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A new approach to the cosmological constant problem (with an update from February 6, 2018)

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1. Introduction

The main **Cosmological Constant Problem** (CCP1) can be phrased as follows (Pauli, 1933; Bohr, 1948; Veltman, 1974; see [1, 2] for two reviews):

why do the quantum fields in the vacuum not produce naturally a large cosmological constant Λ in the Einstein field equations?

The magnitude of the problem is enormous:

$$|\Lambda^{\text{theory}}|/|\Lambda^{\text{experiment}}| \geq 10^{54},$$

where the large number on the RHS will be explained on the next slide.

From now on, $\hbar = 1 = c$.

1. Introduction

With the ATLAS and CMS results [3, 4] in support of the Higgs mechanism, it is clear that the EWSM in the laboratory involves a vacuum energy density of order

$$|\epsilon_V^{(\text{EWSM})}| \sim (100 \text{ GeV})^4 \sim 10^{44} \text{ eV}^4 .$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_V^{(\text{EWSM})} = \epsilon_V^{(\text{EWSM})}(T) .$$

How can the Universe then end up with a vacuum energy density

$$|\Lambda^{(\text{obs})}| < 10^{-28} \text{ g cm}^{-3} \sim 10^{-10} \text{ eV}^4 ?$$

Here, there are 54 orders of magnitude to explain:

$$\begin{aligned} & \left| \Lambda^{(\text{obs})} / \epsilon_V^{(\text{EWSM})} \right| \\ & \leq 0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 001 . \end{aligned}$$

1. Introduction

In short, the main cosmological constant problem is

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{Planck}})^4$?

Still more CCPs after the discovery of the “accelerating Universe”:

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}|_{\text{present}} \sim +10^{-11} \text{ eV}^4$?

Hundreds of papers have been published on CCP2. But, most likely:

CCP1 needs to be solved first before CCP2 can even be addressed.

1. Introduction

Here, a discussion of one particular approach to CCP1 by Volovik and the speaker, which goes under the name of q -theory [5, 6, 7] (a brief review appears in [8]).

It is instructive to consider two explicit realizations of q -theory:

1. with a three-form gauge field [9, 10, 11, 12],
2. with a massless vector-field [13, 14].

The vector-field realization, in particular, is found to give

Minkowski spacetime as an **attractor** of the field equations.

(But a new problem arises: the danger of ruining the standard Newtonian physics of small self-gravitating systems [15]. This disaster can, however, be avoided by a special model with two vector fields [16, 17].)

In this talk, we first focus on the 3-form-gauge-field realization of q -theory, which keeps the Newtonian physics in tact, and then briefly turn to CCP2.

1. Introduction

OUTLINE:

1. Introduction
2. Basics of q -theory
3. Two realizations
4. Remnant ρ_V : Electroweak-kick mechanism
5. Conclusion
6. Update from February 6, 2018
7. References

2. Basics of q -theory

Crucial insight [5]: there is vacuum energy and vacuum energy.

More specifically and introducing an appropriate notation:

the vacuum energy density ϵ appearing in the action

need not be the same as

the vacuum energy density ρ_V in the Einstein field equations.

How can this happen concretely ...

2. Basics of q -theory

Consider the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Study, then, the **macroscopic** equations of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy:

- Take the mass density ρ of a liquid, for example, liquid Argon.
- This ρ describes microscopic quantities ($\rho = m_{\text{Ar}} n_{\text{Ar}}$ with number density n_{Ar} and mass m_{Ar} of the atoms).
- Still, ρ obeys the macroscopic equations of hydrodynamics, because of particle-number and mass conservation.

However, is the quantum vacuum just like a normal liquid?

2. Basics of q -theory

No, as the quantum vacuum is known to be **Lorentz invariant** (cf. experimental limits at the 10^{-15} level in the photon sector [18]).

The Lorentz invariance of the vacuum rules out the standard type of charge density, which arises from the time component j_0 of a conserved vector current j_μ .

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q .

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material liquids.

2. Basics of q -theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{nonconstant}}(q), \quad (1)$$

including a possible constant term Λ_{bare} from the zero-point energies of the fields of the Standard Model (SM).

From ① thermodynamics and ② Lorentz invariance follows that [5]

$$P_V \stackrel{\textcircled{1}}{=} - \left(\epsilon - q \frac{d\epsilon}{dq} \right) \stackrel{\textcircled{2}}{=} -\rho_V, \quad (2)$$

where the first equality corresponds to an integrated form of the Gibbs–Duhem equation for chemical potential $\mu \equiv d\epsilon/dq$.

