Taming the Big Bang

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On the MWW2022 webpage:

“What is it about? Why is it important? Perspectives.”
1. Introduction

We are motivated by the following question:

what happened at the birth of the Universe?

Or, at a more technical level:

what replaces the big bang singularity of Friedman cosmology?

How do we deal with infinities in QFT?
Easy: first, regularize and, then, find a better theory...

Let’s try to do something similar. First, we stay within GR and get a “regularized” big bang.

Then, we look for a theory beyond GR (here, a nonperturbative formulation of superstring theory) to get a “new phase” that replaces the regularized big bang and gives the “emergence of spacetime.”
2. Big bang of Friedmann cosmology

**Metric** from the spatially flat Robertson–Walker (RW) Ansatz:

\[
\frac{\mathrm{d}s^2}{(\text{RW})} = g_{\mu\nu}(x) \, \mathrm{d}x^\mu \, \mathrm{d}x^\nu \bigg|_{(\text{RW})} = -\, \mathrm{d}t^2 + a^2(t) \, \delta_{mn} \, \mathrm{d}x^m \, \mathrm{d}x^n , \quad (1)
\]

with \( x^0 = c \, t \) and \( c = 1 \). The spatial indices \( m, n \) run over \( \{1, 2, 3\} \).

**Matter** as a homogeneous perfect fluid with energy density \( \rho_M(t) \) and pressure \( P_M(t) \); define equation-of-state (EOS) parameter \( w_M \equiv P_M/\rho_M \).

**Dynamics** from Einstein’s gravitational field equation in GR.

\[ \Rightarrow \text{three ODEs, the Friedmann equations} [1922, 1924]. \]
2. Big bang of Friedmann cosmology

For relativistic matter with constant EOS parameter $w_M = 1/3$, the Friedmann–Lemaître–Robertson–Walker (FLRW) solution is given by

$$a(t) \bigg|_{\text{FLRW}}^{(w_M=1/3)} = \sqrt{t/t_0}, \quad \text{for } t > 0, \quad (2a)$$

$$\rho_M(t) \bigg|_{\text{FLRW}}^{(w_M=1/3)} = \rho_{M0}/a^4(t) \propto 1/t^2, \quad \text{for } t > 0, \quad (2b)$$

with normalization $a(t_0) = 1$ at $t_0 > 0$.

The FLRW solution displays the **big bang singularity** for $t \to 0^+$,

$$\lim_{t \to 0^+} a(t) = 0, \quad (3)$$

where the curvature and the energy density diverge.

But, at $t = 0$, the theory (GR + SM) is no longer valid and we can ask: what happens **really** at the big bang?
3. Regularized big bang

Let us try to control the divergences by considering a new Ansatz for a “regularized” big bang [1]:

\[ ds^2 \big|^{\text{(RWK)}} \equiv g_{\mu\nu}(x) \, dx^\mu \, dx^\nu \big|^{\text{(RWK)}} = -\frac{t^2}{t^2 + b^2} \, dt^2 + a^2(t) \, \delta_{mn} \, dx^m \, dx^n, \quad (4a) \]

where we set \( x^0 = c \, t \) and \( c = 1 \).

This metric \( g_{\mu\nu}(x) \) is degenerate, with a vanishing determinant at \( t = 0 \). In physical terms, there is a spacetime defect at \( t = 0 \).
2. Regularized big bang

With the standard Einstein equation and a homogeneous perfect fluid, get **modified** spatially flat Friedmann equations:

\[
\left[ 1 + \frac{b^2}{t^2} \right] \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_M, 
\]

\[
\left[ 1 + \frac{b^2}{t^2} \right] \left( \frac{\ddot{a}}{a} + \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 \right) - \frac{b^2}{t^3} \frac{\dot{a}}{a} = -4\pi G P_M, 
\]

\[
\dot{\rho}_M + 3 \frac{\dot{a}}{a} \left[ \rho_M + P_M \right] = 0, 
\]

\[
P_M = P_M(\rho_M), 
\]

where the overdot stands for differentiation with respect to \( t \).

