International workshop on CPT and Lorentz Symmetry in Field Theory
University of the Algarve, Faro, Portugal July 6-7, 2017

On an anomalous origin of Lorentz and CPT violation

Frans R. Klinkhamer

Institute for Theoretical Physics

Karlsruhe Institute of Technology (KIT)

76128 Karlsruhe, Germany

Email: frans.klinkhamer@kit.edu

(presented by Marco Schreck)

Introduction

1. INTRODUCTION

Experiment has shown the violation of P, C, CP, and T, but not CPT.

Indeed, there is the well-known CPT “theorem” [Lüders, 1954–57; Pauli, 1955; Bell, 1955; Jost, 1957]:

any local relativistic quantum field theory is invariant under the combined operation of charge conjugation (C), parity reflection (P), and time reversal (T).

The main inputs of this “theorem,” by itself, are:

- flat Minkowski spacetime $(M,\, g) = \big(\mathbb{R}^4,\, \eta_{\mu\nu}^{\text{Minkowski}}\big)$;
- invariance under proper orthochronous Lorentz transformations and spacetime translations;
- normal spin-statistics connection;
- locality and Hermiticity of the Hamiltonian.

Introduction

BUT CAN CPT INVARIANCE BE VIOLATED AT ALL IN A PHYSICAL THEORY AND, IF SO, IS IT IN THE REAL WORLD?

It was widely believed that only quantum-gravity or superstring effects could give CPT violation.

A different result has, however, been obtained several years ago [1]:

for certain spacetime topologies and classes of chiral gauge theories, CPT invariance is broken anomalously, that is, by quantum effects.

Crucial ingredients of this “CPT anomaly” are:

- chiral fermions and gauge interactions;
- nontrivial spacetime topology.

---

[1] Klinkhamer, NPB 578 (2000) 277 [arXiv:hep-th/9912169].

Introduction

Possible applications:

- *optical activity of the vacuum*, e.g., for the CMB [2abc];
- *fundamental arrow-of-time*, e.g., for the Big Bang [3];
- *spacetime foam*, with CPT anomaly as diagnostic tool [4].

In this brief talk, we focus on the CPT anomaly and skip possible applications (see [2b]).

[2a] Carroll, Field & Jackiw, PRD 41 (1990) 1231.

[2b] Klinkhamer, in: GustavoFest Proceedings [arXiv:hep-ph/0511030].

[2c] Komatsu et al. [WMAP], ApJSuppl 180 (2009) 330, arXiv:0803.054.

[3] Klinkhamer, PRD 66 (2002) 047701 [gr-qc/0111090].

[4] Klinkhamer & Rupp, PRD 70 (2004) 045020 [arXiv:hep-th/0312032].

Outline

Outline of the rest of this talk:

1. Introduction
2. CPT anomaly – Heuristics
3. Perturbative result
4. Nonperturbative result
5. Summary

CPT anomaly – Heuristics

2. CPT ANOMALY – HEURISTICS

The main ingredients of the CPT anomaly for 4D manifold $M = \mathbb{R}^3 \times S^1$ with vierbeins $e^a_\mu(x) = \delta^a_\mu$ and appropriate gauge fields:

- compact space dimension, coordinate $x^3 \in [0, L]$, with periodic spin structure (fermions can have momentum component $p_3 = 0$);
- a single chiral fermion with $p_3 = 0$ corresponds to a single massless Dirac fermion in 3D;
- a single massless Dirac fermion in 3D has a "parity anomaly," provided gauge invariance is maintained exactly [5,6];
- this "parity" violation corresponds to T violation in 4D, which, in turn, implies CPT violation in 4D.

---

[5] Redlich, PRL 52 (1984) 18; PRD 29 (1984) 2366.

[6] Alvarez-Gaumé & Witten, NPB 234 (1984) 269.

Perturbative result

3 PERTURBATIVE RESULT IN THE CONTINUUM

Consider the chiral gauge theory with

$$\begin{aligned} G &= SO(10)\,, \quad R_{\text{left}} = N_{\text{fam}} \times (\mathbf{16})\,, \quad N_{\text{fam}} = 1\,, \\ M &= \mathbb{R}^3 \times S^1_{\text{PSS}}\,, \quad e^a_\mu(x) = \delta^a_\mu\,, \quad g_{\mu\nu}(x) = \eta_{\mu\nu}\,, \end{aligned}$$

where PSS stands for periodic spin structure. (Similar results for $N_{\text{fam}} = 3$.)

