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A new approach to the cosmological constant problem*

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* the adjective “new” refers to the time scale of the CCP and GR, 100 years.

1. Introduction

The main **Cosmological Constant Problem** (CCP1) can be phrased as follows (Pauli, 1933; Bohr, 1948; Veltman, 1974; see [1, 2] for two reviews):

why do the quantum fields in the vacuum not produce naturally a large cosmological constant Λ in the Einstein field equations?

The magnitude of the problem is enormous:

$$|\Lambda^{\text{theory}}|/|\Lambda^{\text{experiment}}| \geq 10^{54}\,,$$

where the large number on the RHS will be explained on the next slide.

From now on, $\hbar = 1 = c$.

1. Introduction

With the ATLAS and CMS results [3, 4] in support of the Higgs mechanism, it is clear that the EWSM in the laboratory involves a vacuum energy density of order

$$\left|\epsilon_V^{\text{(EWSM)}}\right| \sim \big(100\ \text{GeV}\big)^4 \sim 10^{44}\ \text{eV}^4\,.$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_V^{\text{(EWSM)}} = \epsilon_V^{\text{(EWSM)}}(T)\,.$$

How can the Universe then end up with a vacuum energy density

$$\left|\Lambda^{\text{(obs)}}\right| < 10^{-28}\ \text{g cm}^{-3} \sim 10^{-10}\ \text{eV}^4\ ?$$

Here, there are 54 orders of magnitude to explain:

$$\left|\Lambda^{\text{(obs)}}/\epsilon_V^{\text{(EWSM)}}\right|$$
$$\leq 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,001\,.$$

1. Introduction

In short, the main cosmological constant problem is

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{Planck}})^4$?

Still more CCPs after the discovery of the "accelerating Universe":

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{present}} \sim +10^{-11}\ \text{eV}^4$?

Hundreds of papers have been published on CCP2. But, most likely:

$\boxed{\text{CCP1 needs to be solved first before CCP2 can even be addressed.}}$

1. Introduction

Here, a discussion of one particular approach to CCP1 by Volovik and the speaker, which goes under the name of **q–theory** [5, 6, 7] (a brief review appears in [8]).

It is instructive to consider two explicit realizations of q–theory:

1. with a three-form gauge field [9, 10, 11, 12],
2. with a massless vector-field [13, 14].

The vector-field realization, in particular, is found to give **Minkowski spacetime** as an **attractor** of the field equations.
(But a new problem arises: the danger of ruining the standard Newtonian physics of small self-gravitating systems [15]. This disaster can, however, be avoided by a special model with two vector fields [16, 17].)

In this talk, we first focus on the 3-form-gauge-field realization of q–theory, which keeps the Newtonian physics in tact, and then briefly turn to CCP2.

1. Introduction

OUTLINE:

1. Introduction
2. Basics of q–theory
3. Two realizations
4. Remnant ρ_V: Electroweak-kick mechanism
5. Conclusion
6. References

2. Basics of q–theory

Crucial insight [5]: $\boxed{\text{there is vacuum energy and vacuum energy.}}$

More specifically and introducing an appropriate notation:

the vacuum energy density $\boxed{\epsilon}$ appearing in the action

need not be the same as

the vacuum energy density $\boxed{\rho_V}$ in the Einstein field equations.

How can this happen concretely . . .

2. Basics of q–theory

Consider the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Study, then, the **macroscopic** equations of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy:

- Take the mass density ρ of a liquid, for example, liquid Argon.
- This ρ describes microscopic quantities ($\rho = m_{\text{Ar}}\, n_{\text{Ar}}$ with number density n_{Ar} and mass m_{Ar} of the atoms).
- Still, ρ obeys the macroscopic equations of hydrodynamics, because of particle-number and mass conservation.

However, is the quantum vacuum just like a normal liquid?

2. Basics of q–theory

No, as the quantum vacuum is known to be **Lorentz invariant** (cf. experimental limits at the 10^{-15} level in the photon sector [18]).

The Lorentz invariance of the vacuum rules out the standard type of charge density, which arises from the time component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material liquids.

2. Basics of q–theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{nonconstant}}(q)\,, \tag{1}$$

including a possible constant term Λ_{bare} from the zero-point energies of the fields of the Standard Model (SM).

From ① thermodynamics and ② Lorentz invariance follows that [5]

$$P_V \overset{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \overset{②}{=} -\rho_V\,, \tag{2}$$

where the first equality corresponds to an integrated form of the Gibbs–Duhem equation for chemical potential $\mu \equiv d\epsilon/dq$.

