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A new approach to the cosmological constant problem

(with updates from February 6, 2018 and January 29, 2019)

Frans R. Klinkhamer

Institute for Theoretical Physics,

Karlsruhe Institute of Technology (KIT), Germany

Email: frans.klinkhamer@kit.edu

1. Introduction

The main **Cosmological Constant Problem** (CCP1) can be phrased as follows (Pauli, 1933; Bohr, 1948; Veltman, 1974; see [1, 2] for two reviews):

why do the quantum fields in the vacuum not produce naturally a large cosmological constant Λ in the Einstein field equations?

The magnitude of the problem is enormous:

$$|\mathbf{\Lambda}^{\text{theory}}|/|\mathbf{\Lambda}^{\text{experiment}}| \geq \mathbf{10^{54}}\,,$$

where the large number on the RHS will be explained on the next slide.

From now on, $\hbar = 1 = c$.

1. Introduction

With the ATLAS and CMS results [3, 4] in support of the Higgs mechanism, it is clear that the EWSM in the laboratory involves a vacuum energy density of order

$$\left|\epsilon_V^{\text{(EWSM)}}\right| \sim \left(100\ \text{GeV}\right)^4 \sim 10^{44}\ \text{eV}^4\,.$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_V^{\text{(EWSM)}} = \epsilon_V^{\text{(EWSM)}}(T)\,.$$

How can the Universe then end up with a vacuum energy density

$$\left|\Lambda^{\text{(obs)}}\right| < 10^{-28}\ \text{g cm}^{-3} \sim 10^{-10}\ \text{eV}^4\,?$$

Here, there are 54 orders of magnitude to explain:

$$\left|\Lambda^{\text{(obs)}}/\epsilon_V^{\text{(EWSM)}}\right|$$
$$\leq 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,001\,.$$

1. Introduction

In short, the main cosmological constant problem is

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{Planck}})^4$?

Still more CCPs after the discovery of the “accelerating Universe”:

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{present}} \sim +10^{-11}\ \text{eV}^4$?

Hundreds of papers have been published on CCP2. But, most likely:

CCP1 needs to be solved first, before CCP2 can even be addressed.

1. Introduction

Here, a discussion of one particular approach to CCP1 by Volovik and the speaker, which goes under the name of **q–theory** [5, 6, 7] (a brief review appears in [8]).

It is instructive to consider four explicit realizations of q–theory:

1. with a three-form gauge field [9, 10, 11, 12],
2. with a massless vector-field [13, 14].
3. with a brane (update from January 29, 2019).
4. with tetrads (update from January 29, 2019).

1. Introduction

The vector-field realization, in particular, is found to give Minkowski spacetime as an **attractor** of the field equations.

But a new problem arises: the danger of ruining the standard Newtonian physics of small self-gravitating systems [15]. This disaster can, however, be avoided by a special model with two vector fields [16, 17].

In this talk, we first focus on the 3-form-gauge-field realization of q–theory, which keeps the Newtonian physics in tact, and then briefly turn to CCP2.

1. Introduction

OUTLINE:

1. Introduction
2. Basics of q–theory
3. Two realizations
4. Remnant ρ_V: Electroweak-kick mechanism
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2. Basics of q–theory

Crucial insight [5]: there is vacuum energy and vacuum energy.

More specifically and introducing an appropriate notation:

the vacuum energy density $\boxed{\epsilon}$ appearing in the action

need not be the same as

the vacuum energy density $\boxed{\rho_V}$ in the Einstein field equations.

How can this happen concretely . . .

2. Basics of q–theory

Consider the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Study, then, the **macroscopic** equations of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy:

- Take the mass density ρ of a liquid, for example, liquid Argon.
- This ρ describes microscopic quantities ($\rho = m_{\text{Ar}}\, n_{\text{Ar}}$ with number density n_{Ar} and mass m_{Ar} of the atoms).
- Still, ρ obeys the macroscopic equations of hydrodynamics, because of particle-number conservation and mass conservation.

2. Basics of q–theory

However, is the quantum vacuum similar to a “normal” liquid?

No, the quantum vacuum behaves like a liquid but not like a “normal” liquid.

2. Basics of q–theory

In fact, the quantum vacuum is known to be **Lorentz invariant** (cf. experimental limits at the 10^{-15} level in the photon sector [18]).

The Lorentz invariance of the vacuum rules out the standard type of charge density, which arises from the time component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material liquids.