Recall GD eq: $N d\mu = V dP - S dT \Rightarrow dP = (N/V) d\mu$ for $dT = 0$.

2. Basics of q -theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{Planck}})^4$, but they cancel exactly for an appropriate value q_0 of the vacuum variable q .

Hence, for a generic function $\epsilon(q)$,

$$\exists q_0 = \text{const} : \quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q \frac{d\epsilon(q)}{dq} \right]_{q=q_0} = 0 , \quad (3)$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle ...

But, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q (see Sec. 3).

3.0 Two realizations for two questions

Explicit realizations of q -theory provide answers to the following two obvious questions:

Q1: How does the adjustment-type solution (3) of CCP1 circumvent Weinberg's no-go "theorem" [2]?

Answer: q is a non-fundamental scalar field (see Sec. 3.1).

Q2: How did the Universe get the right value q_0 ?

One possible answer is that q_0 (or the corresponding chemical potential μ_0) is fixed globally as an integration constant, being conserved throughout the history of the Universe [6].

Another possible answer uses a generalization of q -theory, for which the 'correct' value q_0 arises dynamically (Sec. 3.2).

Update: q -theory with quantum dissipative effects included (Sec. 6.1).

3.1 Four-form realization

Vacuum variable q may arise from a 3–form gauge field A [9, 10].

Start from the effective action of GR+SM,

$$S^{\text{eff}}[g, \psi] = \int_{\mathbb{R}^4} d^4x \sqrt{-\det g} \left(K_N R[g] + \Lambda_{\text{SM}} + \mathcal{L}_{\text{SM}}^{\text{eff}}[\psi, g] \right), \quad (4)$$

with gravitational coupling constant $K_N \equiv 1/(16\pi G_N)$ and $\hbar = c = 1$.

Change this theory by the introduction of one field, A , to get [6, 7]:

$$\tilde{S}^{\text{eff}}[A, g, \psi] = \int_{\mathbb{R}^4} d^4x \sqrt{-\det g} \left(K(q) R[g] + \epsilon(q) + \mathcal{L}_{\text{SM}}^{\text{eff}}[\psi, g] \right), \quad (5a)$$

$$q \equiv \boxed{-\frac{1}{24} \epsilon^{\alpha\beta\gamma\delta} \nabla_{\alpha} A_{\beta\gamma\delta} / \sqrt{-g}}, \quad (5b)$$

where q arises from the four-form field strength $F = dA$.

Variational principle gives generalized Einstein and Maxwell equations:

3.1 Four-form realization

$$2K(q) (R_{\alpha\beta} - g_{\alpha\beta} R/2) = -2 (\nabla_{\alpha} \nabla_{\beta} - g_{\alpha\beta} \square) K(q) + \rho_V(q) g_{\alpha\beta} - T_{\alpha\beta}^M, \quad (6a)$$

$$\frac{d\rho_V(q)}{dq} + R \frac{dK(q)}{dq} = 0, \quad (6b)$$

with a vacuum energy density,

$$\rho_V = \epsilon - q \left(\frac{d\epsilon}{dq} + R \frac{dK}{dq} \right) = \epsilon - q \mu, \quad (7)$$

for integration constant (chemical potential) μ . Eq. (7) is precisely of the Gibbs–Duhem form (2) in Minkowski spacetime ($R = 0$). Technically, the extra $g_{\alpha\beta}$ term on the RHS of (6a) appears because $q = q(A, \underline{g})$.

Hence, an answer to Q1: (5b) shows that q is a non-fundamental scalar field, which invalidates Weinberg's argument (see [7] for details).

3.2 Vector-field realization

Vacuum variable q comes from an aether-type velocity field u_β [13, 14], setting $E_{UV} = E_{\text{Planck}}$. For a flat RW metric with cosmic time t , there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \rightarrow q_0 t, \quad u_b(t) = 0, \quad H(t) \rightarrow 1/t, \quad (8a)$$

$$u_\alpha{}^\beta \equiv \nabla_\alpha u^\beta \rightarrow \boxed{q_0 \delta_\alpha{}^\beta}. \quad (8b)$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. With an action quadratic in the variable $u_\alpha{}^\beta$, the field equations are [13]:

$$\ddot{v} + 3h\dot{v} - 3h^2v = 0, \quad (9a)$$

$$2\lambda - (\dot{v})^2 - 3(hv)^2 = 6h^2, \quad (9b)$$

with the overdot standing for differentiation with respect to τ . Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\hat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\hat{q}_0 = \sqrt{\lambda/2}$.