Of course, the effective action $\Gamma[A]$, for $A \in \text{so}(10)$, is not known exactly (there are, however, exact results [7] in 2D).

---

[7] Klinkhamer & Nishimura, PRD 63 (2001) 097701 [arXiv:hep-th/0006154].

Perturbative result

But the crucial term has been identified perturbatively [1,8] for a gauge field with trivial holonomy (e.g., $A_3 = 0$):

$$\Gamma^{\mathbb{R}^3\times S^1}_{\text{anom}}[A] = \frac{1}{32\pi} \int_{\mathbb{R}^3} \mathrm{d}x^0 \mathrm{d}x^1 \mathrm{d}x^2 \int_0^L \mathrm{d}x^3 \; \frac{x^3}{L} \, \epsilon^{\kappa\lambda\mu\nu} \, \mathrm{tr}\, \left[A_{\kappa\lambda}(x)\, A_{\mu\nu}(x)\right] ,$$

for a Lie-algebra-valued gauge potential $A_\mu(x) \equiv g\, A^a_\mu(x)\, T^a$ and Yang-Mills field strength

$$A_{\kappa\lambda}(x) \equiv \partial_\kappa A_\lambda(x) - \partial_\lambda A_\kappa(x) + [A_\kappa(x), A_\lambda(x)] \, .$$

This local gauge-invariant term Γ_{anom} is Lorentz-noninvariant, because of the spacetime-dependent "coupling constant" x^3/L, and also CPT-odd (whereas the Yang-Mills action term is CPT-even).

---

[8] Ghosh & Klinkhamer, arXiv:1706.07025.

Perturbative result

Technical remarks:

1. After a partial integration of the anomalous term, the integrand contains a Chern-Simons-like term

$$\frac{1}{L}\,\epsilon^{\kappa\lambda\mu 3}\ \text{tr}\ \left[A_{\kappa\lambda}A_{\mu}-\frac{2}{3}A_{\kappa}A_{\lambda}A_{\mu}\right]\,,$$

where the explicit spacetime index ‘3’ makes clear that Lorentz invariance is broken.

2. For non-Abelian gauge fields, the anomalous term has, strictly speaking, only been derived for a special class of gauge fields, namely x^3-independent fields,

$$A_3=0\,,\quad A_\mu=A_\mu(x^0,\,x^1,\,x^2)\,,\ \ \text{for}\ \ \mu=0,\,1,\,2\,.$$

Nonperturbative result

4 NONPERTURBATIVE RESULT ON THE LATTICE

Consider the 4D Abelian chiral gauge theory with

$$G = U(1),\ R_{\text{left}} = 6\times\left(\frac{1}{3}\right)+3\times\left(\frac{-4}{3}\right)+3\times\left(\frac{2}{3}\right)+2\times(-1)+1\times(2)+1\times(0)\ ,$$

where perturbative chiral gauge anomalies cancel out, $U(1) \subset SO(10)$.
[N.B. vectorlike QED has $R_{\text{left}} = (+1) + (-1)$, a real representation.]

Define a chiral lattice gauge theory over a finite hypercubic lattice, with

- periodic spin structure in one direction;
- Ginsparg–Wilson fermions;
- Neuberger's lattice Dirac operator;
- Lüscher's chiral constraints.

Nonperturbative result

The goal is to establish that the Euclidean effective gauge-field action $\Gamma[U]$ changes under a CPT transformation,

$$\Gamma[U] \neq \Gamma[U^{\text{CPT}}]\,,$$

where U denotes the set of link variables.

This result has been obtained [9] (see also [8]) for Abelian gauge fields with a vanishing holonomy (e.g., $U_3 = 1$).

Moreover, the origin of the CPT anomaly has been identified as an ambiguity in the choice of basis vectors for the fermion integration measure; cf. the path-integral derivation of the triangle anomaly [10].

---

[9] Klinkhamer & Schimmel, NPB 639 (2002) 241 [arXiv:hep-th/0205038].
[10] Fujikawa, PRD 21 (1980) 2848.

Summary

5. SUMMARY

The subtle role of topology on the local properties of quantum field theory is well-known (e.g., the Casimir effect).

For certain chiral gauge theories, the interplay of UV and IR effects may also lead to

Lorentz and CPT noninvariance,

even for flat spacetime manifolds, that is, without gravity.

The basic idea is quite simple, but, as always, there are subtleties . . .