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Nontrivial topology and CPT violation

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1. Introduction
2. CPT anomaly
3. Large-scale topology
4. Small-scale topology
5. Summary

Introduction

1. INTRODUCTION

Experiment has shown the violation of P, C, CP, and T, but not CPT.

Indeed, there is the well-known CPT "theorem" [Lüders, 1954–57; Pauli, 1955; Bell, 1955; Jost, 1957]:

any local relativistic quantum field theory is invariant under the combined operation of charge conjugation (C), parity reflection (P), and time reversal (T).

The main inputs of this "theorem," by itself, are:

- flat spacetime $(M,\, g) = \left(\mathbb{R}^4,\, \eta_{\mu\nu}^{\text{Minkowski}}\right)$;
- invariance under proper orthochronous Lorentz transformations and spacetime translations;
- normal spin-statistics connection;
- locality and Hermiticity of the Hamiltonian.

Introduction

BUT CAN CPT INVARIANCE BE VIOLATED AT ALL IN A PHYSICAL THEORY AND, IF SO, IS IT IN THE REAL WORLD?

It was widely believed that only quantum-gravity or superstring effects could give CPT violation.

A different result was, however, obtained a few years ago [1]:

for certain spacetime topologies and classes of chiral gauge theories, CPT invariance is broken anomalously, that is, by quantum effects.

Crucial ingredients of the "CPT anomaly" are:

- chiral fermions and gauge interactions;
- nontrivial spacetime topology.

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[1] Klinkhamer, NPB 578 (2000) 277 [hep-th/9912169].

Introduction

Possible applications:

- *optical activity of the vacuum*, e.g., for the CMB [2];
- *fundamental arrow-of-time*, e.g., for the Big Bang [3];
- *spacetime foam*, with CPT anomaly as diagnostic tool [4].

In this talk, we summarize the basic idea of the CPT anomaly and, time permitting, discuss the last two applications.

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[2] K, in: GustavoFest Proceedings [hep-ph/0511030].
[3] K, PRD 66 (2002) 047701 [gr-qc/0111090].
[4] K & Rupp, PRD 70 (2004) 045020 [hep-th/0312032].

Outline

Outline rest of this talk:

1. Introduction
2. CPT anomaly (main focus)
3. Large-scale topology (some details)
4. Small-scale topology (very briefly)
5. Summary

CPT anomaly

2. CPT ANOMALY

2.1 HEURISTICS

The main ingredients of the CPT anomaly for 4D manifold $M = \mathbb{R}^3 \times S^1$ with vierbeins $e^a_\mu(x) = \delta^a_\mu$ and appropriate gauge fields:

- compact space dimension, coordinate $x^3 \in [0, L]$, with periodic spin structure (fermions can have momentum component $p_3 = 0$);
- a single chiral fermion with $p_3 = 0$ corresponds to a single massless Dirac fermion in 3D;
- a single massless Dirac fermion in 3D has a “parity anomaly,” provided gauge invariance is maintained exactly [5,6];
- this “parity” violation corresponds to T violation in 4D, which, in turn, leads to CPT violation in 4D.

$[5]$ Redlich, PRL 52 (1984) 18; PRD 29 (1984) 2366.
$[6]$ Alvarez-Gaumé & Witten, NPB 234 (1984) 269.

CPT anomaly

2.2 RIGOROUS RESULT ON THE LATTICE

Consider the 4D Abelian chiral gauge theory with

$$G = U(1),\ R_{\text{left}} = 6\times\left(\frac{1}{3}\right)+3\times\left(\frac{-4}{3}\right)+3\times\left(\frac{2}{3}\right)+2\times(-1)+1\times(2)+1\times(0)\ ,$$

where perturbative chiral gauge anomalies cancel out, $U(1) \subset SO(10)$.
(N.B. vectorlike QED has $R_{\text{left}} = (+1) + (-1)$, a real representation.)

Define a chiral lattice gauge theory over a finite hypercubic lattice, with

- periodic spin structure in one direction;
- Ginsparg–Wilson fermions;
- Neuberger's lattice Dirac operator;
- Lüscher's chiral constraints.

CPT anomaly

The goal is to establish that the Euclidean effective gauge field action $\Gamma[U]$ changes under a CPT transformation,

$$\Gamma[U] \neq \Gamma[U^{\mathsf{CPT}}]\,,$$

where U denotes the set of link variables.

This result has been obtained [7] for an arbitrary odd number N of links in the periodic direction and for arbitrary values of the lattice spacing a (the compact dimension has $L = Na$).

Moreover, the origin of the CPT anomaly has been identified as an ambiguity in the choice of basis vectors for the fermion integration measure; cf. the path-integral derivation of the triangle anomaly [8].

