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Cosmological constant problem, q–theory, and new TeV-scale physics

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Introduction

Consider the biggest problem of modern physics:

the **Cosmological Constant Problem** (CCP).

The 'biggest' problem for, at least, two reasons:

1. magnitude of the problem: $|\Lambda^{\text{theo}}|/|\Lambda^{\text{exp}}| \geq 10^{42}$,
2. size of the problem: the Universe.

The main Cosmological Constant Problem (CCP1) can be phrased as follows (see, e.g., [1] for a review):

why does the zero-point energy of the vacuum not produce naturally a large cosmological constant Λ *in the Einstein field equations?*

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[1] S. Weinberg, RMP 61, 1 (1989).

Introduction

Indeed, it is known that QCD involves a vacuum energy density (e.g., gluon condensate or bag constant) of order

$$|\epsilon_{\text{QCD}}| \sim (100\ \text{MeV})^4 \sim 10^{32}\ \text{eV}^4\,.$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_{\text{QCD}} = \epsilon_{\text{QCD}}(T)\,.$$

How can it, then, be that the Universe ends up with a vacuum energy density certainly less than

$$|\epsilon_{\text{present}}| < 10^{-28}\ \text{g cm}^{-3} \sim 10^{-10}\ \text{eV}^4\,?$$

Here, there are 42 orders of magnitude to explain!

Introduction

Even more CCPs after the discovery of the "accelerating Universe" :

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{UV}})^4$?

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{present}} \sim 10^{-11}\ \text{eV}^4$?

Hundreds of papers have been published on CCP2, but, most likely, CCP1 needs to be <u>solved first</u> before CCP2 can even be <u>addressed</u>.

Introduction

Here, a brief review of a particular approach to CCP1, which goes under the name of q**–theory** [2,3].

Then, turn to CCP2, describe a possible mechanism, and discuss a hint for new **TeV–scale** physics [4,5].

Outline talk:

A1. Basics of q–theory

B1. *Coup d'envoi*

B2. Electroweak kick

B3. Effective Λ and E_{ew}

B4. Recap mechanism

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[2] F.R. Klinkhamer & G.E. Volovik, PRD 77, 085015 (2008), arXiv:0711.3170.

[3] F.R. Klinkhamer & G.E. Volovik, JETPL 91, 259 (2010), arXiv:0907.4887.

[4] F.R. Klinkhamer and G.E. Volovik, PRD 80, 083001 (2009), arXiv:0905.1919.

[5] F.R. Klinkhamer, arXiv:1001.1939v7.

A1. Basics of q–theory

Crucial insight [2]: *there is vacuum energy and vacuum energy.*

More specifically and introducing an appropriate notation:

the vacuum energy density $\boxed{\epsilon}$ appearing in the action need not be the same as the vacuum energy density $\boxed{\rho_V}$ in the Einstein field equations.

How can this happen concretely . . .

A1. Basics of q–theory

Consider the full quantum vacuum to be a **self-sustained medium** (as is a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Then, consider **macroscopic equations** of this conserved **microscopic** variable (later called q), whose precise nature need not be known.

An analogy: the mass density in liquids, which describes a microscopic quantity – the number density of atoms – but obeys the macroscopic equations of hydrodynamics, because of particle-number conservation.

However, is the quantum vacuum just like a normal fluid?

A1. Basics of q–theory

No, as the vacuum is known to be Lorentz invariant
(cf. experimental limits at the 10^{-15} level in the photon sector [6–8]).

The Lorentz invariance of the vacuum rules out the standard type of charge density which arises from the <u>time</u> component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved charge**, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material fluids.

[6] A. Kostelecký and M. Mewes, PRD 66, 056005 (2002), arXiv:hep-ph/0205211.

[7] F.R. Klinkhamer and M. Risse, PRD 77, 117901 (2008), arXiv:0709.2502

[8] F.R. Klinkhamer and M. Schreck, PRD 78, 085026 (2008), arXiv:0809.3217.

A1. Basics of q–theory

With such a variable $q(x)$, the vacuum energy density of the effective action can be a generic function

$$\epsilon = \epsilon(q)\,, \tag{1}$$

which may include a constant term due to the zero-point energies of the fields of the Standard Model (SM), $\epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{var}}(q)$.

