{cases} \phi & \text{for} \quad 0 < \phi < \pi\,, \\ \phi - \pi & \text{for} \quad \pi < \phi < 2\pi\,, \end{cases} \tag{6a}$$

$$Y_2 = \begin{cases} r - b & \text{for} \quad 0 < \phi < \pi\,, \\ b - r & \text{for} \quad \pi < \phi < 2\pi\,, \end{cases} \tag{6b}$$

$$Z_2 = \begin{cases} \theta & \text{for} \quad 0 < \phi < \pi\,, \\ \pi - \theta & \text{for} \quad \pi < \phi < 2\pi\,, \end{cases} \tag{6c}$$

with ranges

$$X_2 \in (0,\, \pi)\,, \quad Y_2 \in (-\infty,\, \infty)\,, \quad Z_2 \in (0,\, \pi)\,. \tag{6d}$$

The other two sets, $(X_1,\, Y_1,\, Z_1)$ and $(X_3,\, Y_3,\, Z_3)$, are defined similarly. In the following, we explicitly give only one coordinate chart, which we take to be (6), and drop the suffix '2'.

2.2 Skyrmion spacetime defect – Fields

Consider a Skyrme-type scalar field $\Omega(X) \in SO(3)$, which propagates over the spacetime manifold (1) and has this action ($c = \hbar = 1$):

$$
\begin{aligned}
S &= \int_{\widetilde{M}_4} d^4X \, \sqrt{-g} \, \Big(\mathcal{L}_{\text{grav}} + \mathcal{L}_{\text{mat}} \Big) \,, && \text{(7a)} \\
\mathcal{L}_{\text{grav}} &= \frac{1}{16\pi \, G_N} \, R \,, && \text{(7b)} \\
\mathcal{L}_{\text{mat}} &= \frac{f^2}{4} \, \text{tr}\Big(\omega_\mu \, \omega^\mu \Big) + \frac{1}{16 \, e^2} \, \text{tr}\Big(\left[\omega_\mu, \, \omega_\nu\right] \left[\omega^\mu, \, \omega^\nu\right] \Big) \\
&\quad + \frac{1}{2} \, m^2 \, f^2 \, \text{tr}\Big(\Omega - \mathbb{1}_3 \Big) \,, && \text{(7c)} \\
\omega_\mu &\equiv \Omega^{-1} \, \partial_\mu \, \Omega \,. && \text{(7d)}
\end{aligned}
$$

2.2 Skyrmion spacetime defect – Fields

With "pions" π^a defined by the following expansion:

$$\Omega(X) = \exp\left[S^a\,\pi^a(X)/f\right], \tag{8}$$

for 3×3 matrices S^a given by

$$S^1 \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad S^2 \equiv \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$S^3 \equiv \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

we have

$$\mathcal{L}_{\text{mat}} = -\frac{1}{2}\,\partial_\mu \pi^a \partial^\mu \pi^a - \frac{1}{2}\,m^2\,\pi^a \pi^a + \cdots\,. \tag{10}$$

2.2 Skyrmion spacetime defect – Fields

Dimensional parameters of the theory:

$$G_N \geq 0\,, \tag{11a}$$

$$f > 0\,, \tag{11b}$$

$$m \geq 0\,. \tag{11c}$$

Real dimensionless parameters:

$$\widetilde{\eta} \equiv 8\pi\, G_N\, f^2 \geq 0\,, \tag{12a}$$

$$\widetilde{m}^2 \equiv \frac{m^2}{e^2\, f^2} \geq 0\,, \tag{12b}$$

$$e > 0\,. \tag{12c}$$

2.3 Skyrmion spacetime defect – Ansätze

The self-consistent *Ansätze* for the metric and the $SO(3)$ matter field are

$$
\begin{aligned}
ds^2\Big|_{\widetilde{M}_4\,,\,\text{chart}-2} &= -\big[\mu(W)\big]^2\,dT^2 + \big(1-b^2/W\big)\,\big[\sigma(W)\big]^2\,(dY)^2 \\
&\quad +W\Big[(dZ)^2+\sin^2 Z\,(dX)^2\Big]\,, && \text{(13a)}\\
\Omega(X) &= \cos\big[F(W)\big]\,\mathbb{1}_3 - \sin\big[F(W)\big]\,\widehat{x}\cdot\vec{S} \\
&\quad +\big(1-\cos\big[F(W)\big]\big)\,\widehat{x}\otimes\widehat{x}\,, && \text{(13b)}\\
W &\equiv b^2+Y^2\,, && \text{(13c)}
\end{aligned}
$$

with unit 3-vector $\widehat{x} \equiv \vec{x}/|\vec{x}|$ from the Cartesian coordinates $\vec{x}$ defined in terms of the chart-2 coordinates X, Y, and Z.

Note that the sine-F term in (13b) displays the hedgehog behavior.

2.3 Skyrmion spacetime defect – Ansätze

The boundary conditions (BCS) on the three *Ansatz* functions are:

$$F(b^2) = \pi\,, \quad F(\infty) = 0\,, \tag{14a}$$

$$\sigma(b^2) \in (0,\,\infty)\,, \tag{14b}$$

$$\mu(b^2) \in (0,\,\infty)\,. \tag{14c}$$

The BCS (14b)–(14c) will be discussed later.

2.3 Skyrmion spacetime defect – Ansätze

Two remarks. First, with finite *Ansatz* functions $\mu(W)$ and $\sigma(W)$, the metric from (13a) is **degenerate** at $W = b^2$:

$$\text{det}(g_{\mu\nu})\Big|_{W=b^2} = 0\,, \tag{15}$$

and the standard elementary-flatness property does not apply [K, 2014b].

Second, the matter-field *Ansatz* (13b) corresponds to a topologically nontrivial scalar field configuration, a Skyrmion-like configuration with **unit winding number**,

$$N \equiv \text{deg}[\Omega] = -\frac{2}{\pi}\int_{\pi}^{0} dF\,\sin^2(F/2) = 1\,, \tag{16}$$

where the endpoints of the integral on the right-hand side correspond to the boundary conditions (14a).

