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Taming the Big Bang

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On the MWW2022 webpage:

“What is it about? Why is it important? Perspectives.” $\rightarrow$ slide 2

1. Introduction

We are motivated by the following question:

what happened at the birth of the Universe?

Or, at a more technical level:

what replaces the big bang singularity of Friedman cosmology?

How do we deal with infinities in QFT?
Easy: first, regularize and, then, find a better theory...

Let's try to do something similar. First, we stay within GR and get a "regularized" big bang.

Then, we look for a theory beyond GR (here, a nonperturbative formulation of superstring theory) to get a "new phase" that replaces the regularized big bang and gives the "emergence of spacetime."

2. Big bang of Friedmann cosmology

Metric from the spatially flat Robertson–Walker (RW) *Ansatz*:

$$ds^2\Big|^{\text{(RW)}} \equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu \Big|^{\text{(RW)}} = -dt^2 + a^2(t)\, \delta_{mn}\, dx^m\, dx^n\,, \qquad (1)$$

with $x^0 = c\,t$ and $c = 1$. The spatial indices m, n run over $\{1,\, 2,\, 3\}$.

Matter as a homogeneous perfect fluid with energy density $\rho_M(t)$ and pressure $P_M(t)$; define equation-of-state (EOS) parameter $w_M \equiv P_M/\rho_M$.

Dynamics from Einstein's gravitational field equation in GR.

$\Rightarrow$ three ODEs, the **Friedmann equations** [1922, 1924].

2. Big bang of Friedmann cosmology

For relativistic matter with constant EOS parameter $w_M = 1/3$, the Friedmann–Lemaître–Robertson–Walker (FLRW) solution is given by

$$a(t)\Big|^{(w_M=1/3)}_{\text{FLRW}} = \sqrt{t/t_0}\,, \qquad \text{for} \quad t > 0\,, \tag{2a}$$

$$\rho_M(t)\Big|^{(w_M=1/3)}_{\text{FLRW}} = \rho_{M0}/a^4(t) \propto 1/t^2\,, \qquad \text{for} \quad t > 0\,, \tag{2b}$$

with normalization $a(t_0) = 1$ at $t_0 > 0$.

The FLRW solution displays the **big bang singularity** for $t \to 0^+$,

$$\lim_{t\to 0^+} a(t) = 0\,, \tag{3}$$

where the curvature and the energy density diverge.

But, at $t = 0$, the theory (GR + SM) is no longer valid and we can ask: what happens **really** at the big bang?

3. Regularized big bang

Let us try to control the divergences by considering a new *Ansatz* for a "regularized" big bang [1]:

$$
\begin{aligned}
ds^2 \Big|^{\text{(RWK)}} &\equiv g_{\mu\nu}(x)\, dx^\mu\, dx^\nu \Big|^{\text{(RWK)}} \\
&= -\frac{t^2}{t^2+b^2}\, dt^2 + a^2(t)\, \delta_{mn}\, dx^m\, dx^n\,, && \text{(4a)} \\
b^2 &> 0\,, && \text{(4b)} \\
a^2(t) &> 0\,, && \text{(4c)} \\
t &\in (-\infty,\, \infty)\,, \quad x^m \in (-\infty,\, \infty)\,, && \text{(4d)}
\end{aligned}
$$

where we set $x^0 = c\,t$ and $c = 1$.

This metric $g_{\mu\nu}(x)$ is **degenerate**, with a vanishing determinant at $t = 0$.
In physical terms, there is a spacetime defect at $t = 0$.

2. Regularized big bang

With the standard Einstein equation and a homogeneous perfect fluid, get **modified** spatially flat Friedmann equations:

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho_M\,, \tag{5a}$$

$$\left[1+\frac{b^2}{t^2}\right]\left(\frac{\ddot{a}}{a}+\frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2\right) - \frac{b^2}{t^3}\,\frac{\dot{a}}{a} = -4\pi G\,P_M\,, \tag{5b}$$

$$\dot{\rho}_M + 3\,\frac{\dot{a}}{a}\left[\rho_M + P_M\right] = 0\,, \tag{5c}$$

$$P_M = P_M\big(\rho_M\big)\,, \tag{5d}$$

where the overdot stands for differentiation with respect to t.

The standard Friedmann equations are recovered by setting the singular b^2/t^2 terms to zero.