From ① thermodynamics and ② Lorentz invariance, it then follows that

$$P_V \stackrel{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \stackrel{②}{=} -\rho_V\,, \tag{2}$$

with the first equality corresponding to an integrated form of the Gibbs–Duhem equation (for chemical potential $\mu \equiv d\epsilon/dq$).

Recall GD-eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

A1. Basics of q–theory

Both terms entering ρ_V from (2) can be of order $(E_{\text{UV}})^4$, but they can cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence, for a generic function $\epsilon(q)$,

$$\exists\, q_0 \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\,\frac{d\,\epsilon(q)}{d\,q}\right]_{q=q_0} = 0\,, \tag{3}$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved, in principle ...

But, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which contain such a q (see later).

A1. Basics of q–theory

To summarize, the q–theory approach to the main Cosmological Constant Problem (CCP1) provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the "beyond-the-Standard-Model" physics does have an appropriate q–type variable.

Still, better to have one possible solution than none.

(Two remarks in Appendix A.)

B1. *Coup d'envoi*

Now, the remaining puzzles:

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\Big|_{\text{now}} \sim 10^{-29}$ g cm^{-3} $\sim 10^{-11}$ eV^4 ?

CCP2b also goes under the name of 'cosmic coincidence puzzle' (ccp).

Here, consider a possible realization of q operative at an UV (Planck) energy scale.

In the very early Universe, the vacuum energy density $\rho_V(t)$ rapidly drops to zero and stays there, but small effects are found to occur at cosmic temperatures T of the order of the TeV scale . . .

B2. Electroweak kick

Explicit realization of vacuum variable q via a 3–form gauge field A [9,10].

Effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = \int_{\mathbb{R}^4} d^4x \, \sqrt{-\det g} \, \Big(K_N \, R[g] + \Lambda_{\text{SM}} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi, g] \Big), \tag{4}$$

with $K_N \equiv 1/(16\pi G_N)$ and $\hbar = c = 1$, is replaced by [3]

$$\widetilde{S}^{\text{eff}}[A,g,\psi] = \int_{\mathbb{R}^4} d^4x \, \sqrt{-\det g} \, \Big(K(q) \, R[g] + \widetilde{\epsilon}(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi, g] \Big), \tag{5a}$$

$$q^2 \equiv -\frac{1}{24} F_{\alpha\beta\gamma\delta} \, F^{\alpha\beta\gamma\delta} \, . \tag{5b}$$

$$F_{\alpha\beta\gamma\delta} = \nabla_{[\alpha} A_{\beta\gamma\delta]} \, , \tag{5c}$$

[9] M.J. Duff and P. van Nieuwenhuizen, PLB 94, 179 (1980).

[10] A. Aurilia, H. Nicolai, and P.K. Townsend, NPB 176, 509 (1980).

B2. Electroweak kick

Then, variational principle produces generalized Einstein equations with a vacuum energy density term

$$\rho_V = \widetilde{\epsilon} - q\,\frac{d\,\widetilde{\epsilon}}{d\,q}\,, \tag{6}$$

which is precisely of the Gibbs–Duhem form (2). Technically, the extra term on the RHS of (6) appears because of the fact that $q = q(A,\,g)$.

Specifically, the generalized Einstein and Maxwell equations give:

$$\begin{aligned} 2K(q)\left(R_{\alpha\beta} - g_{\alpha\beta}\,R/2\right) \quad &= \quad -2\left(\nabla_\alpha \nabla_\beta - g_{\alpha\beta}\,\Box\right) K(q) \\ &\qquad +\rho_V(q)\,g_{\alpha\beta} - T^M_{\alpha\beta}\,, \end{aligned} \tag{7a}$$

$$\frac{d\rho_V(q)}{dq} + R\,\frac{dK(q)}{dq} \quad = \quad 0\,. \tag{7b}$$

Eqs. (6)–(7) are generic, i.e., independent of scale and dimension of q.