2. Basics of q–theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{nonconstant}}(q)\,, \tag{1}$$

including a possible constant term Λ_{bare} from the zero-point energies of the fields of the Standard Model (SM).

From ① thermodynamics and ② Lorentz invariance follows that [5]

$$P_V \overset{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \overset{②}{=} -\rho_V\,, \tag{2}$$

where the first equality corresponds to an integrated form of the Gibbs–Duhem equation for chemical potential $\mu \equiv d\epsilon/dq$.

Recall GD eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

2. Basics of q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{Planck}})^4$, but they cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 = \text{const} \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\,\frac{d\,\epsilon(q)}{d\,q}\right]_{q=q_0} = 0\,, \qquad (3)$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle . . .

But, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q (see Sec. 3).

3.0 Two realizations for two questions

Explicit realizations of q-theory provide answers to the following two obvious questions:

Q1: How does the adjustment-type solution (3) of CCP1 circumvent Weinberg's no–go "theorem" [2]?

Answer: q is a non-fundamental scalar field (see Sec. 3.1).

Q2: How did the Universe get the right value q_0?

One possible answer is that q_0 (or the corresponding chemical potential μ_0) is fixed globally as an integration constant, being conserved throughout the history of the Universe [6].

Another possible answer uses a generalization of q–theory, for which the 'correct' value q_0 arises dynamically (Sec. 3.2).

Update: q–theory with quantum dissipative effects included (Sec. 6.1).

3.1 Four-form realization

Vacuum variable q may arise from a 3–form gauge field A [9, 10].

Start from the effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = \int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K_N\,R[g] + \Lambda_{\text{SM}} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \qquad (4)$$

with gravitational coupling constant $K_N \equiv 1/(16\pi G_N)$ and $\hbar = c = 1$.

Change this theory by the introduction of one field, A, to get [6, 7]:

$$\widetilde{S}^{\text{eff}}[A,g,\psi] \;=\; \int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K(q)\,R[g] + \epsilon(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big), \qquad (5a)$$

$$q \;\equiv\; \boxed{-\tfrac{1}{24}\,\epsilon^{\alpha\beta\gamma\delta}\,\nabla_\alpha A_{\beta\gamma\delta}\,/\,\sqrt{-g}}\,, \qquad (5b)$$

where q arises from the four-form field strength $F = d\,A$.

Variational principle gives generalized Einstein and Maxwell equations:

3.1 Four-form realization

$$2K(q)\left(R_{\alpha\beta}-g_{\alpha\beta}\,R/2\right) = -2\left(\nabla_\alpha\nabla_\beta-g_{\alpha\beta}\,\Box\right)K(q) +\rho_V(q)\,g_{\alpha\beta}-T^M_{\alpha\beta}\,, \tag{6a}$$

$$\frac{d\rho_V(q)}{dq}+R\,\frac{dK(q)}{dq} = 0\,, \tag{6b}$$

with a vacuum energy density,

$$\rho_V=\epsilon-q\left(\frac{d\epsilon}{dq}+R\,\frac{dK}{dq}\right)=\epsilon-q\,\mu\,, \tag{7}$$

for integration constant (chemical potential) μ. Eq. (7) is precisely of the Gibbs–Duhem form (2) in Minkowski spacetime ($R=0$). Technically, the extra $g_{\alpha\beta}$ term on the RHS of (6a) appears because $q=q(A,\,\underline{\underline{g}})$.

Hence, an answer to Q1: (5b) shows that q is a non-fundamental scalar field, which invalidates Weinberg's argument (see [7] for details).

3.2 Vector-field realization

Vacuum variable q comes from an aether-type velocity field u_β [13, 14], setting $E_{\text{UV}} = E_{\text{Planck}}$. For a flat RW metric with cosmic time t, there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \quad\to\quad q_0\, t\,, \quad u_b(t) = 0\,, \quad H(t) \to 1/t\,, \tag{8a}$$

$$u_\alpha{}^\beta \equiv \nabla_\alpha\, u^\beta \quad\to\quad \boxed{q_0\ \delta_\alpha{}^\beta}\,. \tag{8b}$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t\, E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. With an action quadratic in the variable $u_\alpha{}^\beta$, the field equations are [13]:

$$\ddot{v} + 3\,h\,\dot{v} - 3\,h^2\,v \quad=\quad 0\,, \tag{9a}$$

$$2\,\lambda - (\dot{v})^2 - 3\,(h\,v)^2 \quad=\quad 6\,h^2\,, \tag{9b}$$

with the overdot standing for differentiation with respect to τ. Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\widehat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\widehat{q}_0 = \sqrt{\lambda/2}$.