The standard Friedmann equations are recovered by setting the singular \( b^2/t^2 \) terms to zero.
3. Regularized big bang

For constant EOS parameter $w_M = 1/3$, the new solution is

$$a(t) \bigg|_{\text{FLRWK}}^{(w_M=1/3)} = \frac{4}{t^2 + b^2} / \left( t_0^2 + b^2 \right), \quad (6a)$$

$$\rho_M(t) \bigg|_{\text{FLRWK}}^{(w_M=1/3)} = \frac{\rho_{M0} (t_0^2 + b^2)}{t^2 + b^2}, \quad (6b)$$

which is perfectly smooth at $t = 0$ as long as $b \neq 0$. Figure compares with the singular FLRW solution, as shown by the dashed curve.
3. Regularized big bang

Two possible scenarios:

1. **nonsingular bouncing cosmology** from $t = -\infty$ to $t = \infty$
   (valid for $b \gg l_{\text{Planck}}$?) [gravitational waves generated in the pre-bounce epoch keep on propagating into the postbounce epoch];

2. **new phase** at $t = 0$ pair-produces a “universe” for $t > 0$ and an “antiuniverse” for $t < 0$ (valid for $b \sim l_{\text{Planck}}$).
4. Emergent spacetime from IIB matrix model

For an explicit description of such a new phase, we can use the IIB matrix model [Kawai and collaborators, 1997-1999], suggested as a nonperturbative definition of superstring theory (M-theory).

The IIB matrix model has a finite number of $N \times N$ traceless Hermitian matrices: ten bosonic matrices $A^\mu$ and eight fermionic (Majorana–Weyl) matrices $\Psi_\alpha$.

The partition function $Z$ of the IIB matrix model is defined by the following “path” integral:

$$Z = \int dA d\Psi \ e^{-S(A, \Psi)} = \int dA d\Psi \ e^{-S_{\text{bos}}(A) - S_{\text{ferm}}(\Psi, A)},$$

(7)

where the bosonic action $S_{\text{bos}}(A)$ is quartic in $A$ and the fermionic action $S_{\text{ferm}}(\Psi, A)$ is quadratic in $\Psi$ and linear in $A$, i.e., $S_{\text{ferm}} = \overline{\Psi} \mathcal{M}(A) \Psi$. 
4. Emergent spacetime from IIB matrix model

The fermionic matrices $\Psi$ can be integrated out exactly (Gaussian integrals) and give the Pfaffian of $\mathcal{M}$:

$$Z = \int dA \text{Pf}[\mathcal{M}(A)] e^{-S_{\text{bos}}(A)} \equiv \int dA e^{-S_{\text{eff}}(A)}.$$  \hspace{1cm} (8)

For the bosonic observable

$$w^{\mu_1 \ldots \mu_m} = \text{Tr} \left( A^{\mu_1} \ldots A^{\mu_m} \right),$$ \hspace{1cm} (9)

and arbitrary strings thereof, the expectation values are defined by the same integral as in (8):

$$\langle w^{\mu_1 \ldots \mu_m} w^{\nu_1 \ldots \nu_n} \ldots w^{\omega_1 \ldots \omega_z} \rangle = \frac{1}{Z} \int dA \left( w^{\mu_1 \ldots \mu_m} w^{\nu_1 \ldots \nu_n} \ldots w^{\omega_1 \ldots \omega_z} \right) e^{-S_{\text{eff}}}.$$ \hspace{1cm} (10)
4. **Emergent spacetime from IIB matrix model**

But the IIB matrix model just gives **numbers**, $Z$ and the expectation values $\langle w w \cdots w \rangle$, and the matrices $A^\mu$ and $\Psi_\alpha$ in the “path” integral are merely integration variables.