B2. Electroweak kick

Consider flat FRW universe with two types of matter, massive (‘type 1’) and massless (‘type 2’) particles. Resulting ODEs:

$$6\left(H\,\frac{dK}{dq}\,\frac{dq}{dt}+K\,H^2\right) = \rho_V+\rho_{M1}+\rho_{M2}\,, \tag{8a}$$

$$6\,\frac{dK}{dq}\left(\frac{dH}{dt}+2H^2\right) = \frac{d\rho_V}{dq}\,, \tag{8b}$$

$$\frac{d\rho_{M1}}{dt}+\left[4-\kappa_{M1}(t/t_{\text{ew}})\right]H\,\rho_{M1} = 0\,, \tag{8c}$$

$$\frac{d\rho_{M2}}{dt}+4\,H\,\rho_{M2} = 0\,, \tag{8d}$$

with prescribed equation-of-state (EOS) function $\kappa_{M1}(x)$ peaking at $x=1$.

B2. Electroweak kick

Analytically, it has been shown [4] that there exists a solution which

- starts from a standard radiation-dominated FRW universe with $\rho_V = 0$,
- is perturbed around $t = t_{\text{ew}} \sim E_{\text{Planck}}/(E_{\text{ew}})^2$ with $\rho_V \neq 0$,
- resumes the standard radiation-dominated expansion with $\rho_V = 0$.

Specifically, the vacuum energy density for $t \sim t_{\text{ew}}$ is given by

$$\rho_V(t) \sim \kappa^2_{M1}(t)\, H(t)^4 \,, \tag{9}$$

which has a peak value of order $(t_{\text{ew}})^{-4} \sim \left((E_{\text{ew}})^2 / E_{\text{Planck}}\right)^4$ but vanishes as $t \to \infty$.

$\Rightarrow$ standard (nondissipative) dynamic equations of q–theory do not produce a constant $\rho_{V,\,\text{remnant}} > 0$ from the electroweak kick.

B3. Effective Λ and E_{ew}

As argued in [4], quantum-dissipative effects of the vacuum energy density may lead to a finite remnant value of order

$$\Lambda \equiv \rho_{V,\,\text{remnant}} \sim \left((E_{\text{ew}})^2 \,/ E_{\text{Planck}} \right)^4 \sim (10^{-3}\ \text{eV})^4\,, \tag{10}$$

for $E_{\text{ew}} \sim 1$ TeV and $E_{\text{Planck}} \sim 10^{15}$ TeV. In fact, expression (10) was already suggested by Arkani-Hamed, Hall, Kolda, and Murayama [11].

In [5], it was then shown that it is possible to modify the "classical" q–theory equations (8) in such a way as to recover (10).
This approach is in the spirit of Kepler rather than Newton.

Details of these ODEs and their numerical solution in Appendix B.
Now, focus on the physics implications.

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[11] N. Arkani-Hamed et al., PRL 85, 4434 (2000), arXiv:astro-ph/0005111.

B3. Effective Λ and E_{ew}

Theoretical value of the effective cosmological constant given by

$$\Lambda^{\text{theory}} \equiv \lim_{t\to\infty} \rho_V^{\text{theory}}(t) = r_V^{\text{num}} \, (E_{\text{ew}})^8 \,/ (E_{\text{Planck}})^4 \,, \tag{11}$$

with a number $r_V^{\text{num}} \equiv r_V(\tau_{\text{freeze}})$ from the solution of the ODEs.

Equating this to the experimental value $\Lambda^{\text{exp}} \approx (2\ \text{meV})^4$ gives

$$E_{\text{ew}} = \left(\frac{\Lambda^{\text{exp}}}{r_V^{\text{num}}}\right)^{1/8} (E_{\text{Planck}})^{1/2} \approx 3.8\ \text{TeV} \left(\frac{0.013}{r_V^{\text{num}}}\right)^{1/8} . \tag{12}$$

Analytic bound: $r_V^{\text{num}} \lesssim 1 \,\Rightarrow E_{\text{ew}} \gtrsim 2$ TeV.

Numerical results for r_V^{num} give E_{ew} estimates of Table 1.

B3. Effective Λ and E_{ew}

Table 1: Preliminary estimates [5] of the energy scale E_{ew} for hierarchy parameter $\xi \gg 1$. Both massive type–1 and massless type–2 particles are assumed to have been in thermal equilibrium before the “kick” and the number of type–2 particles is taken as $N_{\text{eff},2} = 10^2$. Left: prescribed kick with type–1 particles of equal mass $M = E_{\text{ew}}$ and, for fixed coupling constant $\zeta = 2$, E_{ew} shown as a function of the number of degrees of freedom $N_{\text{eff},1}$. Right: dynamic kick with case–A type–1 mass spectrum $(N_{1a},\, M_{1a}\,;\, N_{1b},\, M_{1b}) = (40,\ 2 \times E_{\text{ew}};\ 60,\ 1/3 \times E_{\text{ew}})$ and $E_{\text{ew}} = \langle M_{1i} \rangle$ shown as a function of ζ.