3.2 Vector-field realization

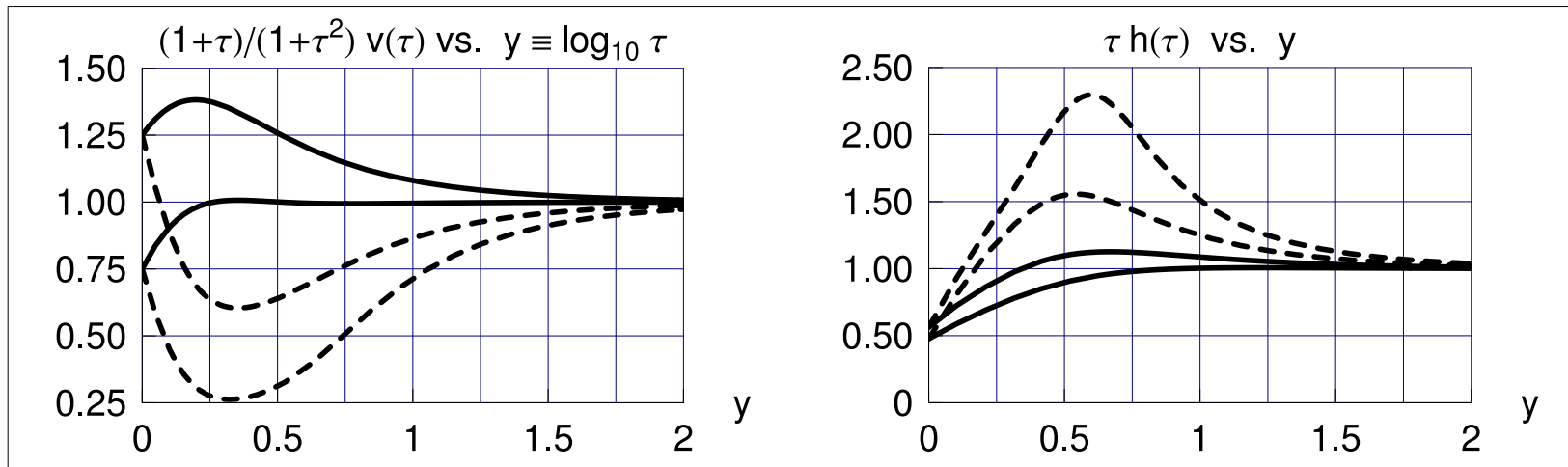


Fig. 1: Four numerical solutions of ODEs (9ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25$.

- \Rightarrow Minkowski value $\hat{q}_0 = \sqrt{\lambda/2} = 1$ arises dynamically [see left panel].
- \Rightarrow Minkowski spacetime is an attractor in this aether-type theory [7].

Hence, an answer to Q2. But, as mentioned above, there is serious collateral damage [15] which needs to be avoided [16, 17].

3.3 Recap

To summarize, the q -theory approach to the main Cosmological Constant Problem (CCP1) provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics contains such a q -type variable.

GENERAL REMARK: it is clear that the SM harbors huge vacuum energy densities, which somehow need to be cancelled by new d.o.f., possibly related to the fundamental theory of spacetime and gravity.

BAD NEWS: nothing is known for sure about these fundamental d.o.f.

GOOD NEWS: even though the detailed (high-energy) microphysics is unknown, it may be possible to describe the macroscopic (low-energy) effects along the lines of q -theory, just as for the hydrodynamics of water.

4.1 Remnant ρ_V

Now, briefly the remaining problems (or puzzles, rather):

CCP2a – why $\Lambda_{\text{eff}} \neq 0$?

CCP2b – why $\Lambda_{\text{eff}} \sim \rho_{\text{matter}} \big|_{\text{now}} \sim 10^{-29} \text{ g cm}^{-3} \sim 10^{-11} \text{ eV}^4$?

Last one also goes under the name of ‘cosmic coincidence puzzle’ (ccp).

In the framework of q -theory, we have given speculative discussions of the remnant vacuum energy density from the physics of QCD [19] or massive neutrinos [8]. But, here, our speculations will focus on the electroweak (TeV) scale.