3.2 Vector-field realization

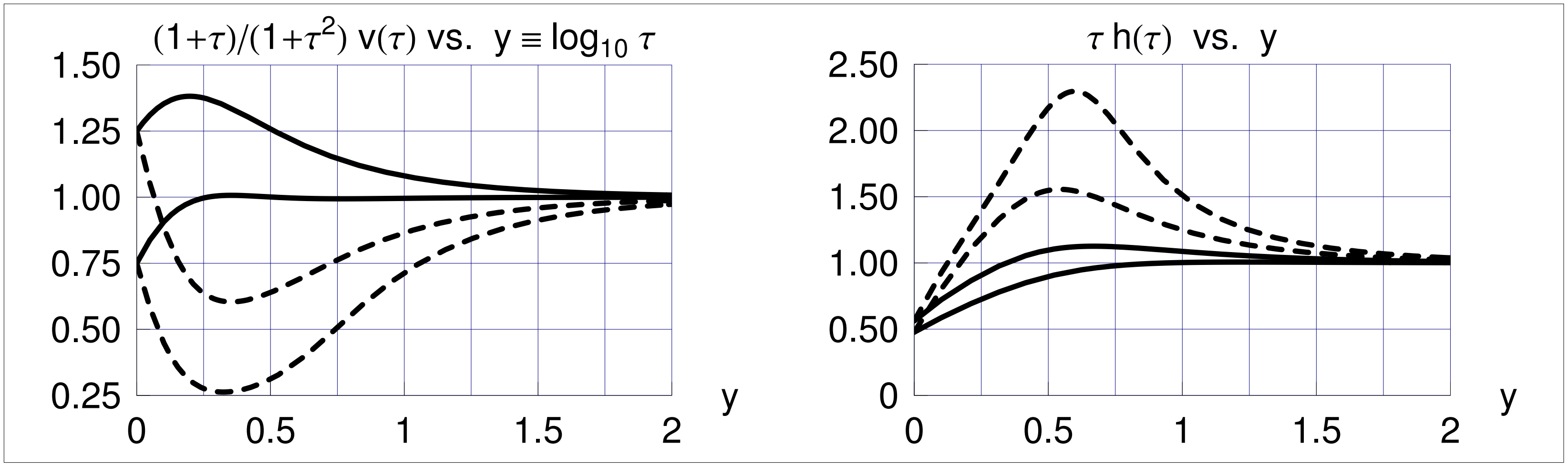

Fig. 1: Four numerical solutions of ODEs (9ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25$.

$\Rightarrow$ Minkowski value $\widehat{q}_0 = \sqrt{\lambda/2} = 1$ arises dynamically [see left panel].

$\Rightarrow$ Minkowski spacetime is an attractor in this aether-type theory [7].

Hence, an answer to Q2. But, as mentioned above, there is serious collateral damage [15] which needs to be avoided [16, 17].

3.3 Recap

To summarize, the q–theory approach to the main Cosmological Constant Problem (CCP1) provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics contains such a q–type variable.

GENERAL REMARK: it is clear that the SM harbors huge vacuum energy densities, which somehow need to be cancelled by new d.o.f., possibly related to the fundamental theory of spacetime and gravity.

BAD NEWS: nothing is known for sure about these fundamental d.o.f.

GOOD NEWS: even though the detailed (high-energy) microphysics is unknown, it may be possible to describe the macroscopic (low-energy) effects along the lines of q–theory, just as for the hydrodynamics of water.

4.1 Remnant ρ_V

Now, briefly the remaining problems (or puzzles, rather):

CCP2a – why $\Lambda_{\text{eff}} \neq 0$?

CCP2b – why $\Lambda_{\text{eff}} \sim \rho_{\text{matter}}\big|_{\text{now}} \sim 10^{-29}$ g cm^{-3} $\sim 10^{-11}$ eV^4 ?

Last one also goes under the name of ‘cosmic coincidence puzzle’ (ccp).

In the framework of q–theory, we have given speculative discussions of the remnant vacuum energy density from the physics of QCD [19] or massive neutrinos [8]. But, here, our speculations will focus on the electroweak (TeV) scale.

4.2 Remnant ρ_V – Electroweak kick

Reconsider the four-form realization of q, taken to be operative at a UV (Planckian) energy scale.