Recall GD eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

2. Basics of q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{Planck}})^4$, but they cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 = \text{const} \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\,\frac{d\,\epsilon(q)}{d\,q}\right]_{q=q_0} = 0\,, \tag{3}$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle . . .

But, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q (see Sec. 3).

3.0 Two realizations for two questions

Explicit realizations of q-theory provide answers to the following two obvious questions:

Q1: How does the adjustment-type solution (3) of CCP1 circumvent Weinberg's no–go "theorem" [2]?

Answer: q is a non-fundamental scalar field; see Sec. 3.1.

Q2: How did the Universe get the right value q_0?

One possible answer is that q_0 (or the corresponding chemical potential μ_0) is fixed globally as an integration constant, being conserved throughout the history of the Universe [6].

Another possible answer uses a generalization of q–theory, for which the 'correct' value q_0 arises dynamically; see Sec. 3.2.

3.1 Four-form realization

Vacuum variable q may arise from a 3–form gauge field A [9, 10].

Start from the effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = \int_{\mathbb{R}^4} d^4x\, \sqrt{-\det g}\, \Big(K_N\, R[g] + \Lambda_{\text{SM}} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g] \Big), \qquad (4)$$

with gravitational coupling constant $K_N \equiv 1/(16\pi G_N)$ and $\hbar = c = 1$.

Change this theory by the introduction of one field, A, to get [6, 7]:

$$\widetilde{S}^{\text{eff}}[A,g,\psi] \;=\; \int_{\mathbb{R}^4} d^4x\, \sqrt{-\det g}\, \Big(K(q)\, R[g] + \epsilon(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g] \Big), \qquad \text{(5a)}$$

$$q \;\equiv\; \boxed{-\tfrac{1}{24}\, \epsilon^{\alpha\beta\gamma\delta}\, \nabla_\alpha A_{\beta\gamma\delta} \,/\, \sqrt{-g}}\,, \qquad \text{(5b)}$$

where q arises from the four-form field strength $F = d\,A$.

Variational principle gives generalized Einstein and Maxwell equations:

3.1 Four-form realization

$$2K(q)\left(R_{\alpha\beta} - g_{\alpha\beta}\, R/2\right) = -2\left(\nabla_\alpha \nabla_\beta - g_{\alpha\beta}\,\Box\right) K(q) + \rho_V(q)\, g_{\alpha\beta} - T^M_{\alpha\beta}\,, \tag{6a}$$

$$\frac{d\rho_V(q)}{dq} + R\,\frac{dK(q)}{dq} = 0\,, \tag{6b}$$

with a vacuum energy density,

$$\rho_V = \epsilon - q\left(\frac{d\epsilon}{dq} + R\,\frac{dK}{dq}\right) = \epsilon - q\,\mu\,, \tag{7}$$

for integration constant (chemical potential) μ. Eq. (7) is precisely of the Gibbs–Duhem form (2) in Minkowski spacetime ($R = 0$). Technically, the extra $g_{\alpha\beta}$ term on the RHS of (6a) appears because $q = q(A,\, \underline{\underline{g}})$.

Hence, an answer to Q1: (5b) shows that q is a non-fundamental scalar field, which invalidates Weinberg's argument (see [7] for details).

3.2 Vector-field realization

Vacuum variable q comes from an aether-type velocity field u_β [13, 14], setting $E_{\text{UV}} = E_{\text{Planck}}$. For a flat RW metric with cosmic time t, there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \;\to\; q_0\, t\,, \quad u_b(t) = 0\,, \quad H(t) \to 1/t\,, \tag{8a}$$

$$u_\alpha{}^\beta \equiv \nabla_\alpha\, u^\beta \;\to\; \boxed{q_0\; \delta_\alpha{}^\beta}\,. \tag{8b}$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t\, E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. With an action quadratic in the variable $u_\alpha{}^\beta$, the field equations are [13]:

$$\ddot{v} + 3\, h\, \dot{v} - 3\, h^2\, v \;=\; 0\,, \tag{9a}$$

$$2\,\lambda - (\dot{v})^2 - 3\,(h\, v)^2 \;=\; 6\, h^2\,, \tag{9b}$$

with the overdot standing for differentiation with respect to τ. Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\widehat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\widehat{q}_0 = \sqrt{\lambda/2}$.