2.4 Skyrmion spacetime defect – ODEs

In the following, we will use dimensionless distances:

$$y \equiv e\,f\,Y \in \big(-\infty,\,\infty\big)\,, \tag{17a}$$

$$y_0 \equiv e\,f\,b \in \big(0,\,\infty\big)\,, \tag{17b}$$

$$w \equiv (e\,f)^2\,W \equiv (y_0)^2 + y^2 \in \big[(y_0)^2,\,\infty\big)\,. \tag{17c}$$

The reduced field equations are three ordinary differential equations (ODEs). With the following auxiliary functions:

$$A(w) \equiv 2\,\sin^2\frac{F(w)}{2}\left(\sin^2\frac{F(w)}{2} + w\right)\,, \tag{18a}$$

$$C(w) \equiv 4\,\sin^2\frac{F(w)}{2} + w\,, \tag{18b}$$

2.4 Skyrmion spacetime defect – ODEs

the three ODEs are (the prime stands for differentiation with respect to w):

$$
\begin{aligned}
4\,w\,\sigma'(w) &= +\sigma(w)\,\Big[\big[1-\sigma^2(w)\big]+\widetilde{\eta}\,\frac{2}{w}\Big(A(w)\,\sigma^2(w)+C(w)\,\big[w\,F'(w)\big]^2\Big)\Big] \\
&\quad +2\,w\,\widetilde{m}^2\,\widetilde{\eta}\,\sigma^3(w)\,\sin^2\frac{F(w)}{2}\,, \qquad \text{(19a)}
\end{aligned}
$$

$$
\begin{aligned}
4\,w\,\mu'(w) &= -\mu(w)\,\Big[\big[1-\sigma^2(w)\big]+\widetilde{\eta}\,\frac{2}{w}\Big(A(w)\,\sigma^2(w)-C(w)\,\big[w\,F'(w)\big]^2\Big)\Big] \\
&\quad -2\,w\,\widetilde{m}^2\,\widetilde{\eta}\,\sigma^2(w)\,\sin^2\frac{F(w)}{2}\,, \qquad \text{(19b)}
\end{aligned}
$$

2.4 Skyrmion spacetime defect – ODEs

$$
\begin{aligned}
C(w)\, w^2\, F''(w) \quad = \quad & +\sigma^2(w)\, \sin F(w)\, \left(\sin^2 \frac{F(w)}{2} + \frac{w}{2}\right) \\
& -\frac{1}{2}\, C(w)\, \sigma^2(w)\, w\, F'(w) \\
& \times \left[1 - 4\,\widetilde{\eta}\,\frac{1}{w}\, \sin^2 \frac{F(w)}{2}\, \left(\sin^2 \frac{F(w)}{2} + w\right)\right] \\
& -w\, F'(w)\, \Big[w\, F'(w)\, \sin F(w) + w\Big] \\
& +\frac{\widetilde{m}^2}{2}\, w^2\, \sigma^2(w)\, \sin \frac{F(w)}{2} \\
& \times \left[\cos \frac{F(w)}{2} + 2\,\widetilde{\eta}\, C(w)\, \sin \frac{F(w)}{2} F'(w)\right] .
\end{aligned}
\quad \text{(19c)}
$$

2.5 Skyrmion spacetime defect – Numerics

These ODEs can be solved numerically with BCS from (14).

Specifically, we have $F(y_0^2) = \pi$ and $F(\infty) = 0$ for the matter-field *Ansatz* function $F(w)$ and we take $\sigma(y_0^2) = 1$ for the metric *Ansatz* function $\sigma(w)$ [the value of $\mu(y_0^2)$ can be rescaled arbitrarily].

The next slide shows the *Ansatz* functions $F(w)$, $\sigma(w)$, and $\mu(w)$ of one particular numerical solution. Also shown are:

the dimensionless Ricci curvature scalar $R(w)$,

the dimensionless Kretschmann curvature scalar $K(w)$, and

the negative of the 00 component of the dimensionless Einstein tensor $E^{\mu}{}_{\nu}(w) \equiv R^{\mu}{}_{\nu}(w) - (1/2)\, R(w)\, \delta^{\mu}{}_{\nu}$.

For later use, we define a dimensionless Schwarzschild-type length scale $l(w)$ by setting

$$\sigma^2(w) = \frac{1}{1 - l(w)/\sqrt{w}}\,. \tag{20}$$

2.5 Skyrmion spacetime defect – Numerics

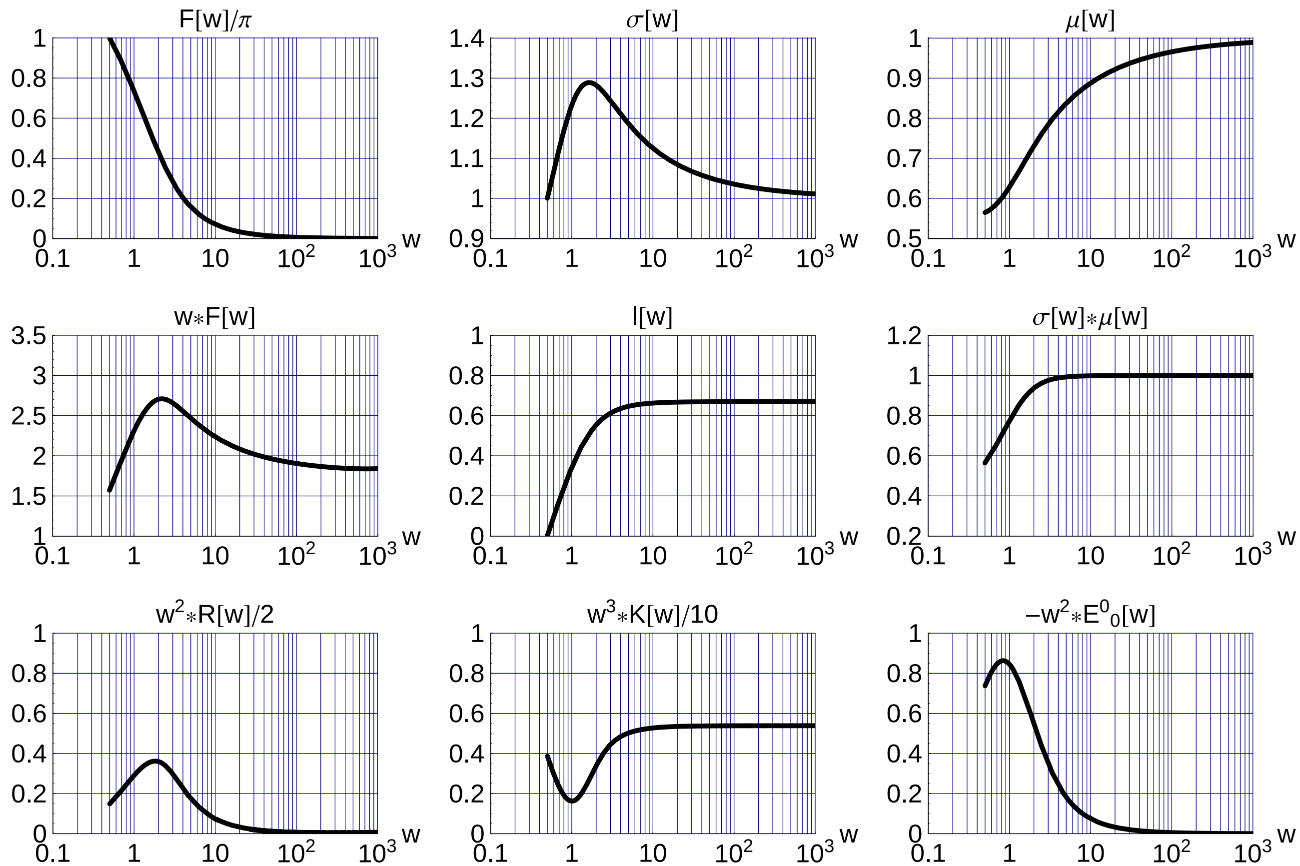


Figure 1: Numerical solution with parameters $\widetilde{\eta} \equiv 8\pi\, G_N\, f^2 = 1/20$, $\widetilde{m} \equiv m/(e\, f) = 0$, and $y_0 \equiv e f b = 1/\sqrt{2}$, and BCS at the defect surface $w = (y_0)^2 = 1/2$: $F = \pi$, $F' = -1.9718377138$, $\sigma = 1$, and $\mu = 0.564337$.