In the very early Universe, the vacuum energy density $\rho_V(t)$ rapidly drops to zero and stays there, but small effects may occur at cosmic temperatures T of the order of the TeV scale ...

Simple picture:

Take a glass of water, hold it steady, and then shake it $\Rightarrow$ water responds.

If vacuum energy density is really like a liquid, then it can be ‘shaken.’ Here, the ‘shaking’ is done by massive particles.

4.2 Remnant ρ_V – Electroweak kick

Key steps of the frozen-electroweak-kick mechanism [20, 21]:

- Presence of massive particles with electroweak interactions [average mass $M \sim$ TeV] changes the Hubble expansion rate $H(t)$ of the Universe compared to the radiation-dominated case.
- Change of the expansion rate kicks $\rho_V(t)$ away from zero.
- Quantum-dissipative effects operating at cosmic time $t_{\text{kick}} \equiv E_P/M^2$ may result in finite remnant value of ρ_V.
[Here, E_P is the reduced Planck energy $\sqrt{1/(8\pi G_N)} \approx 2.44 \times 10^{18}$ GeV.]
- Phenomenological description of this process with a simple field-theoretic model.

4.2 Remnant ρ_V – Electroweak kick

Core formula for the remnant vacuum energy density in a flat Friedmann–Robertson–Walker (FRW) universe:

$$\Lambda \equiv \lim_{t\to\infty} \rho_V(t) = r_{V\infty}\, M^8 / (E_P)^4 \,. \tag{10}$$

Inverting this equation gives

$$\begin{aligned} M &= (r_{V\infty})^{-1/8}\, \Lambda^{1/8}\, (E_P)^{1/2} \\ &\approx 5.56 \text{ TeV} \left(\frac{10^{-3}}{r_{V\infty}}\right)^{1/8} \left(\frac{\Lambda^{1/4}}{2.25 \text{ meV}}\right)^{1/2} . \end{aligned} \tag{11}$$

The outstanding task is to **calculate** the "efficiency factor" $r_{V\infty}$ for producing a remnant vacuum energy density Λ given the energy scales involved, M and E_P. BTW, the parametric behavior of (10) was already discussed by Arkani-Hamed, Hall, Kolda, and Murayama [22].

4.2 Remnant ρ_V – Electroweak kick

Main inputs for a simple model calculation [21]:

- Brans–Dicke-type term in the action density,

$$\mathcal{L}_{\text{grav}} = K[q,\Phi]\, R[g]\,, \qquad (12)$$

  where Φ stands for one or more of the matter fields and we assume the following simplified behavior:

$$K[q,t] = q(t)/2 + \theta(t-t_K)\,\big[q_0/2 - q(t)/2\big]\,. \qquad (13)$$

- Two types of matter: type-1 for ultraheavy particles of mass M; type 2 for massless particles with $N_{\text{eff},\,2} = 10^2$ ($\approx$ SM).
- For the K-freezing-model calculation, set $t_K = O(1)$.
  For the dissipation-model calculation, set $t_K = \infty$.
- Really unsolved question: what **physics** freezes $\rho_V(t)$???