ζ	$N_{\text{eff},1}$	E_{ew} [TeV]
2	1	8.5
2	10^1	4.9
2	10^2	$\boxed{3.2}$
2	10^3	2.8
2	10^4	2.7

ζ	$N_{\text{eff},1}$	E_{ew} [TeV]
0.2	10^2	14.8
2	10^2	$\boxed{3.8}$
20	10^2	5.6

B4. Recap mechanism

- Presence of massive particles with electroweak interactions [average mass $< M > = E_{\text{ew}} \sim$ TeV] changes the expansion rate $H(t)$ of the Universe compared to the radiation-dominated case.
- Change of the expansion rate kicks $\rho_V(t)$ away from zero.
- Quantum-dissipative effects operating at cosmic time t_{ew} set by E_{ew} may result in a finite remnant value of ρ_V.
- Phenomenological description of this process follows from a modification of the classical q–theory cosmological equations.
- Required E_{ew} value ranges from 3 to 15 TeV, depending on the effective number of new particles and assuming the dissipative coupling constant to be of order 1.

Conclusion

CCP1: self-adjustment of a particular type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0$.

CCP2: finite remnant value of $\rho_V(t)$ may result from quantum-dissipative effects operating at a cosmic time t_{ew} set by the scale $E_{\text{ew}} \sim$ TeV of massive particles with $M \sim E_{\text{ew}}$ and electroweak interactions.

Hint: required E_{ew} value ranges from 3 to 15 TeV, which, if correct, implies new TeV–scale physics beyond the SM.

Appendix A: Two remarks

Two remarks [3]:

1. The adjustment-type solution (3) of the CCP1 circumvents Weinberg's no–go theorem [1].

   Crux: q is a non-fundamental scalar field (cf. theory of Sec. B2).

2. Next question is how the Universe got the right value q_0?

   Possible answer via a generalization of q–theory, for which the correct value q_0 arises dynamically (cf. brief summary below).

Appendix A: Two remarks

Realization of vacuum variable q via an aether-type velocity field u_β [12,13], setting $E_{\text{UV}} = E_{\text{Planck}}$. For a flat FRW metric with cosmic time t, there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \to q_0\, t\,, \quad u_b(t) = 0\,, \quad H(t) \to 1/t\,. \tag{A.1}$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t\, E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. Then, the field equations are [12]:

$$\ddot{v} + 3\,h\,\dot{v} - 3\,h^2\,v = 0\,, \tag{A.2a}$$

$$2\,\lambda - (\dot{v})^2 - 3\,(h\,v)^2 = 6\,h^2\,, \tag{A.2b}$$

with the overdot standing for differentiation with respect to τ.
Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\widehat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\widehat{q}_0 = \sqrt{\lambda/2}$.

[12] A.D. Dolgov, PRD 55, 5881 (1997), arXiv:astro-ph/9608175.

[13] T. Jacobson, PoS QG-PH, 020 (2007), arXiv:0801.1547.