3.2 Vector-field realization

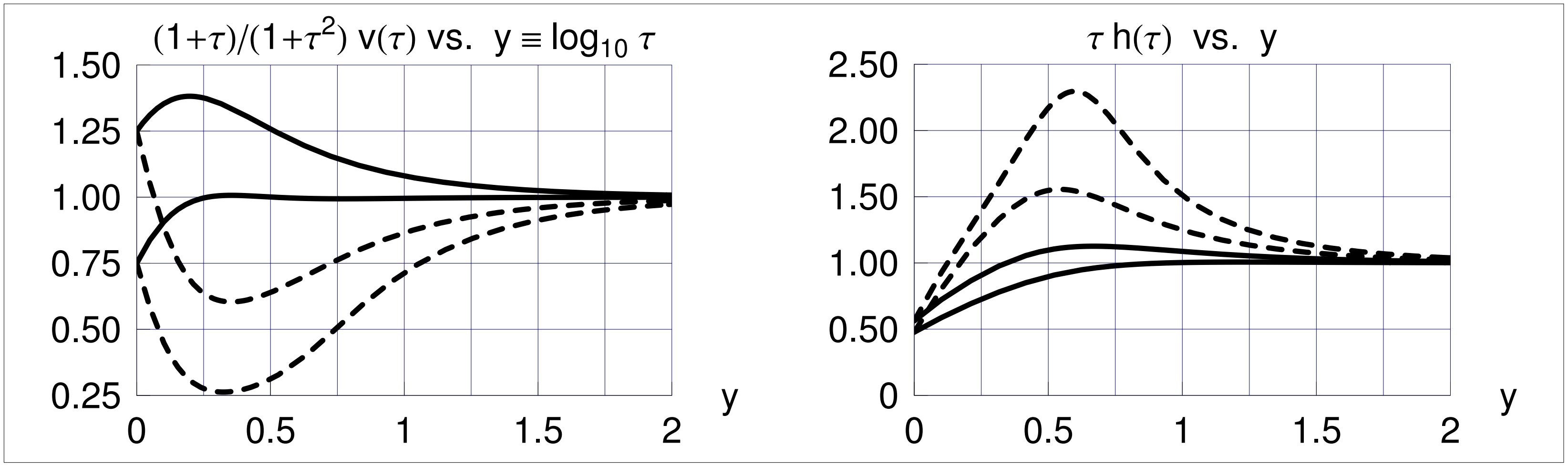

Fig. 1: Four numerical solutions of ODEs (9ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25.$

$\Rightarrow$ Minkowski value $\widehat{q}_0 = \sqrt{\lambda/2} = 1$ arises dynamically [see left panel].

$\Rightarrow$ Minkowski spacetime is an attractor in this aether-type theory [7].

Hence, an answer to Q2. But, as mentioned above, there is serious collateral damage [15] which needs to be avoided [16, 17].

3.3 Recap

To summarize, the q–theory approach to the main Cosmological Constant Problem (CCP1) provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics contains such a q–type variable.

GENERAL REMARK: it is clear that the SM harbors huge vacuum energy densities, which somehow need to be cancelled by new d.o.f., possibly related to the fundamental theory of spacetime and gravity.

BAD NEWS: nothing is known for sure about these fundamental d.o.f.

GOOD NEWS: even though the detailed (high-energy) microphysics is unknown, it may be possible to describe the macroscopic (low-energy) effects along the lines of q–theory, just as for the hydrodynamics of water.

4.1 Remnant ρ_V

Now, briefly the remaining problems (or puzzles, rather):

CCP2a – why $\Lambda_{\text{eff}} \neq 0$?

CCP2b – why $\Lambda_{\text{eff}} \sim \rho_{\text{matter}}\big|_{\text{now}} \sim 10^{-29}$ g cm^{-3} $\sim 10^{-11}$ eV^4 ?

Last one also goes under the name of 'cosmic coincidence puzzle' (ccp).

In the framework of q–theory, we have given speculative discussions of the remnant vacuum energy density from the physics of QCD [19] or massive neutrinos [8]. But, here, our speculations will focus on the electroweak (TeV) scale.

4.2 Remnant ρ_V – Electroweak kick

Reconsider the four-form realization of q, taken to be operative at a UV (Planckian) energy scale.

In the very early Universe, the vacuum energy density $\rho_V(t)$ rapidly drops to zero and stays there, but small effects may occur at cosmic temperatures T of the order of the TeV scale $\ldots$

Simple picture:

Take a glass of water, hold it steady, and then shake it $\Rightarrow$ water responds.

If vacuum energy density is really like a liquid, then it can be ‘shaken.’ Here, the ‘shaking’ is done by massive particles.