4.2 Remnant ρ_V – Electroweak kick

Reconsider the four-form realization of q , taken to be operative at a UV (Planckian) energy scale.

In the very early Universe, the vacuum energy density $\rho_V(t)$ rapidly drops to zero and stays there, but small effects may occur at cosmic temperatures T of the order of the TeV scale . . .

Simple picture:

Take a glass of water, hold it steady, and then shake it \Rightarrow water responds.

If vacuum energy density is really like a liquid, then it can be ‘shaken.’ Here, the ‘shaking’ is done by massive particles.

4.2 Remnant ρ_V – Electroweak kick

Key steps of the frozen-electroweak-kick mechanism [20, 21]:

- Presence of massive particles with electroweak interactions [average mass $M \sim \text{TeV}$] changes the Hubble expansion rate $H(t)$ of the Universe compared to the radiation-dominated case.
- Change of the expansion rate kicks $\rho_V(t)$ away from zero.
- Quantum-dissipative effects operating at cosmic time $t_{\text{kick}} \equiv E_P/M^2$ may result in finite remnant value of ρ_V . [Here, E_P is the reduced Planck energy $\sqrt{1/(8\pi G_N)} \approx 2.44 \times 10^{18} \text{ GeV}$.]
- Phenomenological description of this process with a simple field-theoretic model.

4.2 Remnant ρ_V – Electroweak kick

Core formula for the remnant vacuum energy density in a flat Friedmann–Robertson–Walker (FRW) universe:

$$\Lambda \equiv \lim_{t \rightarrow \infty} \rho_V(t) = r_{V\infty} M^8 / (E_P)^4. \quad (10)$$

Inverting this equation gives

$$\begin{aligned} M &= (r_{V\infty})^{-1/8} \Lambda^{1/8} (E_P)^{1/2} \\ &\approx 5.56 \text{ TeV} \left(\frac{10^{-3}}{r_{V\infty}} \right)^{1/8} \left(\frac{\Lambda^{1/4}}{2.25 \text{ meV}} \right)^{1/2}. \end{aligned} \quad (11)$$

The outstanding task is to **calculate** the “efficiency factor” $r_{V\infty}$ for producing a remnant vacuum energy density Λ given the energy scales involved, M and E_P . BTW, the parametric behavior of (10) was already discussed by Arkani-Hamed, Hall, Kolda, and Murayama [22].

4.2 Remnant ρ_V – Electroweak kick

Main inputs for a simple model calculation [21]:

- Brans–Dicke-type term in the action density,

$$\mathcal{L}_{\text{grav}} = K[q, \Phi] R[g], \quad (12)$$

where Φ stands for one or more of the matter fields and we assume the following simplified behavior:

$$K[q, t] = q(t)/2 + \theta(t - t_K) [q_0/2 - q(t)/2]. \quad (13)$$

- Two types of matter: type-1 for ultraheavy particles of mass M ; type 2 for massless particles with $N_{\text{eff}, 2} = 10^2$ (\approx SM).
- For the K-freezing-model calculation, set $t_K = O(1)$.
For the dissipation-model calculation, set $t_K = \infty$.
- Really unsolved question: what **physics** freezes $\rho_V(t)$???