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[7] K & Schimmel, NPB 639 (2002) 241 [hep-th/0205038].
[8] Fujikawa, PRD 21 (1980) 2848.

CPT anomaly

2.3 PERTURBATIVE RESULT IN THE CONTINUUM

Consider the chiral gauge theory with

$$\begin{aligned} G &= SO(10)\,, \quad R_{\text{left}} = N_{\text{fam}} \times (\mathbf{16})\,, \quad N_{\text{fam}} = 3\,, \\ M &= \mathbb{R}^3 \times S^1_{\text{PSS}}\,, \quad e^a_\mu(x) = \delta^a_\mu\,, \quad g_{\mu\nu}(x) = \eta_{\mu\nu}\,, \end{aligned}$$

where PSS stands for periodic spin structure.

Of course, the effective action $\Gamma[B]$, for $B \in$ so(10), is not known exactly. (Only in 2D, are there exact results [9] .)

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[9] K & Nishimura, PRD 63 (2001) 097701 [hep-th/0006154].

CPT anomaly

But the crucial term has been identified perturbatively [1] for an appropriate class of gauge fields:

$$\Gamma^{\mathbb{R}^3\times S^1}_{\text{anom}}[B] = \int_{\mathbb{R}^3} \mathrm{d}x^0 \mathrm{d}x^1 \mathrm{d}x^2 \int_0^L \mathrm{d}x^3 \; \frac{n\,x^3}{L}\; \frac{1}{32\pi}\; \epsilon^{\kappa\lambda\mu\nu}\; \mathrm{tr}\; F_{\kappa\lambda}(x)\, F_{\mu\nu}(x)\,,$$

with odd integer n (see below) and Yang–Mills field strength

$$F_{\kappa\lambda}(x) \equiv \partial_\kappa B_\lambda(x) - \partial_\lambda B_\kappa(x) + [B_\kappa(x), B_\lambda(x)]\,.$$

This local action term is Lorentz- and CPT-noninvariant, because of the spacetime-dependent “coupling constant” $n\,x^3/L$.

CPT anomaly (skip)

Two remarks:

1. The integer n is a remnant of the UV regularization (for example, Pauli–Villars or lattice):

$$n \equiv \sum_{f=1}^{N_{\text{fam}}} (2\, k_{0f} + 1) \Big|_{N_{\text{fam}}=3} \neq 0 \,, \quad k_{0f} \in \mathbb{Z} \,.$$

The simplest case gives $n = (1 - 1 + 1)\, \Lambda_0 / |\Lambda_0| = \pm 1$, where Λ_0 is a Pauli–Villars mass.

2. For the $SO(10)$ theory with <u>three</u> identical irreps, the CPT anomaly <u>must</u> occur, according to the expression above.

Note that this $SO(10)$ theory incorporates the Standard Model (SM) with $N_{\text{fam}} = 3$ families of quarks and leptons. But, for the SM *per se*, the anomaly <u>may or may not</u> occur, depending on the UV regularization.

The reason is that the SM irreps come in <u>even</u> number (for example, four left-handed isodoublets per family).

Large-scale topology

3. LARGE-SCALE TOPOLOGY

3.1 LIGHT PROPAGATION

Focus on the EM gauge field $a_\mu(x)$ embedded in $B_\mu(x) \in \mathsf{so}(10)$ over $M = \mathbb{R}^3 \times S^1$ and take $n = -1$ in the anomalous term:

$$\begin{aligned} \mathcal{L}_{\mathsf{eff}}[a] &= \mathcal{L}_{\mathsf{MCS}}[a] + \cdots \,, \\ \mathcal{L}_{\mathsf{MCS}}[a] &= \mathcal{L}_{\mathsf{Maxwell}}[a] + \mathcal{L}_{\mathsf{CS-like}}[a] \,, \\ \mathcal{L}_{\mathsf{Maxwell}}[a] &= -\tfrac{1}{4}\, f_{\mu\nu}\, f^{\mu\nu} \,, \\ \mathcal{L}_{\mathsf{CS-like}}[a] &= +\tfrac{1}{4}\, M\, \epsilon^{3\kappa\lambda\mu}\, f_{\kappa\lambda}\, a_\mu \,, \end{aligned}$$

with

$$f_{\mu\nu}[a] \equiv \partial_\mu a_\nu - \partial_\nu a_\mu \,, \qquad M \sim \alpha/L.$$

Large-scale topology

Now consider the Maxwell–Chern–Simons (MCS) theory *per se*.

On the one hand, the action is gauge-invariant, provided the electric and magnetic fields in $f_{\kappa\lambda}$ vanish fast enough at infinity.

On the other hand, there is known to be a close relation between CPT and microcausality (i.e., commutativity of local observables with spacelike separations) [Jost, 1957; Dyson, 1958].