4.2 Remnant ρ_V – Electroweak kick

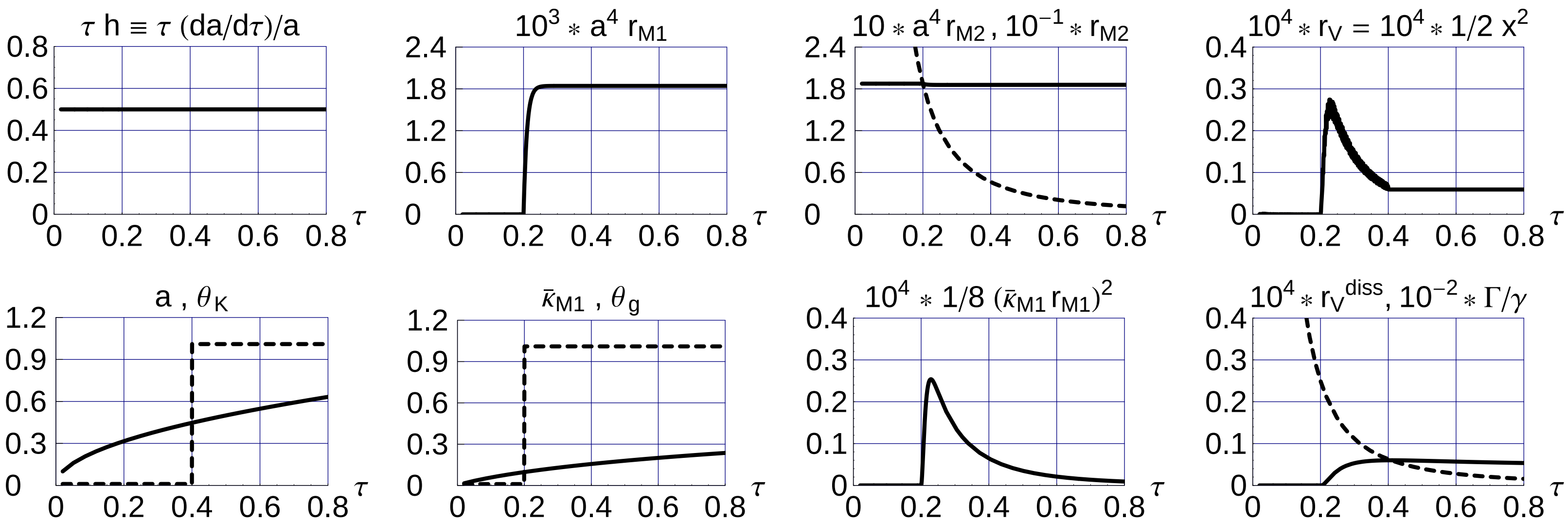

Fig. 2: Numerical results for few new particles (see [21] for details).

For this case ("$N_{\text{eff},\,1} = 1$"): $r_{V\infty}\Big|^{\text{(case-1)}} \leq \max\,\big[r_V(\tau)\big]^{\text{(case-1)}} \sim 10^{-5}$.

From (11), using $\Lambda^{\text{exp}} = (2.25\ \text{meV})^4$, this gives: $M\Big|^{\text{(case-1)}} \gtrsim 10\ \text{TeV}$.

4.2 Remnant ρ_V – Electroweak kick

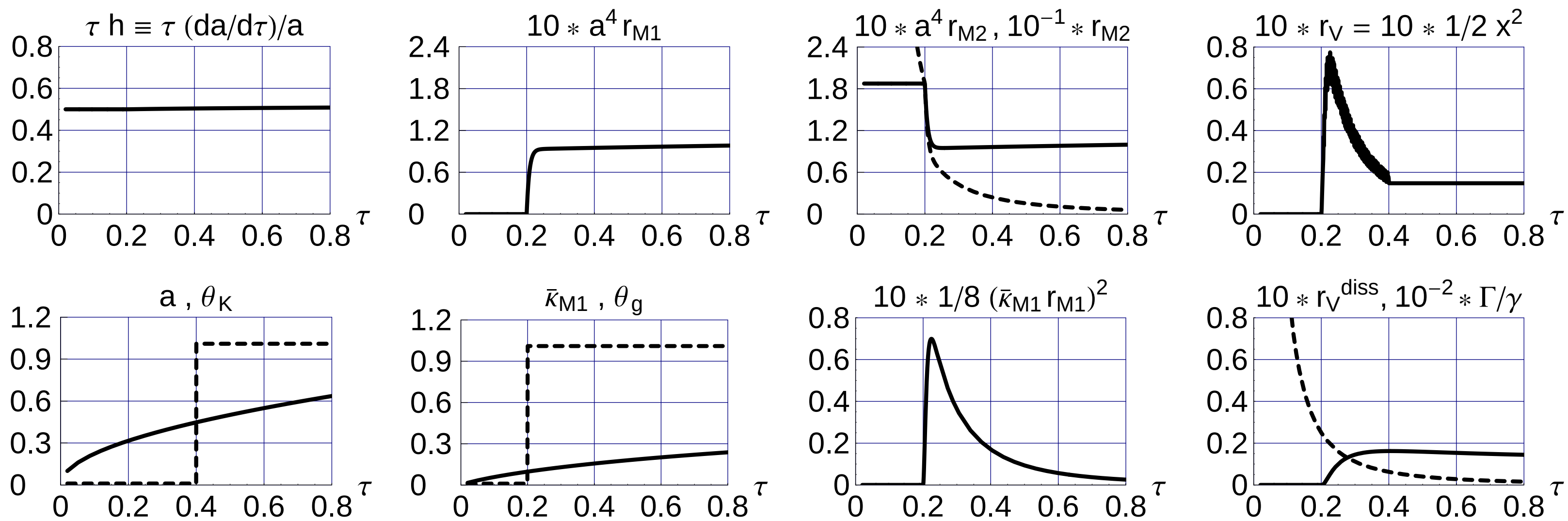

Fig. 3: Numerical results for many new particles (see [21] for details).