Appendix A: Two remarks

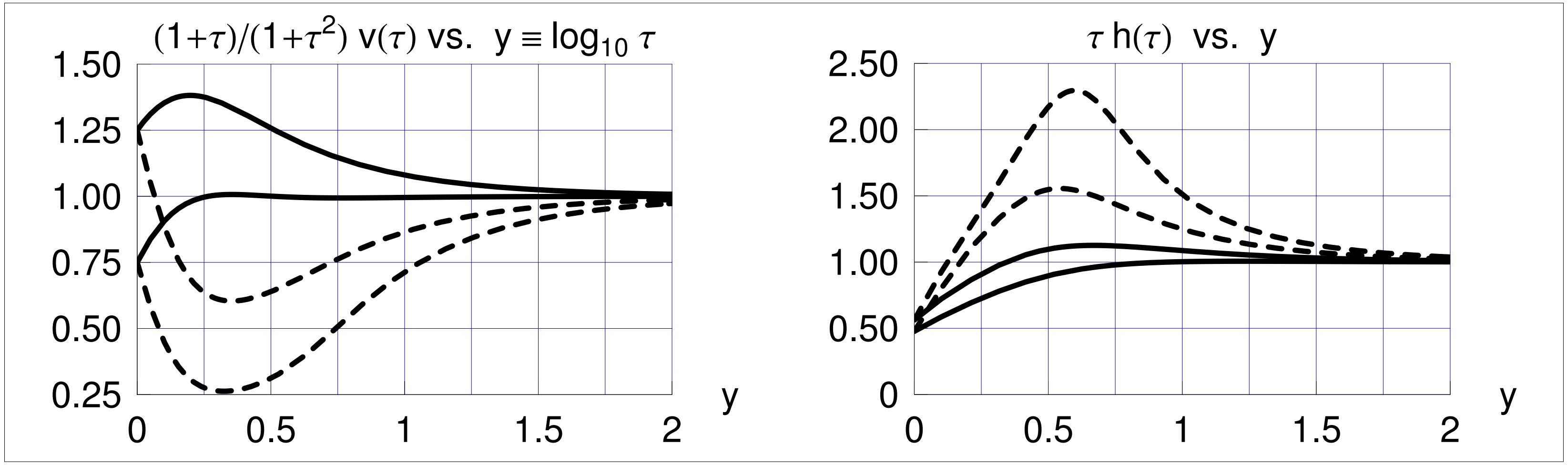

Fig. A1: Four numerical solutions of ODEs (A.2ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25$, with dashed curves for negative $\dot{v}(1)$.

$\Rightarrow$ required Minkowski value $\widehat{q}_0 = \sqrt{\lambda/2} = 1$ arises dynamically [see left panel].

$\Rightarrow$ Minkowski spacetime is an attractor.

Appendix B: Model universe

Model universe with three components (see Appendix A of [5]):

0. Vacuum variable q entering the gravitational coupling $K(q)$.
1. Massive 'type 1' particles (subspecies $i = a, b, c, \ldots$) with masses M_i of order $E_{\text{ew}} \sim 1$ TeV and electroweak interactions.
2. Massless 'type 2' particles with electroweak interactions.

Now, proceed as follows:

- Consider a flat FRW universe with Hubble parameter $H(t)$.
- Allow for energy exchange between the two matter components, so that total type–1 energy density peaks around $t_{\text{ew}} \equiv E_{\text{Planck}}/(E_{\text{ew}})^2$.
- Get function $\overline{\kappa}_{M1i}(t)$ from EOS parameter $w_{M1i}(t)$, with $\overline{\kappa}_{M1i}(t) \sim 0$ for $t \ll t_{\text{ew}}$ in the ultrarelativistic regime.
- Introduce a dissipative coupling constant $\zeta = \mathrm{O}(1)$ and a function $\gamma(t)$ which equals 1 for $t \ll t_{\text{ew}}$ and drops to zero for $t > t_{\text{ew}}$.

Appendix B: Model universe

Modified q–theory ODEs (standard ODEs recovered for $\zeta = 0$ and $\gamma = 1$):

$$6\,\left(H\,K'\,\dot{q} + K\,H^2\right) = \rho_V + \sum_{i=a,b,c,\ldots} \rho_{M1i} + \rho_{M2}\,, \tag{B.1a}$$

$$6\,K'\left(\dot{H} + 2H^2\right) = \gamma\,\rho'_V + (1-\gamma)\,\frac{K'}{K}\,\Big[2\rho_V + \sum_i \tfrac{1}{2}\,\overline{\kappa}_{M1i}\,\rho_{M1i}\Big], \tag{B.1b}$$

$$\dot{\rho}_{M1i} + (4-\overline{\kappa}_{M1i})H\rho_{M1i} = \frac{N_{1i}}{N_1}\Big[\frac{\lambda_{21}}{t_{\text{ew}}}\widehat{\omega}\,\rho_{M2} - \frac{\zeta}{\gamma}q\,\dot{\rho}'_V\Big] - \frac{\lambda_{12}}{t_{\text{ew}}}\widehat{\nu}\,\rho_{M1i}, \tag{B.1c}$$

$$\dot{\rho}_{M2} + 4\,H\,\rho_{M2} = -\frac{\lambda_{21}}{t_{\text{ew}}}\,\widehat{\omega}\,\rho_{M2} + \frac{\lambda_{12}}{t_{\text{ew}}}\,\widehat{\nu}\,\sum_i \rho_{M1i}\,, \tag{B.1d}$$

where the overdot [prime] stands for differentiation with respect to t [q]. Functions γ, $\widehat{\omega}$, and $\widehat{\nu}$ shown in Figs. B1–B4 below.