2.5 Skyrmion spacetime defect – Numerics

The boundary condition $\sigma(y_0^2) = 1$ may be called the "standard" boundary condition, because the limit $b \to 0$ then connects to the standard Minkowski spacetime manifold.

But with $b \neq 0$ and the nontrivial topology $\mathbb{R}P^3$ from (2), the boundary condition on the metric *Ansatz* function can be generalized:

$$\sigma(y_0^2) \in (0,\, \infty)\,, \tag{21}$$

where the value zero has been excluded, in order that the field equations be well-defined at $w = y_0^2$ [Guenther, 2017].

The next two slides give the numerical solutions for two different values of $\sigma(y_0^2) < 1$ with, respectively, positive and negative asymptotic gravitational mass.

Recall the definition of the ADM mass in our context:

$$M_{\text{ADM}} = l_\infty \big/ \big(2\, G_N\, e\, f\big)\,, \qquad l_\infty \equiv \lim_{w\to\infty} l(w)\,. \tag{22}$$

2.5 Skyrmion spacetime defect – Numerics

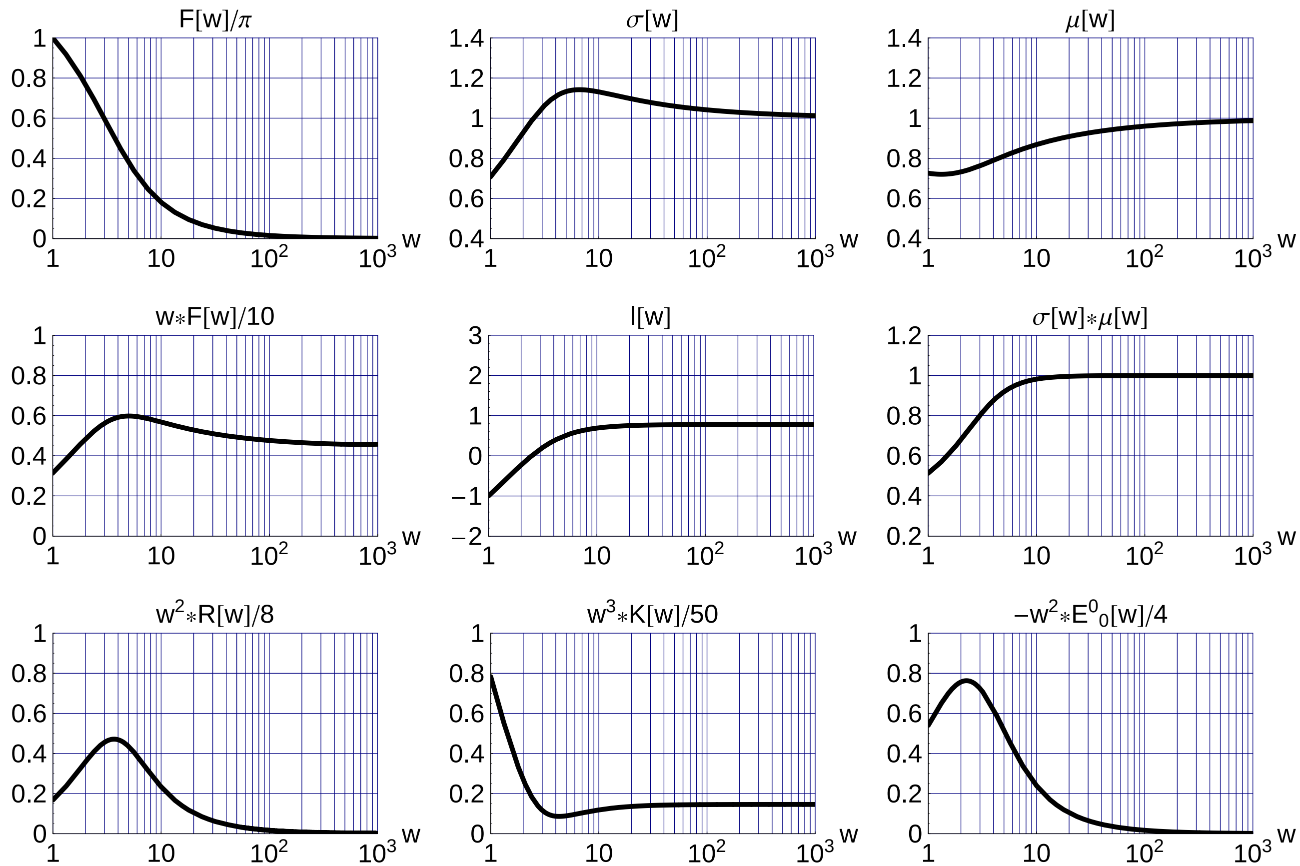


Figure 2: Numerical solution with parameters $\widetilde{\eta} = 1/10$, $\widetilde{m} = 0$, and $y_0 = 1$, and boundary conditions at $w = (y_0)^2 = 1$: $F = \pi$, $F' = -0.82561881304$, $\boldsymbol{\sigma = 1/\sqrt{2}}$, and $\mu = 0.725818$. The solution has $M_{\text{ADM}} > 0$.