For this case ("$N_{\text{eff},\,1} = 10^2$"): $r_{V\infty}\Big|^{\text{(case-2)}} \leq \max\,\big[r_V(\tau)\big]^{\text{(case-2)}} \sim 10^{-1}$.

From (11), using $\Lambda^{\text{exp}} = (2.25\ \text{meV})^4$, this gives $M\Big|^{\text{(case-2)}} \gtrsim 3$ TeV.

5. Conclusions

CCP1: Self-adjustment of a special type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0 =$ const.

CCP2: Within the $\boldsymbol{q}$–theory framework, a finite remnant value of $\rho_V(t)$ may result from:

a “kick” by massive particles with $M \gtrsim 3$ TeV

$\Rightarrow$ new TeV–scale physics beyond the SM?

$\Rightarrow$ surprises at the LHC or a next-generation proton-proton collider?

6. Update from February 6, 2018

6.1 q-theory with quantum dissipative effects

From the abstract of Ref. [23]:

The q-theory approach to the cosmological constant problem is reconsidered. The new observation is that the effective classical q-theory gets modified due to the backreaction of quantum-mechanical particle production by spacetime curvature. Furthermore, a Planck-scale cosmological constant is added to the potential term of the action density, in order to represent the effects from zero-point energies and phase transitions. The resulting dynamical equations of a spatially-flat Friedmann–Robertson–Walker universe are then found to give a steady approach to the Minkowski vacuum, with attractor behavior for a finite domain of initial boundary conditions on the fields. The approach to the Minkowski vacuum is slow and gives rise to an inflation-type increase of the particle horizon.

Cancellation of positive and negative Planck-scale Λ: see Figs. 4–5.

6. Update from February 6, 2018

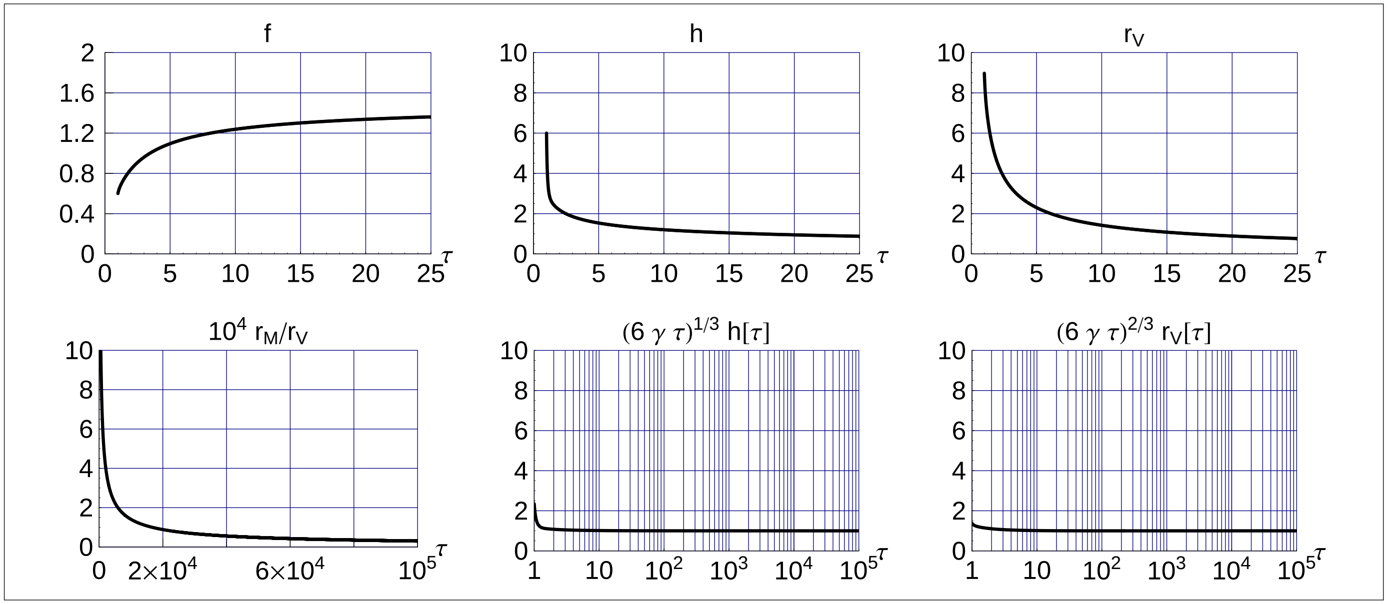

Fig. 4: Numerical solution of the ODEs for positive Planck-scale cosmological constant Λ; from Ref. [23].