Appendix B: Model universe

Use simple *Ansätze*: $\rho_V(q) \propto (q - q_0)^2$ and $K(q) \propto q$.

With t_{ew} and $\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4 \gg 1$, define dimensionless variables:

$$\tau \equiv (t_{\text{ew}})^{-1}\, t\,, \qquad h \equiv t_{\text{ew}}\, H\,, \tag{B.2a}$$

$$r_V \equiv (t_{\text{ew}})^4\, \rho_V\,, \qquad r_{Mn} \equiv \xi^{-1}\,(t_{\text{ew}})^4\, \rho_{Mn}\,, \tag{B.2b}$$

$$x \equiv \xi\,\big(q/q_0 - 1\big)\,. \tag{B.2c}$$

Figures B1–B3 and B4 show numerical results for $\xi = 10^2$ and $\xi = \infty$.

Appendix B: Model universe

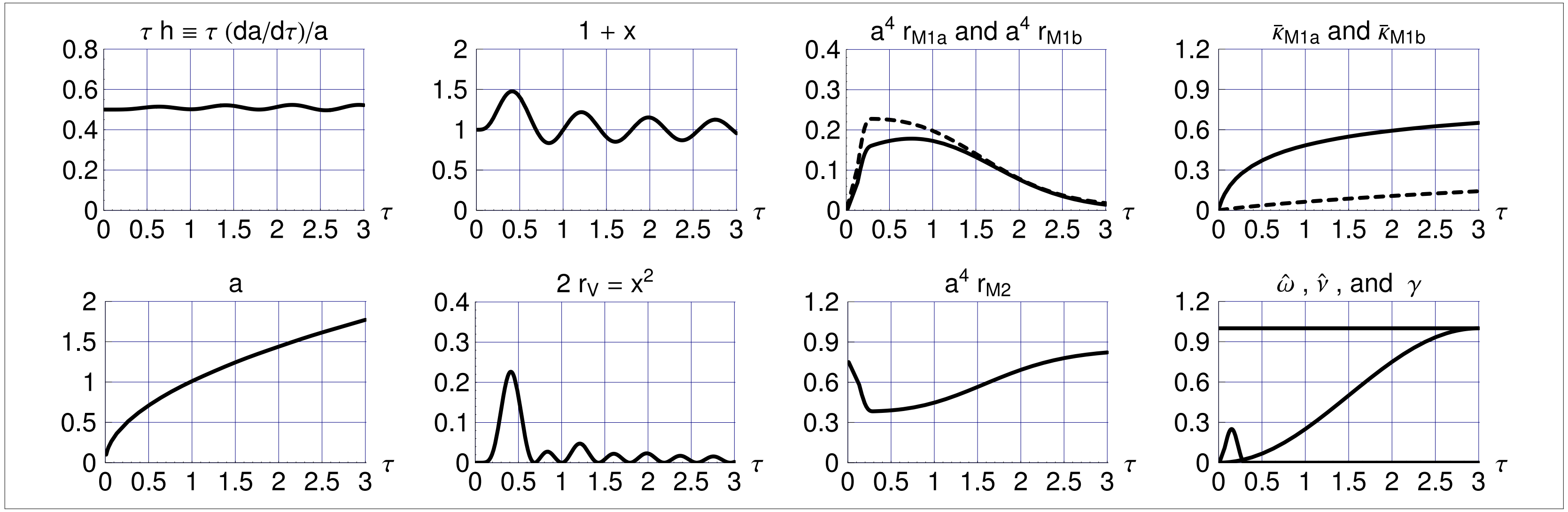

Fig. B1: Numerical solution [5] of standard (nondissipative) q–theory ODEs (B.1) for $\zeta = 0$ and $\gamma = 1$. The hierarchy parameter is $\xi = 10^2$ [oscillatory effects suppressed for larger values of ξ, recovering the smooth behavior of (9)]. Further coupling constants $\{\lambda_{21}, \lambda_{12}\} = \{18, 2\}$ and case–A type–1 mass spectrum $(N_{1a}, M_{1a}\,; N_{1b}, M_{1b}) = (40, 2\,E_{\text{ew}}; 60, 1/3\,E_{\text{ew}})$.