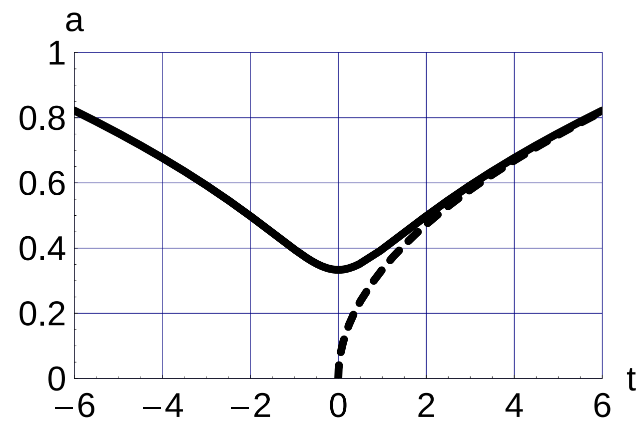
3. Regularized big bang

For constant EOS parameter $w_M = 1/3$, the new solution is

$$a(t)\Big|_{\text{FLRWK}}^{(w_M=1/3)} = \sqrt[4]{\left(t^2+b^2\right)/\left(t_0^2+b^2\right)}\,, \tag{6a}$$

$$\rho_M(t)\Big|_{\text{FLRWK}}^{(w_M=1/3)} = \rho_{M0}\left(t_0^2+b^2\right)/\left(t^2+b^2\right)\,, \tag{6b}$$

which is **perfectly smooth** at $t = 0$ as long as $b \neq 0$. Figure compares with the singular FLRW solution, as shown by the dashed curve.



3. Regularized big bang

Two possible scenarios:

1. **nonsingular bouncing cosmology** from $t = -\infty$ to $t = \infty$ (valid for $b \gg l_{\text{Planck}}$?) [gravitational waves generated in the pre-bounce epoch keep on propagating into the postbounce epoch];

2. **new phase** at $t = 0$ pair-produces a “universe” for $t > 0$ and an “antiuniverse” for $t < 0$ (valid for $b \sim l_{\text{Planck}}$?). $\Leftarrow$ THIS TALK

4. Emergent spacetime from IIB matrix model

For an explicit description of such a new phase, we can use the IIB matrix model [Kawai and collaborators, 1997-1999], suggested as a nonperturbative definition of superstring theory (M-theory).

The IIB matrix model has a **finite number** of $N \times N$ traceless Hermitian matrices: ten bosonic matrices A^μ and eight fermionic (Majorana–Weyl) matrices Ψ_α.

The partition function Z of the IIB matrix model is defined by the following “path” integral:

$$Z = \int dA\, d\Psi\; e^{-\,S(A,\,\Psi)} = \int dA\, d\Psi\; e^{-S_{\text{bos}}(A)\,-\,S_{\text{ferm}}(\Psi,\,A)}, \tag{7}$$

where the bosonic action $S_{\text{bos}}(A)$ is quartic in A and the fermionic action $S_{\text{ferm}}(\Psi,\,A)$ is quadratic in Ψ and linear in A, i.e., $S_{\text{ferm}} = \overline{\Psi}\,\mathcal{M}(A)\,\Psi$.

4. Emergent spacetime from IIB matrix model

The fermionic matrices Ψ can be integrated out exactly (Gaussian integrals) and give the Pfaffian of $\mathcal{M}$:

$$Z \;=\; \int dA\,\text{Pf}[\mathcal{M}(A)]\,e^{-S_{\text{bos}}(A)} \equiv \int dA\,e^{-S_{\text{eff}}(A)}\,. \tag{8}$$

For the bosonic observable

$$w^{\mu_1\,\cdots\,\mu_m} = \text{Tr}\left(A^{\mu_1}\cdots\,A^{\mu_m}\right), \tag{9}$$

and arbitrary strings thereof, the expectation values are defined by the same integral as in (8):

$$\begin{aligned}&\left\langle w^{\mu_1\,\cdots\,\mu_m}\,w^{\nu_1\,\cdots\,\nu_n}\;\ldots\;w^{\omega_1\,\cdots\,\omega_z}\right\rangle\\ &= \frac{1}{Z}\int dA\left(w^{\mu_1\,\cdots\,\mu_m}\,w^{\nu_1\,\cdots\,\nu_n}\;\ldots\;w^{\omega_1\,\cdots\,\omega_z}\right)e^{-\,S_{\text{eff}}}\,.\end{aligned} \tag{10}$$

4. Emergent spacetime from IIB matrix model

But the IIB matrix model just gives **numbers**, Z and the expectation values $\langle w\, w\, \cdots\, w \rangle$, and the matrices A^μ and Ψ_α in the “path” integral are merely integration variables.