4.2 Remnant ρ_V – Electroweak kick

Key steps of the frozen-electroweak-kick mechanism [20, 21]:

- Presence of massive particles with electroweak interactions [average mass $M \sim$ TeV] changes the Hubble expansion rate $H(t)$ of the Universe compared to the radiation-dominated case.
- Change of the expansion rate kicks $\rho_V(t)$ away from zero.
- Quantum-dissipative effects operating at cosmic time $t_{\text{kick}} \equiv E_P/M^2$ may result in finite remnant value of ρ_V. [Here, E_P is the reduced Planck energy $\sqrt{1/(8\pi G_N)} \approx 2.44 \times 10^{18}$ GeV.]
- Phenomenological description of this process with a simple field-theoretic model.

4.2 Remnant ρ_V – Electroweak kick

Core formula for the remnant vacuum energy density in a flat Friedmann–Robertson–Walker (FRW) universe:

$$\Lambda \equiv \lim_{t\to\infty} \rho_V(t) = r_{V\infty}\, M^8 / (E_P)^4 \,. \tag{10}$$

Inverting this equation gives

$$\begin{aligned} M &= (r_{V\infty})^{-1/8}\, \Lambda^{1/8}\, (E_P)^{1/2} \\ &\approx 5.56\ \text{TeV} \left(\frac{10^{-3}}{r_{V\infty}}\right)^{1/8} \left(\frac{\Lambda^{1/4}}{2.25\ \text{meV}}\right)^{1/2} . \end{aligned} \tag{11}$$

The outstanding task is to **calculate** the “efficiency factor” $r_{V\infty}$ for producing a remnant vacuum energy density Λ given the energy scales involved, M and E_P. BTW, the parametric behavior of (10) was already discussed by Arkani-Hamed, Hall, Kolda, and Murayama [22].

4.2 Remnant ρ_V – Electroweak kick

Main inputs for a simple model calculation [21]:

- Brans–Dicke-type term in the action density,

$$\mathcal{L}_{\text{grav}} = K[q,\Phi]\, R[g]\,, \tag{12}$$

where Φ stands for one or more of the matter fields and we assume the following simplified behavior:

$$K[q,t] = q(t)/2 + \theta(t-t_K)\,\big[q_0/2 - q(t)/2\big]\,. \tag{13}$$

- Two types of matter: type-1 for ultraheavy particles of mass M; type 2 for massless particles with $N_{\text{eff},\,2} = 10^2$ ($\approx$ SM).
- For the K-freezing-model calculation, set $t_K = O(1)$. For the dissipation-model calculation, set $t_K = \infty$.
- Really unsolved question: what **physics** freezes $\rho_V(t)$???

4.2 Remnant ρ_V – Electroweak kick

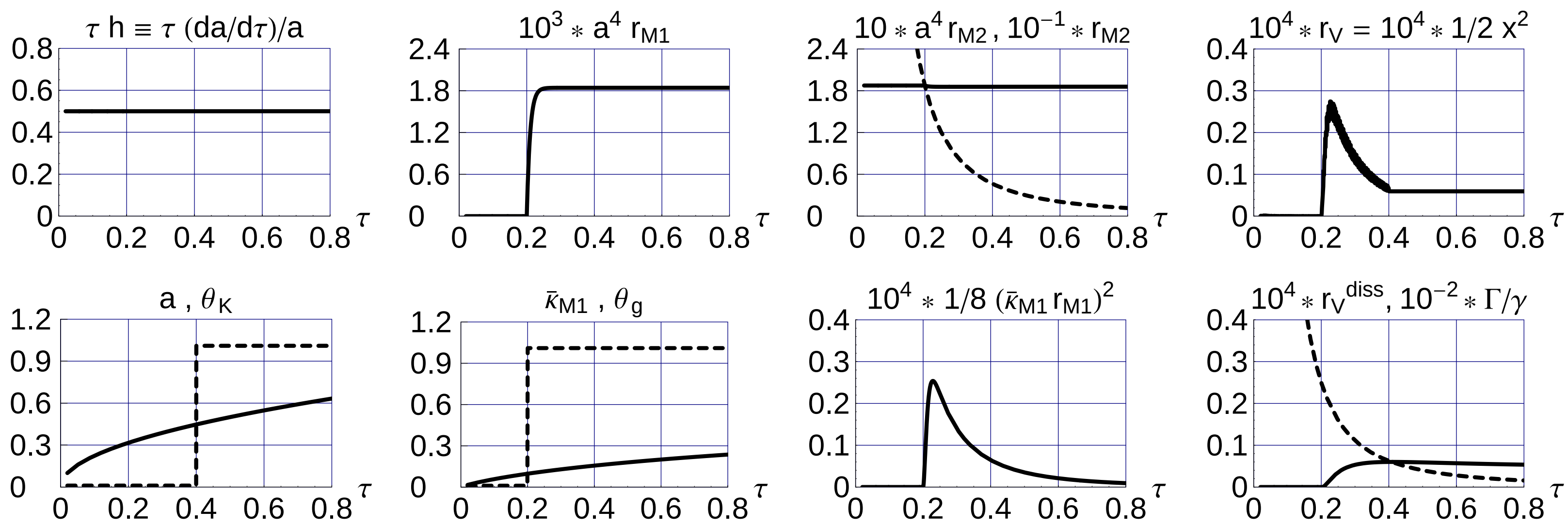

Fig. 2: Numerical results for few new particles (see [21] for details).