2.5 Skyrmion spacetime defect – Numerics

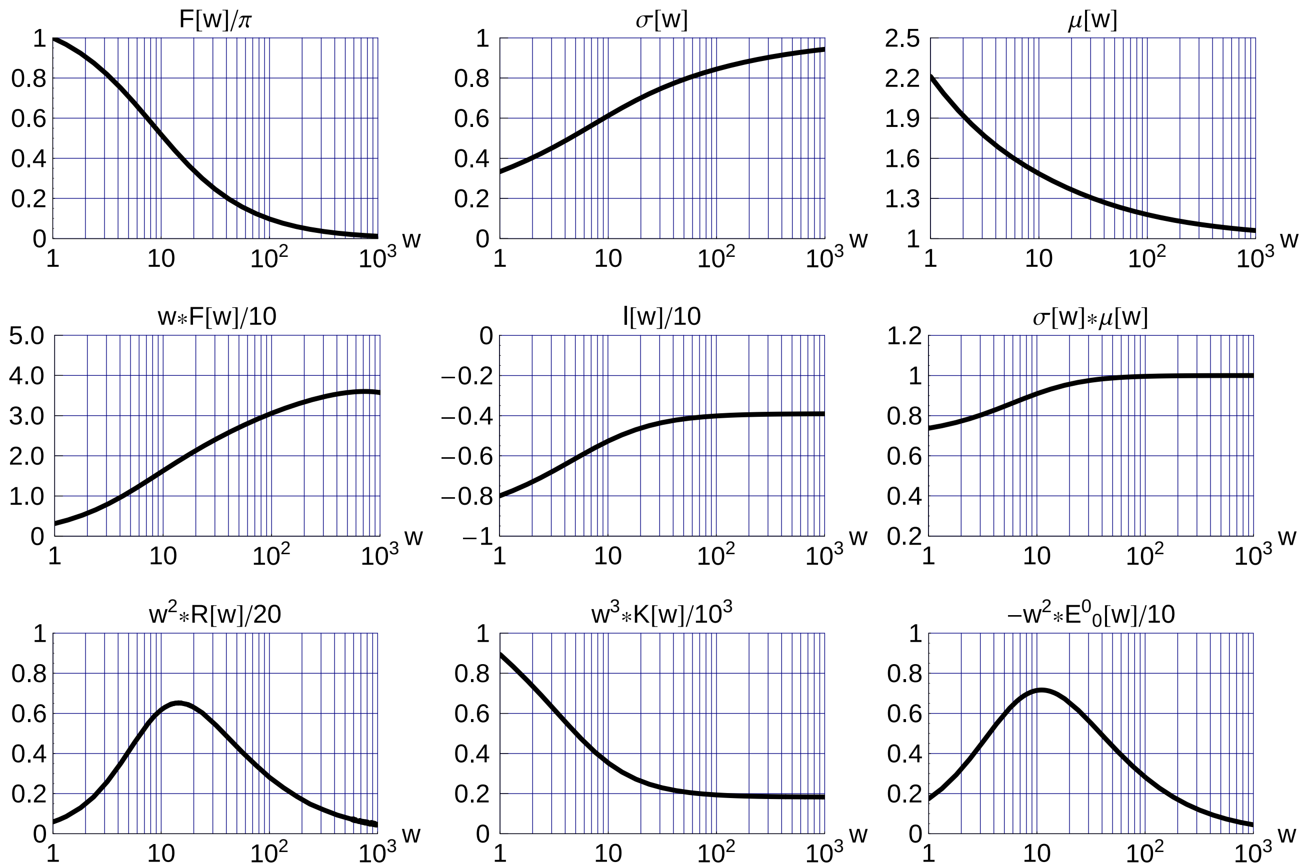


Figure 3: Same as Fig. 2, but with different boundary conditions at $w = 1$: $F = \pi$, $F' = -0.323978148$, $\boldsymbol{\sigma = 1/3}$, and $\mu = 2.21176$. The solution has $M_{\text{ADM}} < 0$.

3. Antigravity

Any localized object made of ponderable matter (e.g., quarks and leptons of the Standard Model) **attracts** a distant test particle. This phenomenon is called gravity and was studied by Newton [1687].

With the solution of Fig 3, we have a localized object which **repels** a distant test particle. The phenomenon may be called "antigravity."

The crucial ingredients of this particular object are, in the framework of Einstein's General Relativity [1915], the nontrivial topology of space [here, $\mathbb{R}P^3$] and the nontrivial gravitational fields at the defect surface [here, $\sigma(b^2) < 1$].

Figures 2 and 3 suggest that a defect can have either a positive or a negative gravitational mass, but we have a further result:

a sufficiently small defect solution exists only if it has a negative gravitational mass [a more precise formulation will be given later].

3. Antigravity

Consider, first, the nature of the solutions with “standard” boundary condition $\sigma(b^2) = 1$. It is, then, found that the solution collapses if it becomes too small (loosely speaking, if $b \lesssim R_{\text{Schwarzschild}}$) :

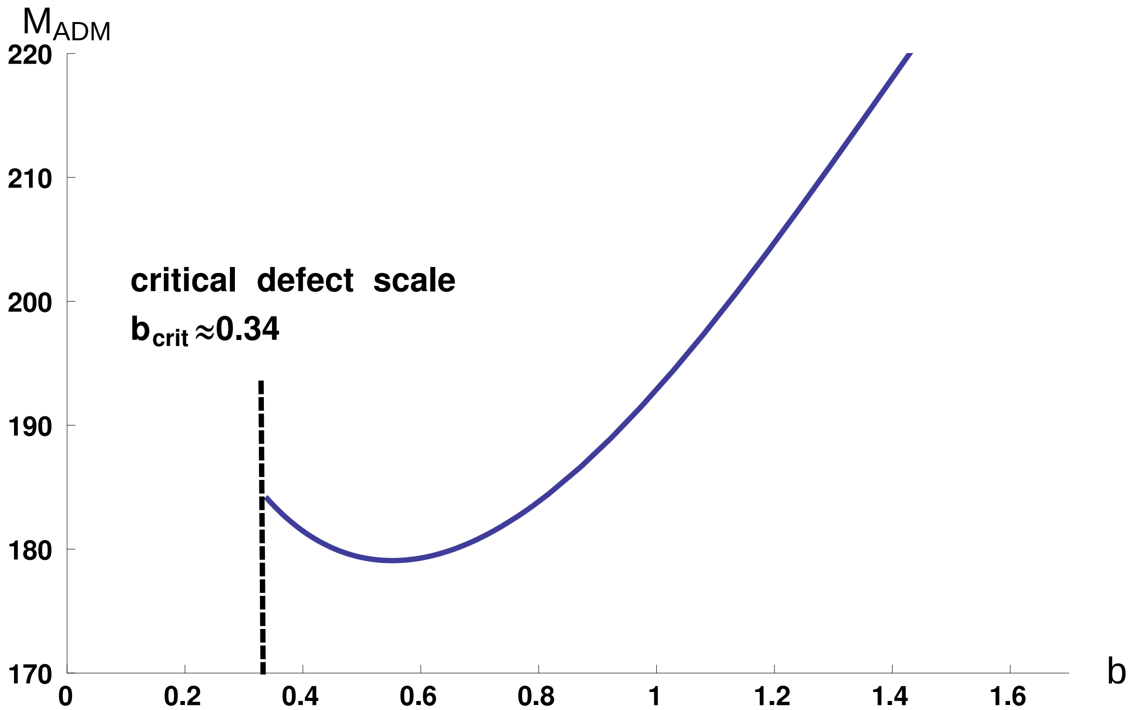


Figure 4: Mass M_{ADM} in units f/e vs. defect scale b in units $1/(ef)$, for parameters $\widetilde{\eta} = 0.033$ and $\widetilde{m} = 0$, and boundary condition $\sigma(b^2) = 1$.