Moreover, there is no obvious small dimensionless parameter to motivate a saddle-point approximation.

Hence, the **conceptual** question: where is the classical spacetime?

Recently, we have suggested to revisit an old idea, the large-$N$ master field of Witten [1979].
According to Witten, the large-$N$ factorization of the expectation values (10) implies that the path integrals are saturated by a single configuration, the so-called **master field** $\hat{A}^\mu$.

To leading order in $N$, the expectation values are then given by

\[
\langle w_{\mu_1 \cdots \mu_m} w_{\nu_1 \cdots \nu_n} \cdots w_{\omega_1 \cdots \omega_z} \rangle \overset{N}{=} \hat{w}_{\mu_1 \cdots \mu_m} \hat{w}_{\nu_1 \cdots \nu_n} \cdots \hat{w}_{\omega_1 \cdots \omega_z}, \quad (11a)
\]

\[
\hat{w}_{\mu_1 \cdots \mu_m} \equiv \text{Tr} (\hat{A}^{\mu_1} \cdots \hat{A}^{\mu_m}). \quad (11b)
\]

Hence, we do not have to perform the integrals on the right-hand side of (10): we just need ten traceless Hermitian matrices $\hat{A}^\mu$ to get all these expectation values from the simple procedure of replacing each $A^\mu$ in the observables by the corresponding $\hat{A}^\mu$. 

4. Emergent spacetime from IIB matrix model

Now, the boxed question on slide 11 can be answered [2]:

classical spacetime may reside in the bosonic master-field matrices $\hat{A}^\mu$ of the IIB matrix model.

The heuristics is as follows [3]:

- The expectation values $\langle w^{\mu_1 \cdots \mu_m} \cdots w^{\omega_1 \cdots \omega_z} \rangle$ from (10), infinitely many numbers, correspond to a large part of the information content of the IIB matrix model (but, of course, not all the information).

- That same information is contained in the master-field matrices $\hat{A}^\mu$, which, to leading order in $N$, give the same numbers from the product $\hat{w}^{\mu_1 \cdots \mu_m} \cdots \hat{w}^{\omega_1 \cdots \omega_z}$, where $\hat{w}$ is the observable $w$ evaluated for $\hat{A}$.

- From these master-field matrices $\hat{A}^\mu$, it appears possible to extract the points and metric of an emergent classical spacetime (recall that the original matrices $A^\mu$ were merely integration variables).
4. Emergent spacetime from IIB matrix model

Assuming that the matrices $\hat{A}^\mu$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by certain numerical results), it is possible [2] to extract a discrete set of spacetime points $\{\hat{x}^\mu_k\}$ and an interpolating metric $g_{\mu\nu}(x)$. See also the review paper [3] for further discussion.

But, instead of assuming the matrices $\hat{A}^\mu$, we want to calculate them.

This project of getting solutions of the master-field equation (an algebraic-equation, in fact) has been started at the beginning of 2021. These are some results but only for very small matrices; see details and references in the review paper [4].
5. Conclusion

It is conceivable that a **new physics phase** gives rise to classical spacetime, gravity, and matter, as described by our current theory (GR + SM).

For an **explicit** calculation, we have considered the **IIB matrix model**, which has been proposed as a nonperturbative formulation of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the **large-N master field** $\hat{A}^\mu$ of the IIB matrix model.

We have now started to **solve** the full bosonic master-field equation of the IIB matrix model: first results are in, but the road ahead is long and arduous ...
6. Two research papers and two reviews

[1] F.R. Klinkhamer,
"Regularized big bang singularity,"

[2] F.R. Klinkhamer,
"IIB matrix model: Emergent spacetime from the master field,"

[3] F.R. Klinkhamer,
"M-theory and the birth of the Universe,"

[4] F.R. Klinkhamer,
"IIB matrix model, bosonic master field, and emergent spacetime,"