Appendix B: Model universe

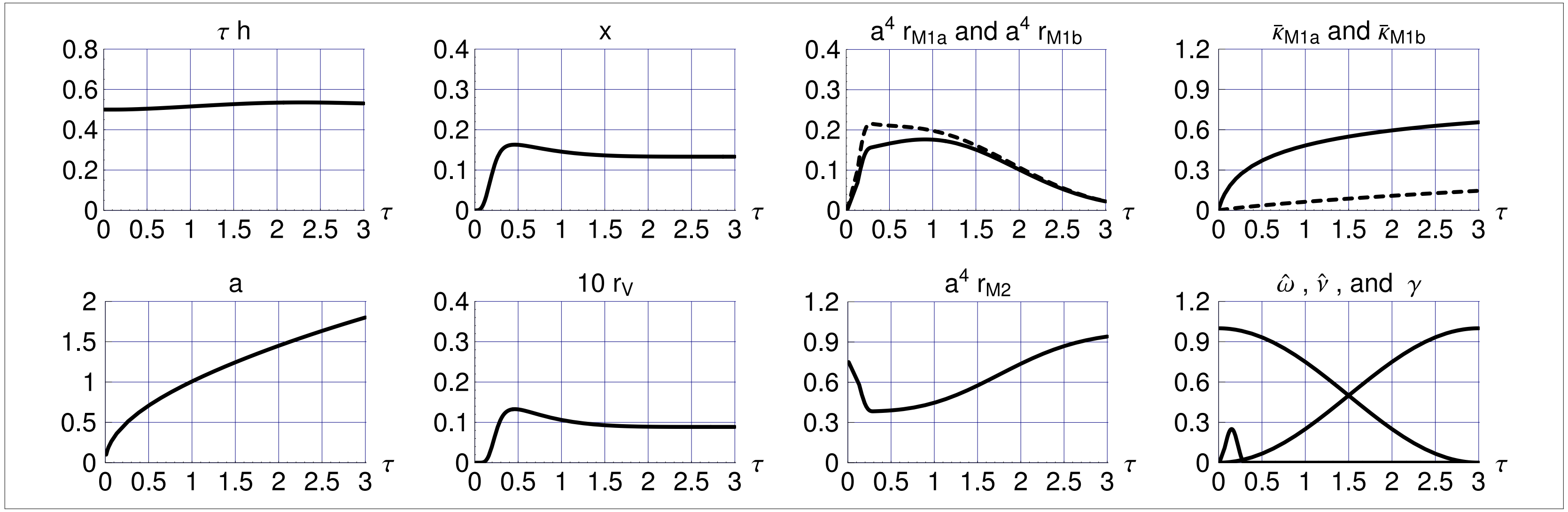

Fig. B2: Same as Fig. B1, but now for the modified q–theory ODEs (B.1) with dissipative coupling constant $\zeta = 2$ and $\gamma(\tau) = 0$ for $\tau \geq \tau_{\text{freeze}} = 3$.