How about microcausality in the MCS theory?

Reassuringly, microcausality (locality) can be established also in the spacelike MCS theory [10].

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[10] Adam & K, NPB 607, 247 (2001) [hep-ph/0101087].

Large-scale topology

Moreover, the propagation of light in the MCS theory makes clear that C and P are conserved, but T not.

In fact, for a left-handed pulse propagating in the "orthogonal" x^2 direction, one has (as long as $M \neq 0$):

$$|\,\vec{v}_{\rm g}^{\,L}(0, p_2, 0)| \neq |\,\vec{v}_{\rm g}^{\,L}(0, -p_2, 0)|\ .$$

Time-reversal noninvariance is manifest, as T flips the momentum and keeps the helicity. See Fig.1.

There may also be observable effects in the cosmic microwave background (CMB). See Fig.2.

Large-scale topology

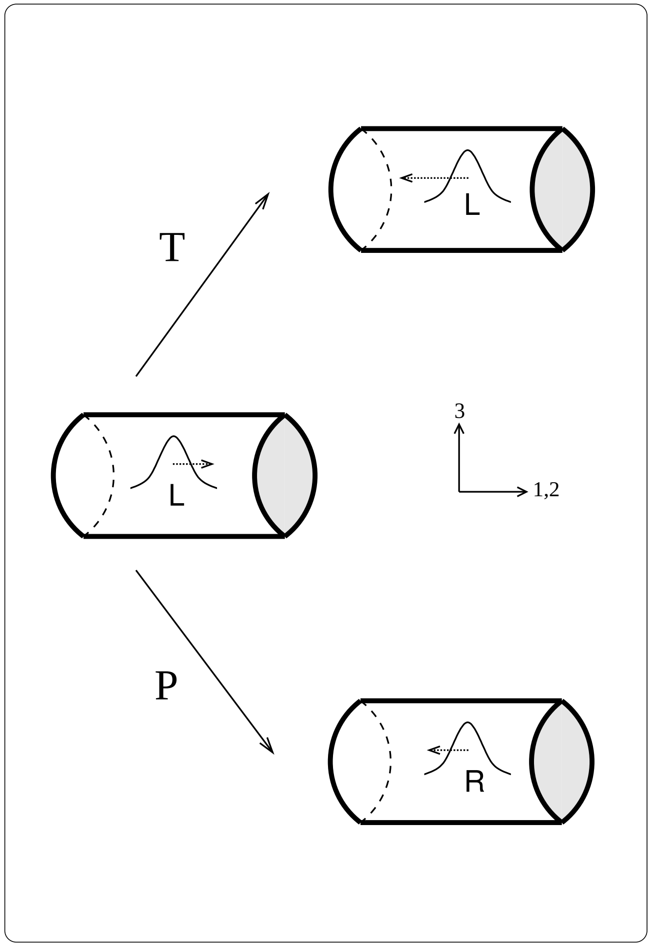

Fig. 1. Sketch of the behavior of a left-handed wave packet under time reversal (T) and parity (P) transformations in MCS theory over $\mathbb{R}^3 \times S^1$.