3. Antigravity

There is then a critical curve in the $(\widetilde{\eta}, b)$ plane, above which there are no globally regular solutions with $\sigma(b^2) = 1$:

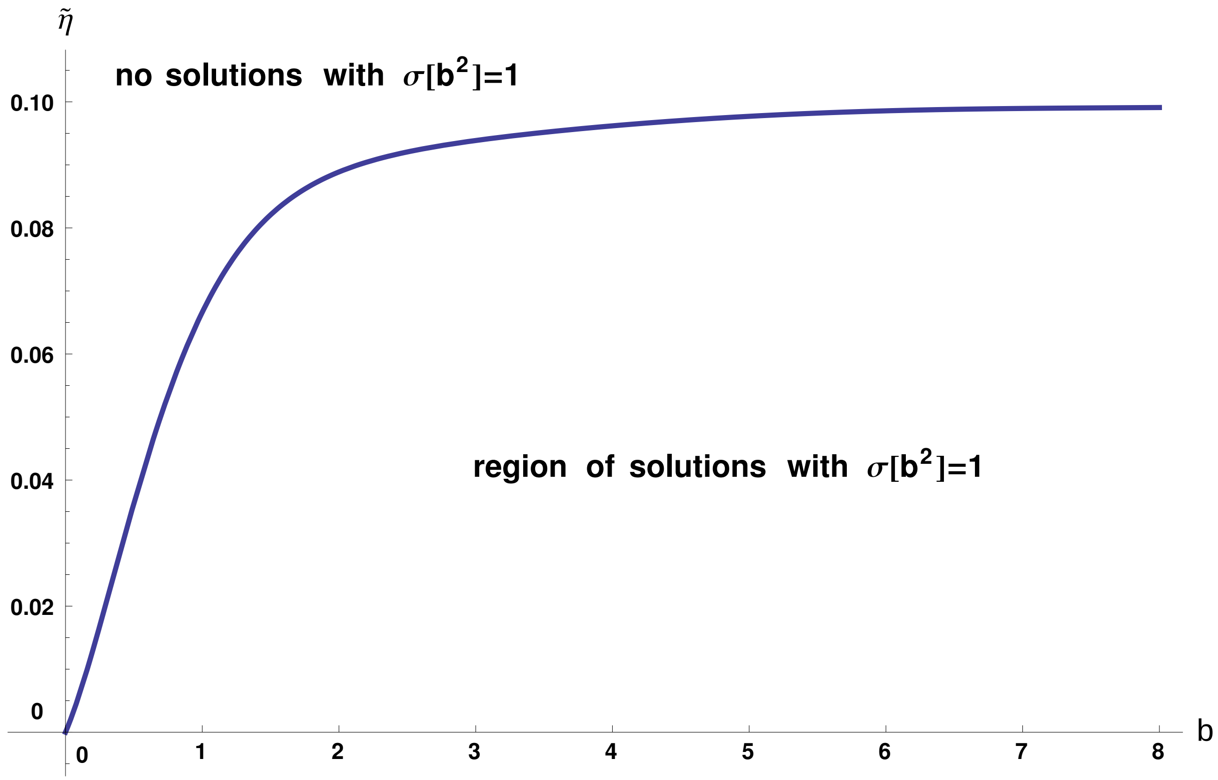


Figure 5: Curve of the critical defect scale b_{crit} [in units of $1/(ef)$] and the corresponding critical coupling constant $\widetilde{\eta}_{\text{crit}}$, with $\widetilde{m} = 0$ and $\sigma(b^2) = 1$.

3. Antigravity

HEURISTICS:

In order to get a globally regular solution in the region *above* the critical curve of Fig. 5, we need to add a sufficiently negative effective mass at the defect surface ($y = 0$).

Now, the dimensionless effective mass from (20) is given by

$$l(w) \equiv \sqrt{w}\,\left[1 - 1/\sigma^2(w)\right]\,, \tag{23}$$

with $w \equiv y_0^2 + y^2$. Then, $l(y_0^2) < 0$ results from $\sigma(y_0^2) < 1$.

For a fixed positive value of $\widetilde{\eta} \equiv 8\pi\, G_N\, f^2$, a sufficiently small globally regular defect solution thus requires a sufficiently negative effective mass at the defect surface $W = b^2$ from a nonstandard boundary condition on one of the metric functions, $\sigma(b^2) < 1$.

For a small enough value of the coupling constant $\widetilde{\eta}$, this boundary condition at the defect surface directly gives a negative ADM mass at spatial infinity.

3. Antigravity

Explicit example with ζ a number of order 1 or larger.

MODEL PARAMETERS:

$$f^2 \ll \left(E_{\text{planck}}\right)^2 \equiv 1/(8\pi G_N) \approx \left(2.44 \times 10^{18}\ \text{GeV}\right)^2, \tag{24a}$$

$$e \leq 1/\zeta\,, \tag{24b}$$

where the first inequality corresponds to $\widetilde{\eta} = \left(f/E_{\text{planck}}\right)^2 \ll 1$.

DEFECT SCALE [remnant of a quantum-spacetime (QST) phase]:

$$b_{\text{QST}} = \zeta\, l_{\text{planck}}\,, \tag{25a}$$

$$l_{\text{planck}} \equiv \sqrt{8\pi G_N\, \hbar/c^3} \approx 8.10 \times 10^{-35}\ \text{m}\,. \tag{25b}$$

DEFECT MASS [from (22) with $l_\infty \sim -1$]:

$$M_{\text{ADM}} \sim -\frac{4\pi}{e}\,\frac{\left(E_{\text{planck}}\right)^2}{f}\,. \tag{26}$$

4. Stealth defect

We have seen that certain soliton-type defect solutions can have positive gravitational mass but also negative gravitational mass.

As the gravitational mass of such a spacetime-defect solution is a continuous variable, there must also be special spacetime defects with vanishing gravitational mass.

These defects with positive energy density of the matter fields and zero asymptotic gravitational mass will be called "stealth defects."

An explicit solution with vanishing gravitational mass and an exponentially-vanishing energy density of the matter fields is given on the next slide.