For this case ("$N_{\text{eff},\,1} = 1$"): $r_{V\infty}\Big|^{\text{(case-1)}} \leq \max\,\big[r_V(\tau)\big]^{\text{(case-1)}} \sim 10^{-5}$.

From (11), using $\Lambda^{\text{exp}} = (2.25\ \text{meV})^4$, this gives: $M\Big|^{\text{(case-1)}} \gtrsim 10$ TeV.

4.2 Remnant ρ_V – Electroweak kick

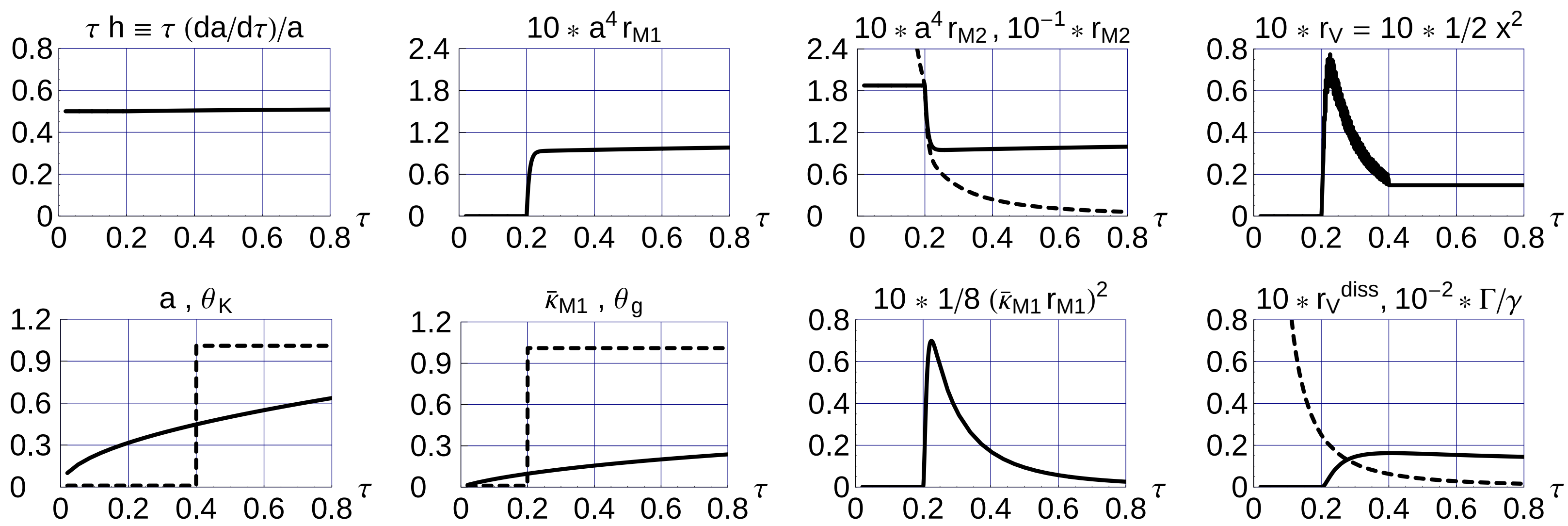

Fig. 3: Numerical results for many new particles (see [21] for details).

For this case ("$N_{\text{eff},\,1} = 10^2$"): $r_{V\infty}\Big|^{\text{(case-2)}} \leq \max\,\left[r_V(\tau)\right]^{\text{(case-2)}} \sim 10^{-1}$.

From (11), using $\Lambda^{\text{exp}} = (2.25\ \text{meV})^4$, this gives $M\Big|^{\text{(case-2)}} \gtrsim 3$ TeV.

5. Conclusions

CCP1: Self-adjustment of a special type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0 =$ const.

CCP2: Within the q–theory framework, a finite remnant value of $\rho_V(t)$ may result from:

a “kick” by massive particles with $M \gtrsim 3$ TeV

$\Rightarrow$ new TeV–scale physics beyond the SM?

$\Rightarrow$ surprises at the LHC or a next-generation proton-proton collider?

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