6. Update from February 6, 2018

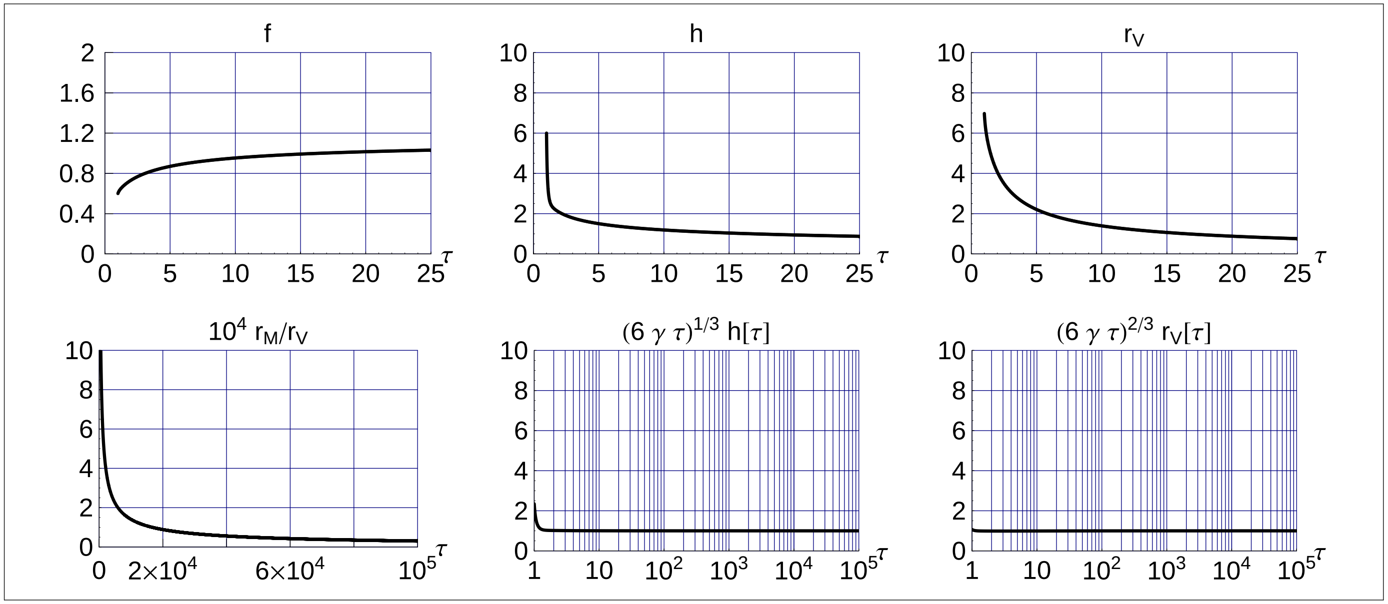

Fig. 5: Numerical solution of the ODEs for negative Planck-scale cosmological constant Λ; from Ref. [23].

6. Update from February 6, 2018

6.2 Dark matter from dark energy in q-theory

From the abstract of Ref. [24]:

A constant (spacetime-independent) q-field may play a crucial role for the cancellation of Planck-scale contributions to the gravitating vacuum energy density. We now show that a small spacetime-dependent perturbation of the equilibrium q-field behaves gravitationally as a pressureless perfect fluid. This makes the fluctuating part of the q-field a candidate for the inferred dark-matter component of the present universe. For a Planck-scale oscillation frequency of the q-field perturbation, the implication would be that direct searches for dark-matter particles would remain unsuccessful in the foreseeable future.

6. Update from February 6, 2018

From the abstract of Ref. [25]:

We consider the rapidly-oscillating part of a q-field in a cosmological context and find that its energy density behaves in the same way as a cold-dark-matter component, namely proportional to the inverse cube of the cosmic scale factor.

Further work on the classical stability of this higher-derivative q-theory in Ref. [26].