Appendix B: Model universe

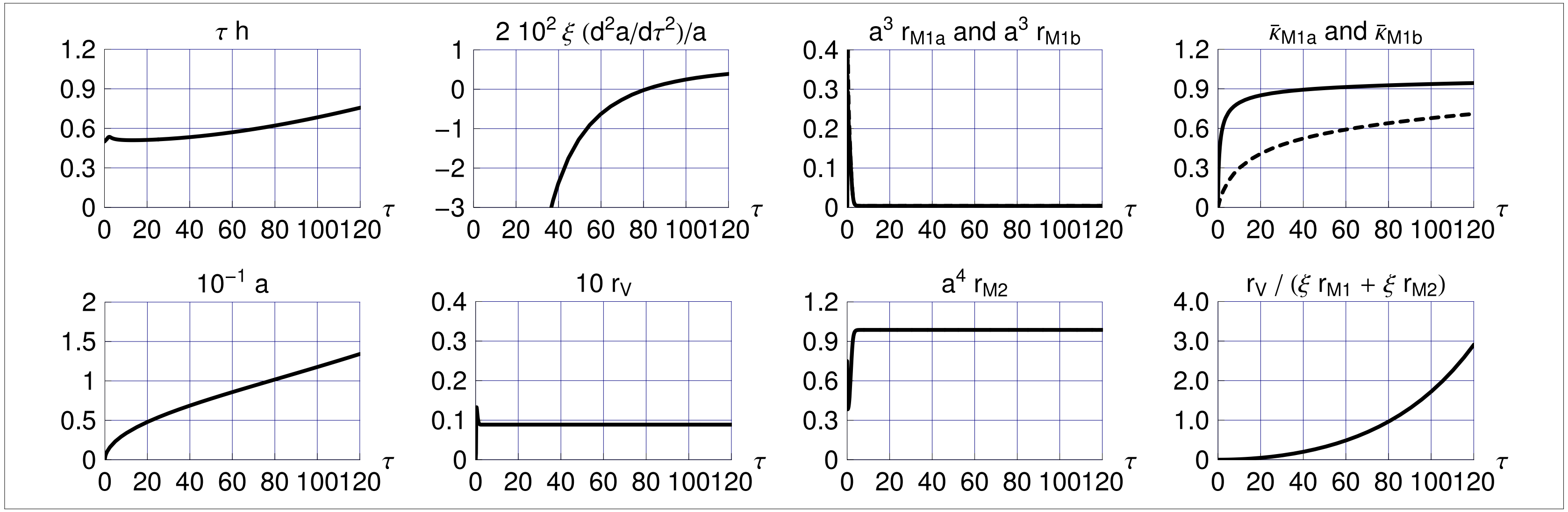

Fig. B3: Same as Fig. B2, but evolved further.

Appendix B: Model universe

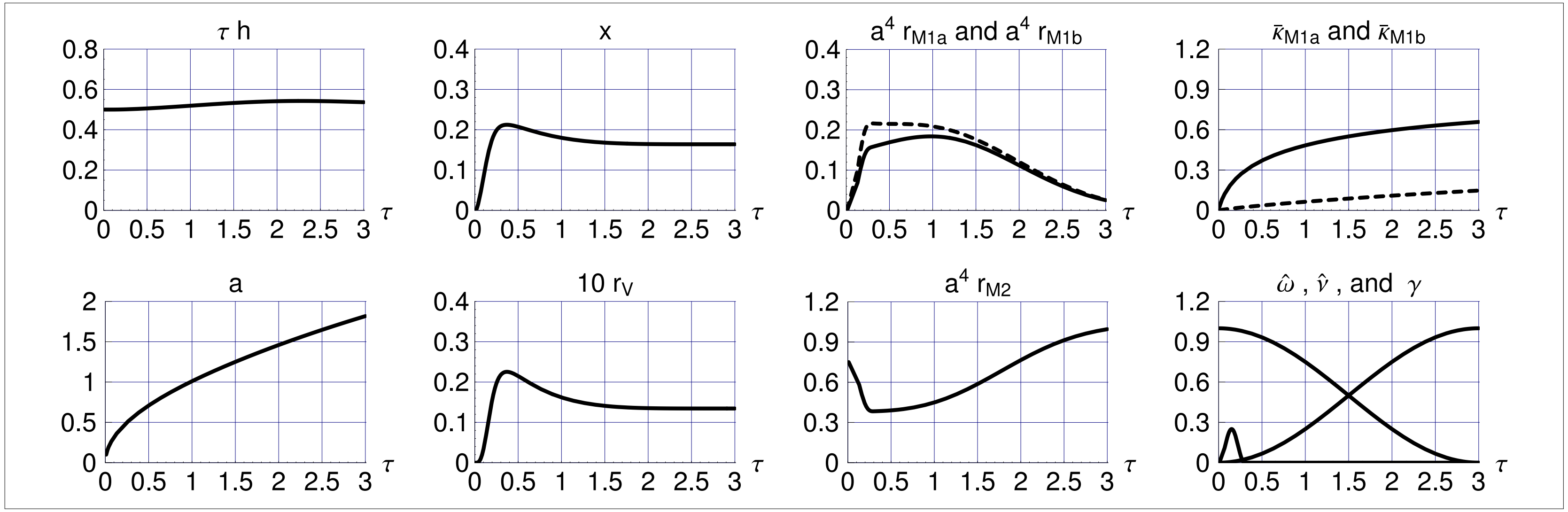

Fig. B4: Same as Fig. B2, but now for $\xi = \infty$.