4. Stealth defect

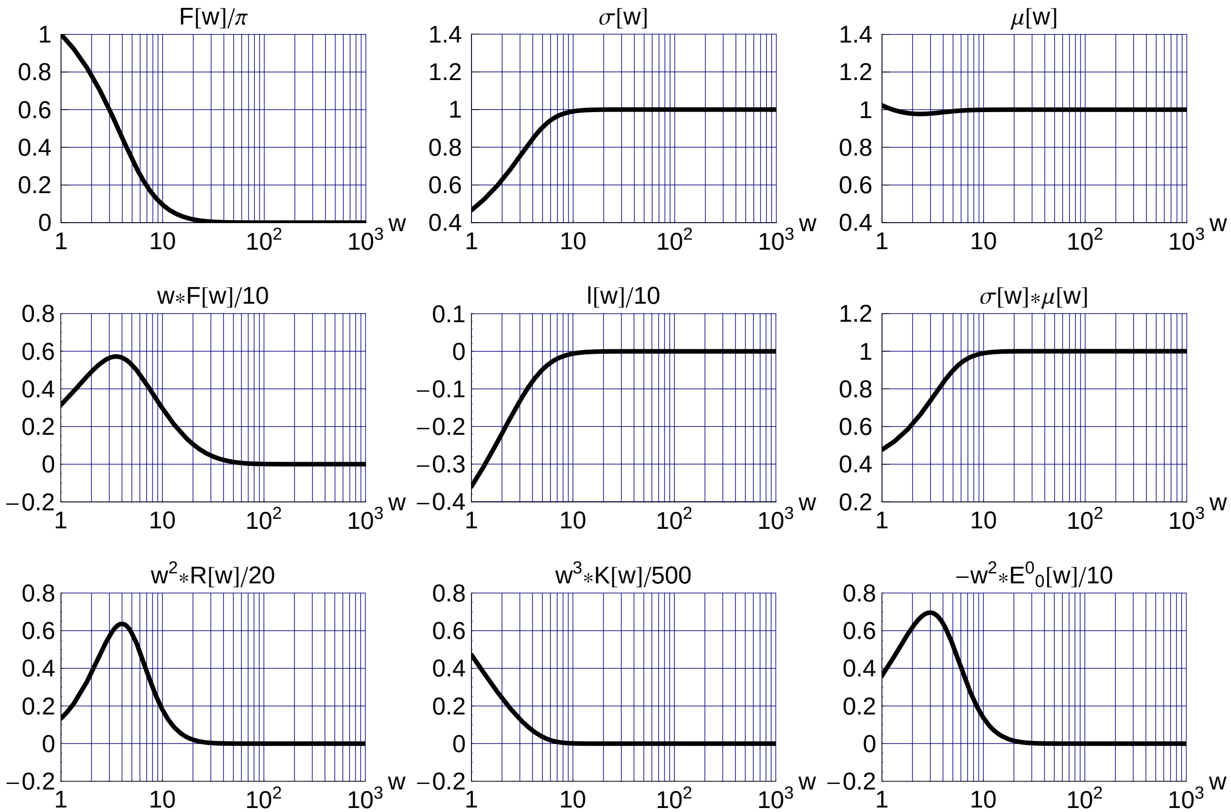


Figure 6: Numerical solution with parameters $\widetilde{\eta} = 1/10$, $\widetilde{m} = 1$, and $y_0 = 1$. The boundary conditions at $w = 1$ are: $F/\pi = 1.00000$, $F' = -0.752388$, $\sigma = 0.466343$, and $\mu = 1.02282$. The value of $|l(10^3)|$ is less than 10^{-11}.

4. Stealth defect (skip)

Now, assume that all matter fields have some form of non-gravitational interaction with each other. If so, there will, in principle, be some interaction between the "pions" of the theory considered in (7) and the elementary particles of the Standard Model.

Then, consider what happens with a head-on collision of a stealth defect and a human observer made of Standard Model particles (mostly up and down quarks, gluons, and electrons).

In close approximation, the observer will have no idea of what is going to happen, until he/she is within a distance of order $h/(mc)$ from the defect, where m is the "pion" mass scale from the matter action (7c).

What happens during the collision itself and afterwards depends on the details of the setup, for example, the size of the observer compared to the defect scale b.

5. Lensing (skip)

Simplified discussion of light rays over a spacetime-defect manifold, by use of an exact vacuum solution [K, 2014b]:

$$ds^2\Big|^{\text{(vac. sol.)}}_{\widetilde{M}_4\,,\,\text{chart}-2} = -\left(1-\widehat{l}/\sqrt{w}\right)(dt)^2 + \frac{1-y_0^2/w}{1-\widehat{l}/\sqrt{w}}\,(dy)^2 + w\left[(dz)^2+\sin^2 z\,(dx)^2\right], \tag{27a}$$

$$w \equiv y_0^2+y^2\,, \tag{27b}$$

$$y_0 \equiv e\,f\,b>0\,, \tag{27c}$$

$$\widehat{l} \in (-\infty,\,y_0)\,. \tag{27d}$$

In the notation of (13a): $[\mu(w)]^2 = 1/[\sigma(w)]^2 = 1-\widehat{l}/\sqrt{w}\,.$

5. Lensing (skip)

Next, specialize to a stealth-defect vacuum solution with

$$\widehat{l} = 0\,, \tag{28}$$

which has a flat spacetime.

Incidentally, *exact* multi-defect solutions of the vacuum Einstein equation are obtained by superposition of these static $\widehat{l} = 0$ defects, as long as the individual defect surfaces do not intersect.

The geodesics for a single $\widehat{l} = 0$ defect are readily calculated:

- straight lines in the ambient Euclidean 3-space, if there are no intersections with the defect surface;
- straight-line segments (with or without a parallel shift in the ambient Euclidean 3-space), if there are intersections with the defect surface.

5. Lensing (skip)

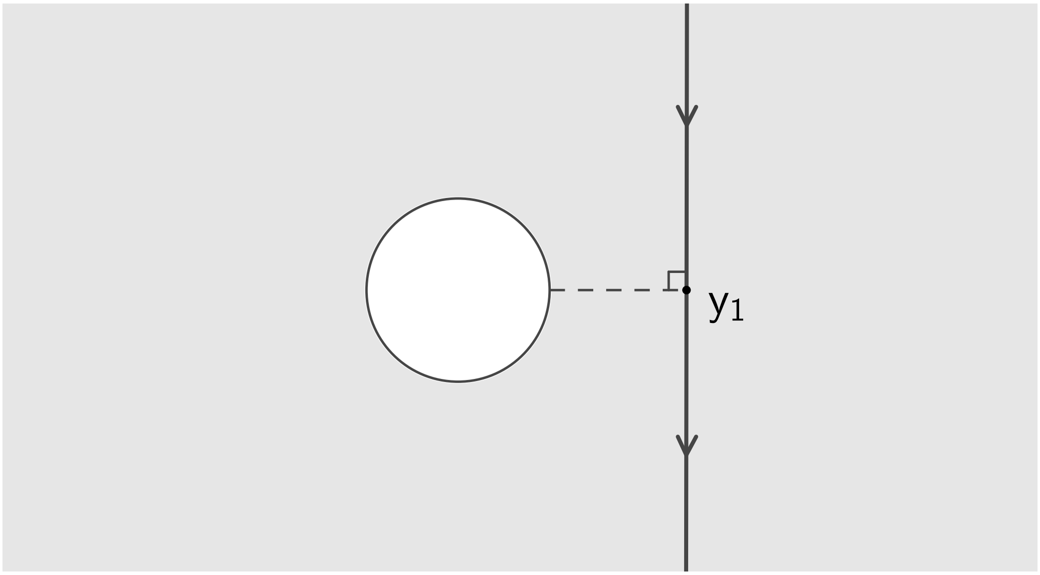


Figure 7: Geodesic which does not cross the defect surface, with part of the 3-space manifold indicated by the shaded area. The dimensionless quasi-radial coordinate y_1 corresponds to an “impact parameter.”