7. Update from January 29, 2019

Brane realization [27]:

We discuss the cosmological constant problem using the properties of a freely-suspended two-dimensional condensed-matter film, i.e., an explicit realization of a 2D brane. The large contributions of vacuum fluctuations to the surface tension of this film are cancelled in equilibrium by the thermodynamic potential arising from the conservation law for particle number. This 2D brane can be generalized to a 4D brane with gravity. For the 4D brane, the analogue of the 2D surface tension is the 4D cosmological constant, which is also nullified in full equilibrium.

Tetrad realization [28]:

We assume that the vacuum can be viewed as a super-elastic crystalline medium and obtain a new realization of the q-variable, involving both the tetrad of standard gravity and the elasticity tetrad of the hypothetical vacuum crystal.

7. Update from January 29, 2019

Vacuum energy decay from a q-bubble [29]:

We consider a finite-size spherical bubble with a nonequilibrium value of the q-field, where the bubble is immersed in an infinite vacuum with the constant equilibrium value q_0 for the q-field. Numerical results are presented for the time evolution of such a q-bubble with gravity turned off ($G_N = 0$) and with gravity turned on ($G_N > 0$ and ratio of energy scales $E_{q-\text{field}}/E_{\text{Planck}} \sim 1/10$). For small enough bubbles and energy scale $E_{q-\text{field}}$ sufficiently below the gravitational energy scale E_{Planck}, the vacuum energy of the q-bubble is found to disperse completely. For large enough bubbles and nonzero G_N, the vacuum energy of the q-bubble disperses only partially and gravitational collapse occurs near the bubble center.

8. References

[1] L. Abbott, Sci. Am. 258, 106 (1988).
[2] S. Weinberg, RMP 61, 1 (1989); arXiv:astro-ph/9610044.
[3] G. Aad *et al.* [ATLAS Collaboration], PLB 716, 1 (2012), arXiv:1207.7214.
[4] S. Chatrchyan *et al.* [CMS Collaboration], PLB 716, 30 (2012), arXiv:1207.7235.
[5] FRK & G.E. Volovik, PRD 77, 085015 (2008), arXiv:0711.3170.
[6] FRK & G.E. Volovik, PRD 78, 063528 (2008), arXiv:0806.2805.
[7] FRK & G.E. Volovik, JETPL 91, 259 (2010), arXiv:0907.4887.
[8] FRK & G.E. Volovik, J. Phys. Conf. Ser. 314, 012004 (2011), arXiv:1102.3152.
[9] M.J. Duff & P. van Nieuwenhuizen, PLB 94, 179 (1980).
[10] A. Aurilia, H. Nicolai, & P.K. Townsend, NPB 176, 509 (1980).
[11] S.W. Hawking, PLB 134, 403 (1984).
[12] M. Henneaux & C. Teitelboim, PLB 143, 415 (1984).
[13] A.D. Dolgov, PRD 55, 5881 (1997), arXiv:astro-ph/9608175.
[14] T. Jacobson, PoS QG-PH, 020 (2007), arXiv:0801.1547.
[15] V.A. Rubakov & P.G. Tinyakov, PRD 61, 087503 (2000), arXiv:hep-ph/9906239.
[16] V. Emelyanov & FRK, PRD 85, 063522 (2012), arXiv:1107.0961.
[17] V. Emelyanov & FRK, PRD 85, 103508 (2012), arXiv:1109.4915.
[18] FRK & M. Schreck, PRD 78, 085026 (2008), arXiv:0809.3217.
[19] FRK & G.E. Volovik, PRD79, 063527 (2009), arXiv:0811.4347.
[20] FRK & G.E. Volovik, PRD 80, 083001 (2009), arXiv:0905.1919.
[21] FRK, MPLA 30, 1550149 (2015), arXiv:1503.03858.
[22] N. Arkani-Hamed *et al.*, PRL 85, 4434 (2000), astro-ph/0005111.

8. References (for the updates)

[23] FRK & G.E. Volovik, MPLA 31, 1650160 (2016), arXiv:1601.00601.
[24] FRK & G.E. Volovik, JETPL105, 74 (2017), arXiv:1612.02326.
[25] FRK & G.E. Volovik, arXiv:1612.04235.
[26] FRK & T. Mistele, IJMPA 32, 1750090 (2017), arXiv:1704.05436.
[27] FRK & G.E. Volovik, JETPL 103, 627 (2016), arXiv:1604.06060.
[28] FRK & G.E. Volovik, to appear in JETPL, arXiv:1812.07046.
[29] FRK, O.P. Santillan, G.E. Volovik & A. Zhou, arXiv:1901.05938.