4.2 Remnant ρ_V – Electroweak kick

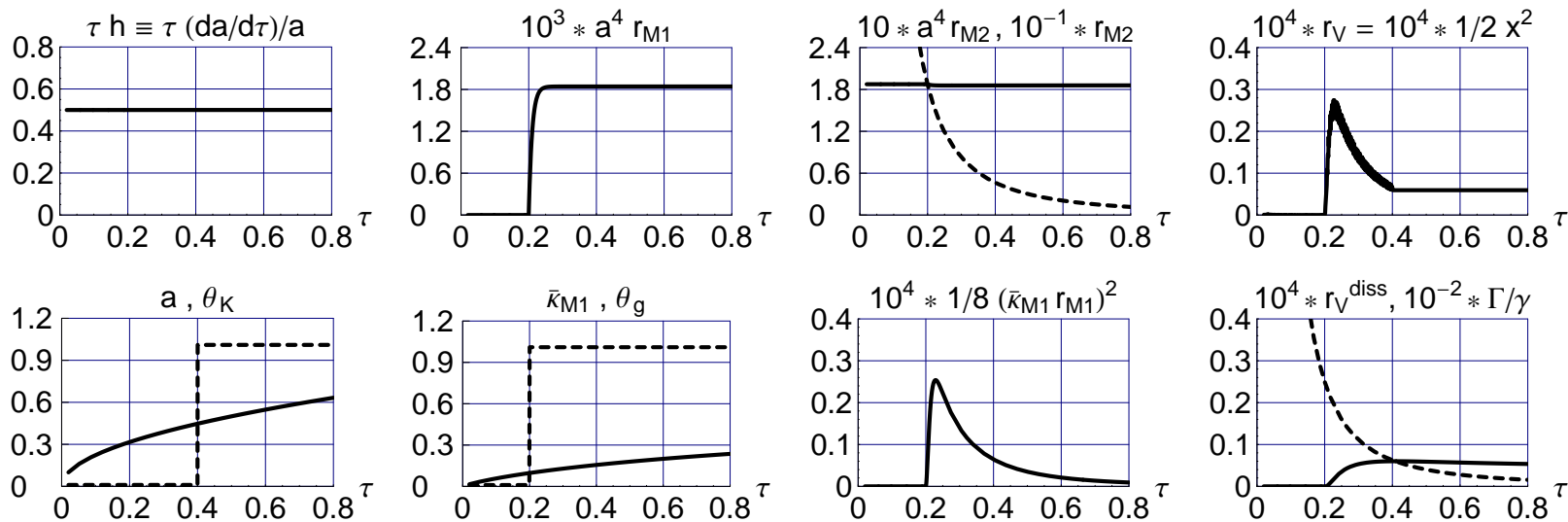


Fig. 2: Numerical results for few new particles (see [21] for details).

For this case (“ $N_{\text{eff}, 1} = 1$ ”): $r_{V\infty} \Big|^{(\text{case-1})} \leq \max [r_V(\tau)]^{(\text{case-1})} \sim 10^{-5}$.

From (11), using $\Lambda^{\text{exp}} = (2.25 \text{ meV})^4$, this gives: $M \Big|^{(\text{case-1})} \gtrsim 10 \text{ TeV}$.

4.2 Remnant ρ_V – Electroweak kick

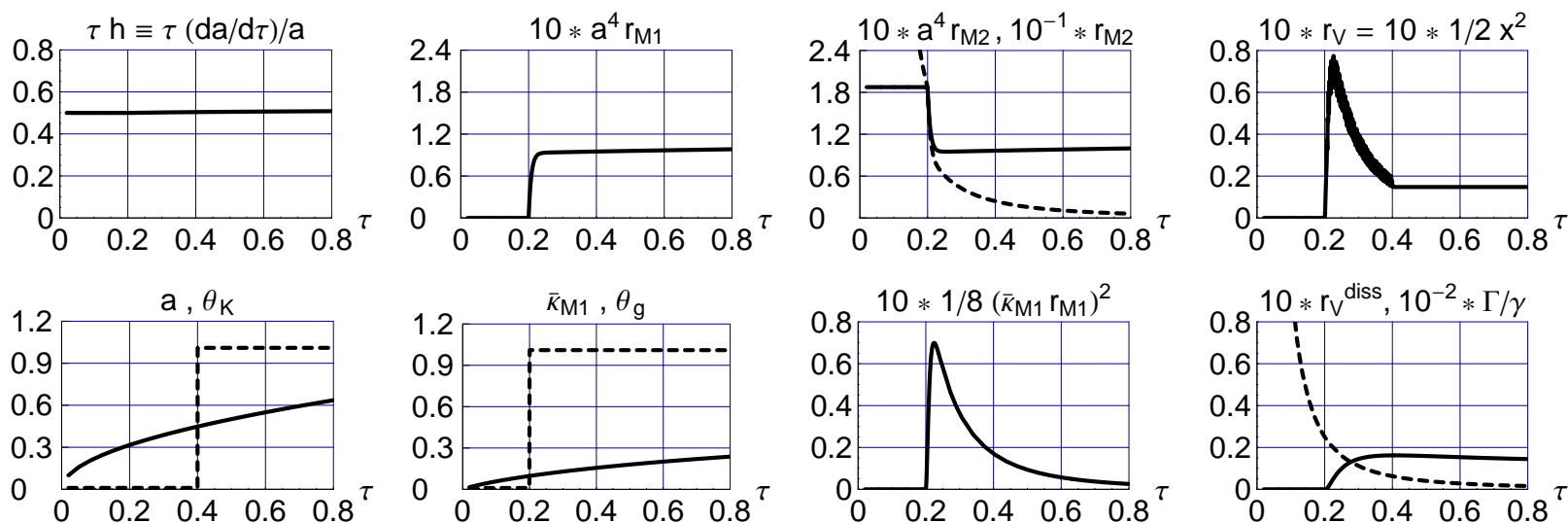


Fig. 3: Numerical results for many new particles (see [21] for details).