5. Lensing (skip)

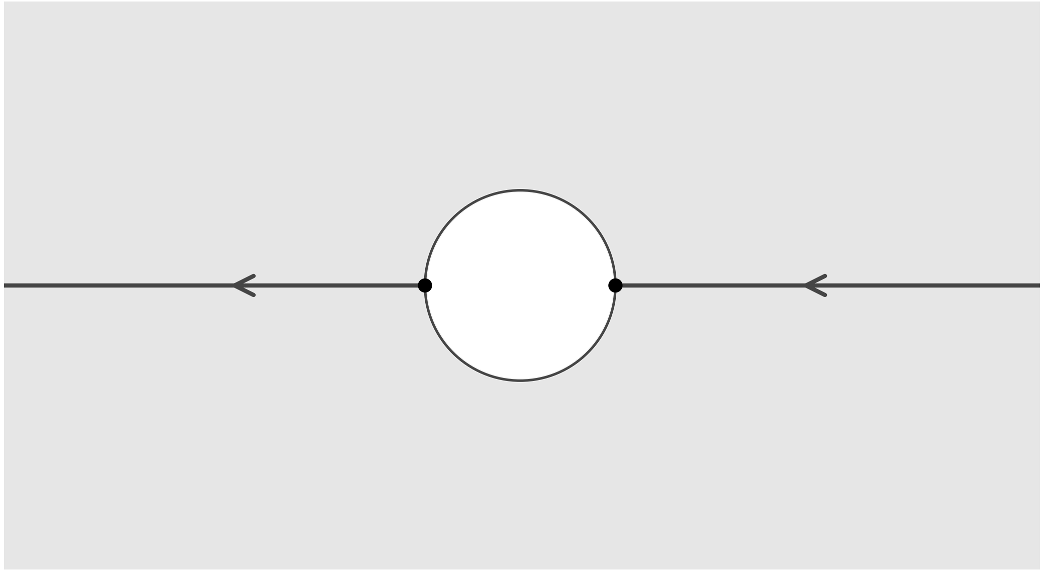


Figure 8: Radial geodesic which crosses the defect surface, where antipodal points (dots) on the defect surface are identified.

5. Lensing (skip)

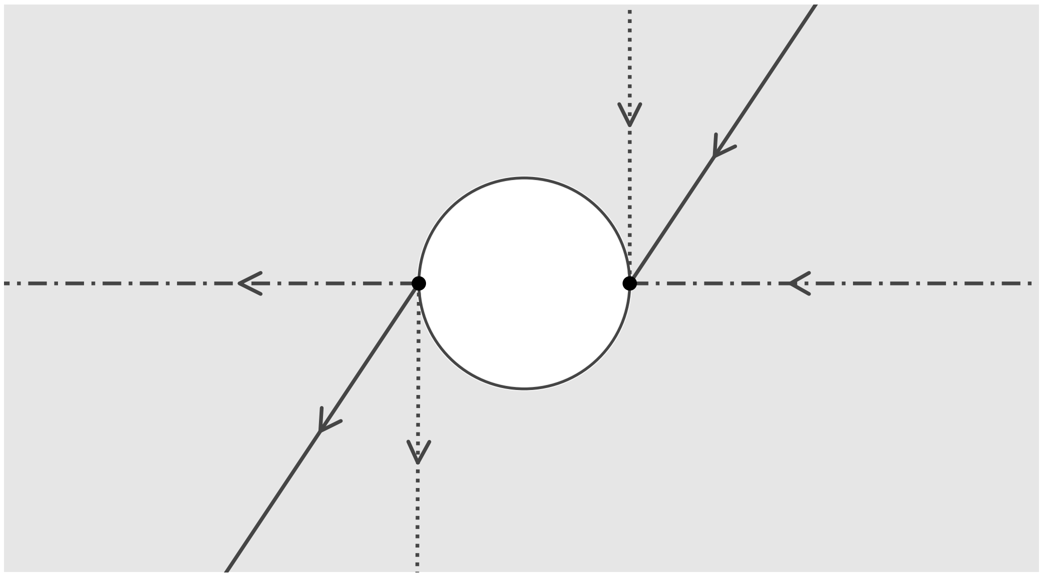


Figure 9: A family of geodesics crossing the defect surface.

5. Lensing (skip)

Due to the parallel shifts at the defect surface, there is a lensing effect:

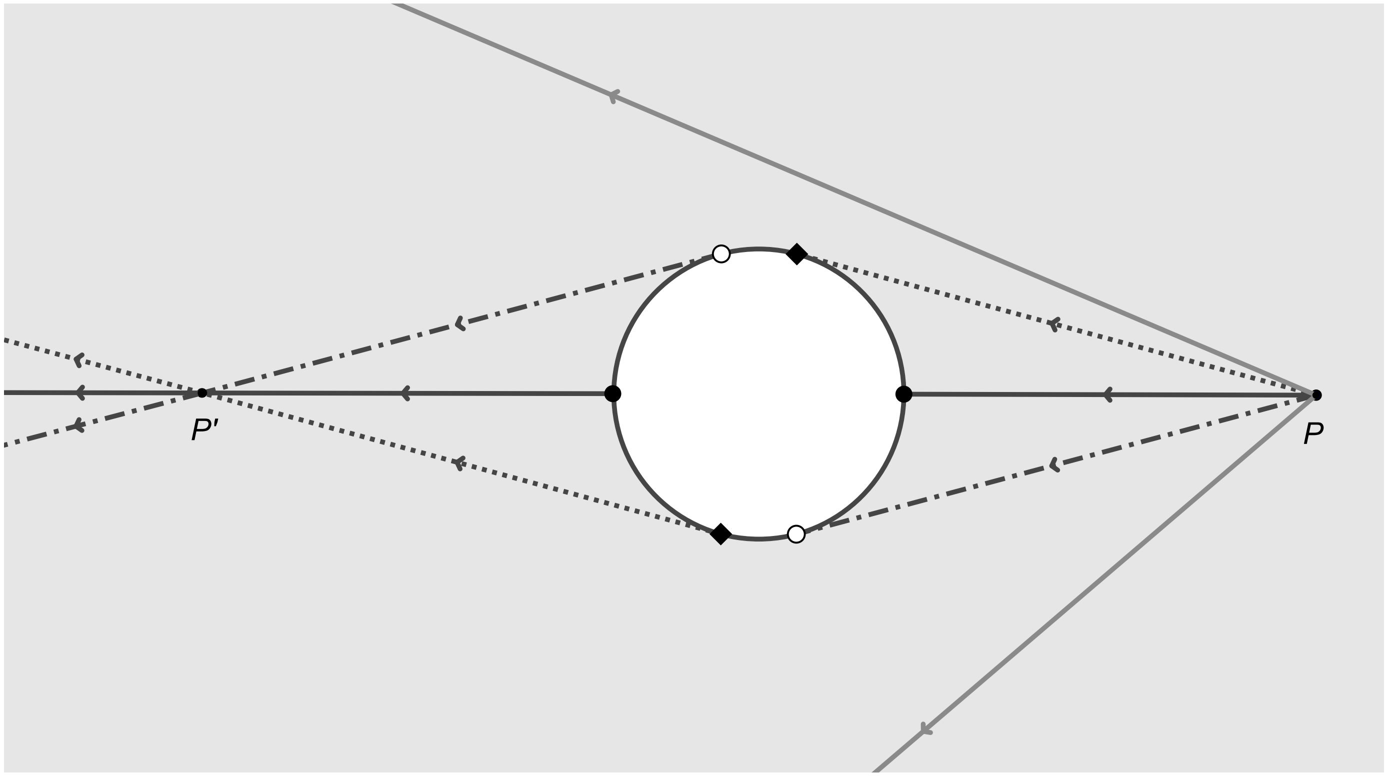


Figure 10: Geodesics with intersection points P and P'.