Moreover, there is no obvious small dimensionless parameter to motivate a saddle-point approximation.

Hence, the **conceptual** question: $\boxed{\text{where is the classical spacetime?}}$

Recently, we have suggested to revisit an old idea, the large-N master field of Witten [1979].

4. Emergent spacetime from IIB matrix model

According to Witten, the large-N factorization of the expectation values (10) implies that the path integrals are saturated by a single configuration, the so-called **master field** $\widehat{A}^{\,\mu}$.

To leading order in N, the expectation values are then given by

$$\left\langle w^{\mu_1\,\cdots\,\mu_m}\, w^{\nu_1\,\cdots\,\nu_n}\,\cdots\, w^{\omega_1\,\cdots\,\omega_z}\right\rangle \stackrel{N}{=} \widehat{w}^{\,\mu_1\,\cdots\,\mu_m}\,\widehat{w}^{\,\nu_1\,\cdots\,\nu_n}\,\cdots\,\widehat{w}^{\,\omega_1\,\cdots\,\omega_z}\,, \tag{11a}$$

$$\widehat{w}^{\,\mu_1\,\cdots\,\mu_m} \equiv \mathrm{Tr}\left(\widehat{A}^{\,\mu_1}\cdots\widehat{A}^{\,\mu_m}\right). \tag{11b}$$

Hence, we do not have to perform the integrals on the right-hand side of (10): we just need ten traceless Hermitian matrices $\widehat{A}^{\,\mu}$ to get *all* these expectation values from the simple procedure of replacing each $A^{\,\mu}$ in the observables by the corresponding $\widehat{A}^{\,\mu}$.

4. Emergent spacetime from IIB matrix model

Now, the boxed question on slide 11 can be answered [2]:

classical spacetime may reside in the bosonic master-field matrices $\widehat{A}^{\mu}$ of the IIB matrix model.

The heuristics is as follows [3]:

- The expectation values $\langle w^{\mu_1 \cdots \mu_m} \,\ldots\, w^{\omega_1 \cdots \omega_z}\rangle$ from (10), infinitely many numbers, correspond to a large part of the **information content** of the IIB matrix model (but, of course, not all the information).
- That **same** information is contained in the master-field matrices $\widehat{A}^{\mu}$, which, to leading order in N, give the same numbers from the product $\widehat{w}^{\,\mu_1 \cdots \mu_m} \,\ldots\, \widehat{w}^{\,\omega_1 \cdots \omega_z}$, where $\widehat{w}$ is the observable w evaluated for $\widehat{A}$.
- From these master-field matrices $\widehat{A}^{\mu}$, it appears possible to **extract** the points and metric of an emergent classical spacetime (recall that the original matrices A^{μ} were merely integration variables).

4. Emergent spacetime from IIB matrix model

Assuming that the matrices $\widehat{A}^{\mu}$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by certain numerical results), it is possible [2] to extract a discrete set of spacetime points $\{\widehat{x}_k^{\mu}\}$ and an interpolating metric $g_{\mu\nu}(x)$. See also the review paper [3] for further discussion.

But, instead of assuming the matrices $\widehat{A}^{\mu}$, we want to **calculate** them.

This project of getting **solutions of the master-field equation** (an algebraic-equation, in fact) has been started at the beginning of 2021. These are some results but only for very small matrices; see details and references in the review paper [4].

5. Conclusion

It is conceivable that a **new physics phase** gives rise to classical spacetime, gravity, and matter, as described by our current theory (GR + SM).

For an explicit calculation, we have considered the **IIB matrix model**, which has been proposed as a nonperturbative formulation of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the **large-N master field** $\widehat{A}^{\,\mu}$ of the IIB matrix model.

We have now started to **solve** the full bosonic master-field equation of the IIB matrix model: first results are in, but the road ahead is long and arduous ...

6. Two research papers and two reviews

[1] F.R. Klinkhamer,
"Regularized big bang singularity,"
Phys. Rev. D **100**, 023536 (2019), arXiv:1903.10450.

[2] F.R. Klinkhamer,
"IIB matrix model: Emergent spacetime from the master field,"
Prog. Theor. Exp. Phys. **2021**, 013B04 (2021), arXiv:2007.08485.

[3] F.R. Klinkhamer,
"M-theory and the birth of the Universe,"
Acta Phys. Polon. B **52**, 1007 (2021), arXiv:2102.11202.

[4] F.R. Klinkhamer,
"IIB matrix model, bosonic master field, and emergent spacetime,"
PoS CORFU2021 259, arXiv:2203.15779.