For this case (“ $N_{\text{eff},1} = 10^2$ ”): $r_{V\infty} \Big|^{(\text{case-2})} \leq \max [r_V(\tau)]^{(\text{case-2})} \sim 10^{-1}$.

From (11), using $\Lambda^{\text{exp}} = (2.25 \text{ meV})^4$, this gives $M \Big|^{(\text{case-2})} \gtrsim 3 \text{ TeV}$.

5. Conclusions

- CCP1:** Self-adjustment of a special type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0 = \text{const.}$
- CCP2:** Within the q -theory framework, a finite remnant value of $\rho_V(t)$ may result from:
- a “kick” by massive particles with $M \gtrsim 3 \text{ TeV}$
 - \Rightarrow new TeV-scale physics beyond the SM?
 - \Rightarrow surprises at the LHC or a next-generation proton-proton collider?

6. Update from February 6, 2018

6.1 q -theory with quantum dissipative effects

From the abstract of Ref. [23]:

The q -theory approach to the cosmological constant problem is reconsidered. The new observation is that the effective classical q -theory gets modified due to the backreaction of quantum-mechanical particle production by spacetime curvature. Furthermore, a Planck-scale cosmological constant is added to the potential term of the action density, in order to represent the effects from zero-point energies and phase transitions. The resulting dynamical equations of a spatially-flat Friedmann–Robertson–Walker universe are then found to give a steady approach to the Minkowski vacuum, with attractor behavior for a finite domain of initial boundary conditions on the fields. The approach to the Minkowski vacuum is slow and gives rise to an inflation-type increase of the particle horizon.

Perfect cancellation of positive and negative Planck-scale Λ : see Figs. 4–5.

6. Update from February 6, 2018

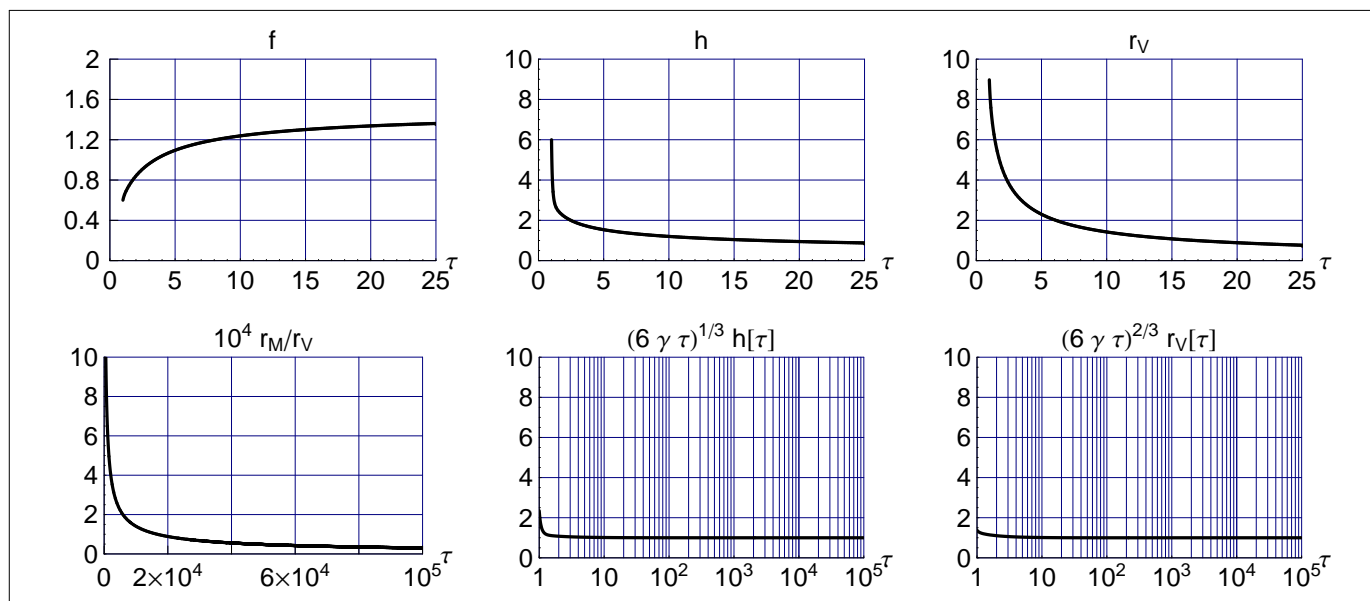


Fig. 4: Numerical solution of the ODEs for positive Planck-scale cosmological constant Λ ; from Ref. [23].