5. Lensing (skip)

The lensing of the flat-spacetime defect results in image formation:

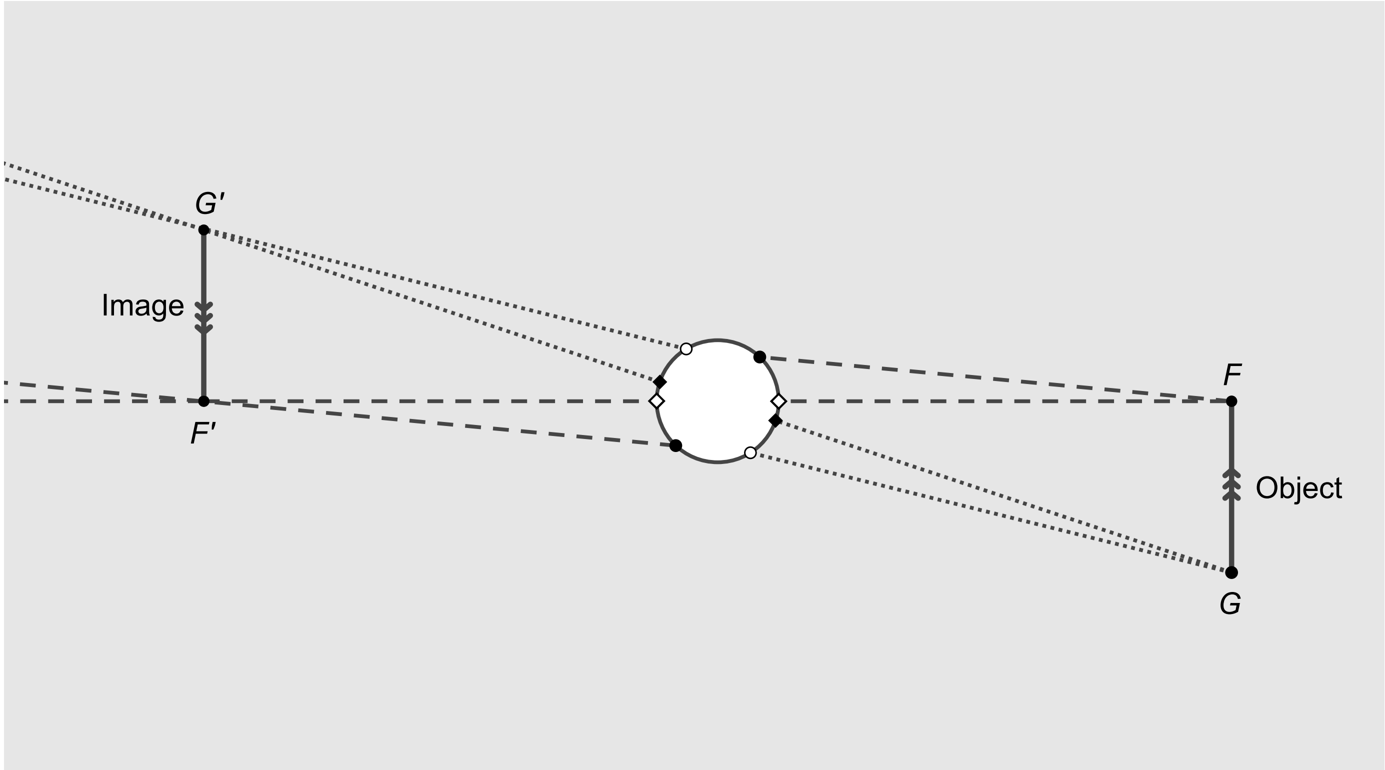


Figure 11: Image formation by a stealth defect.

5. Lensing (skip)

A few remarks:

- The image in Fig. 11 is located at the reflection point on the other side of the defect.
- The image is inverted and the image size is equal to the object size. Note that this is also the case if an object in Minkowski spacetime is located at a $2f$ distance from a thin double-convex lens, where f is the focal length of the lens.
- The brightness of the image depends on the defect scale b and the location of the object: the image will be brighter if b is increased for unchanged object position or if the object is brought closer to the defect for unchanged b.
- If a permanent pointlike light source is placed at point P of Fig. 10, then an observer at point P' in the same figure will see a luminous disk (different from the Einstein ring which the observer would see if the defect were replaced by a patch of Minkowski spacetime with a static spherical star at the center).

6. Discussion

- The new type of Skyrmion solution is rather interesting: it combines the nontrivial topology of spacetime with the nontrivial topology of field-configuration space.

  In fact, the nontrivial topology of the underlying space manifold allows the internal $SO(3)$ space to be covered only once ($N = 1$).

- It remains to be proved that the solution obtained is stable.

  The scalar fields by themselves would be stable because of the topological charge $N = 1$, but, in principle, there could be still more branches of solutions with even lower values of the ADM mass.

- The Skyrmion-spacetime-defect metric from (13) is degenerate: $\det g_{\mu\nu} = 0$ at the defect surface $Y = 0$, which corresponds to a submanifold $\mathbb{R}P^2 \sim S^2/\mathbb{Z}_2$.

6. Discussion

- This degenerate metric makes that the Gannon singularity theorem [1975] and the Schoen–Yau positive-mass theorem [1979] are not directly applicable.

  The special feature of the Skyrmion spacetime defect solution is that certain geodesics at the $\mathbb{R}P^2$ defect surface cannot be continued uniquely (cf. Fig. 7 with $y_1 \to 0^+$ and the dotted line in Fig. 9).

- The negative ADM mass found for a small enough defect scale b is not due to ponderable matter but to the nontrivial gravitational fields at the $\mathbb{R}P^2$ defect surface (area $2\pi b^2$).

- The crucial open question is the origin and role of nontrivial spacetime topology. Specialized to our Skyrmion defect solution, some follow-up questions are:

  1. what sets the constant defect scale b?
  2. can this defect scale become a dynamic variable?

References

[K, 2014a] "Skyrmion spacetime defect,"
Phys. Rev. D **90**, 024007 (2014), arXiv:1402.7048.

[K&Queiruga, 2018a] "Antigravity from a spacetime defect,"
Phys. Rev. D **97**, 124047 (2018), arXiv:1803.09736.

[K&Queiruga, 2018b] "A stealth defect of spacetime,"
Mod. Phys. Lett. A **33**, 1850127 (2018), arXiv:1805.04091.

[K&Wang, 2018] "Lensing and imaging by a stealth defect of spacetime,"
arXiv:1808.02465.

[Schwarz, 2010] "Nontrivial spacetime topology, modified dispersion relations, and an $SO(3)$-Skyrme model," PhD Thesis, KIT, July 2010,
Verlag Dr. Hut, München, Germany, 2010 [ISBN 9783868536232].

[Guenther, 2017] "Skyrmion spacetime defect, degenerate metric, and negative gravitational mass," Master Thesis, KIT, September 2017
[available from https://www.itp.kit.edu/en/publications/diploma].

[K, 2014b] "A new type of nonsingular black-hole solution in general relativity,"
Mod. Phys. Lett. A **29**, 1430018 (2014), arXiv:1309.7011.