6. Update from February 6, 2018

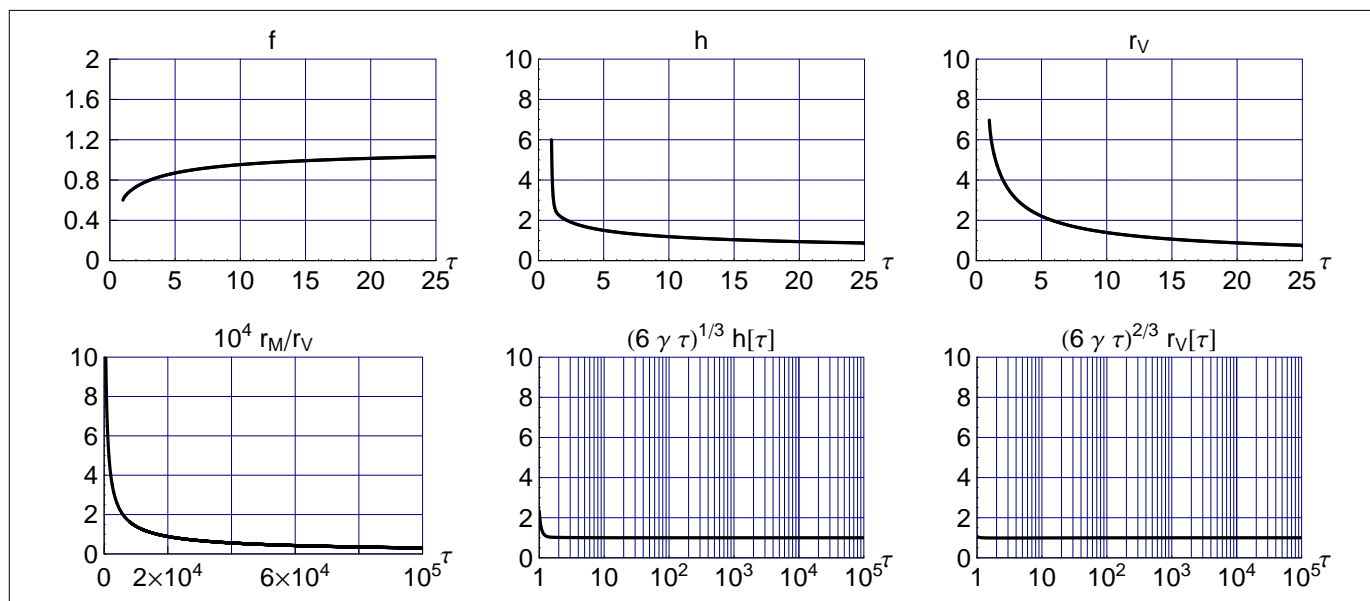


Fig. 5: Numerical solution of the ODEs for negative Planck-scale cosmological constant Λ ; from Ref. [23].

6. Update from February 6, 2018

6.2 Dark matter from dark energy in q -theory

From the abstract of Ref. [24]:

A constant (spacetime-independent) q -field may play a crucial role for the cancellation of Planck-scale contributions to the gravitating vacuum energy density. We now show that a small spacetime-dependent perturbation of the equilibrium q -field behaves gravitationally as a pressureless perfect fluid. This makes the fluctuating part of the q -field a candidate for the inferred dark-matter component of the present universe. For a Planck-scale oscillation frequency of the q -field perturbation, the implication would be that direct searches for dark-matter particles would remain unsuccessful in the foreseeable future.

6. Update from February 6, 2018

From the abstract of Ref. [25]:

We consider the rapidly-oscillating part of a q -field in a cosmological context and find that its energy density behaves in the same way as a cold-dark-matter component, namely proportional to the inverse cube of the cosmic scale factor.

Further work on the classical stability of this higher-derivative q -theory in Ref. [26].

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