Large-scale topology

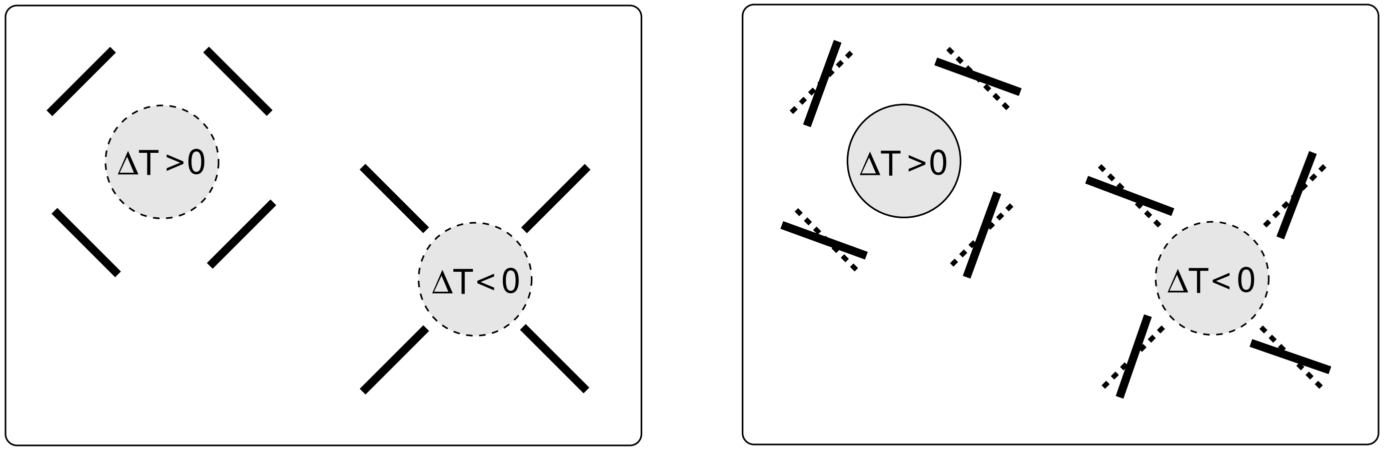

Fig. 2. Sketch of linear polarization pattern (heavy bars) around CMB hot-spots ($\Delta T > 0$) and cold-spots ($\Delta T < 0$), generated by scalar perturbations of the metric.

Left panel: view in a “standard” direction with Cartesian coordinate x^2.

Right panel: view in the “preferred” MCS direction with coordinate x^3.

Large-scale topology

3.2 BIG BANG VS. BIG CRUNCH

Examining the various time-asymmetries present at the macroscopic level, Penrose (1979) asked:

“what special geometric structure did the big bang possess that distinguishes it from the time-reverse of the generic singularities of collapse – and why?”

He, then, proposed a particular condition (the vanishing of the Weyl curvature tensor) to hold at any initial singularity.

Whatever the precise condition may turn out to be, the crucial point is that this condition would not hold for final singularities.

This implies that the unknown physics responsible for the initial singularity necessarily involves T, PT, CT, and CPT violation.

Large-scale topology

But Penrose did not make a concrete proposal for the physical mechanism of this T and CPT noninvariance.

We have suggested [3] the possible relevance of the CPT anomaly which does not involve gravitation directly but does depend on the global structure of space.

Consider the "clock" C of Figs. 3a and 4a below.

The crucial point, now, is that the time-reversed copy C' (Figs. 3b and 4b) runs differently in MCS theory.

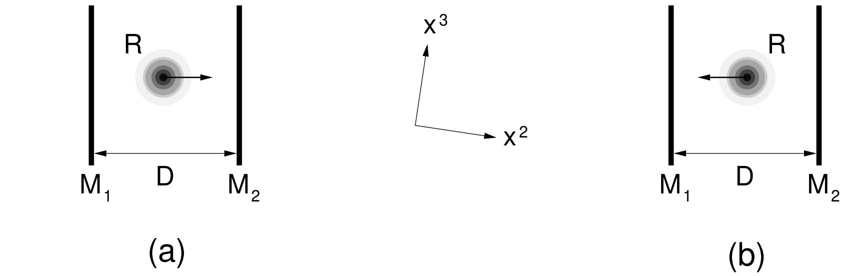

Fig. 3. (a) Sketch of clock C; (b) Sketch of clock C'.

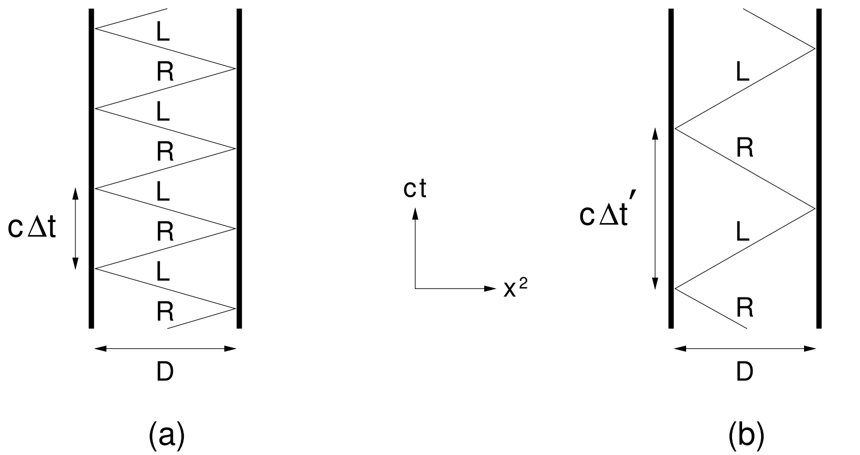

Fig. 4. Spacetime diagrams in MCS theory: (a) C and (b) C'.

Large-scale topology

More specifically, take the $SO(10)$ chiral gauge theory with $N_{\text{fam}} = 3$ in a hypothetical Kantowski–Sachs universe with topology $\mathbb{R} \times S^2 \times S^1{}_{\text{PSS}}$.

The clock C near the big bang and the time-reversed copy of clock C (i.e., clock C') near the big crunch then give a <u>different number of ticks</u> over an equal time interval as determined by a standard clock (or the expansion of the universe).

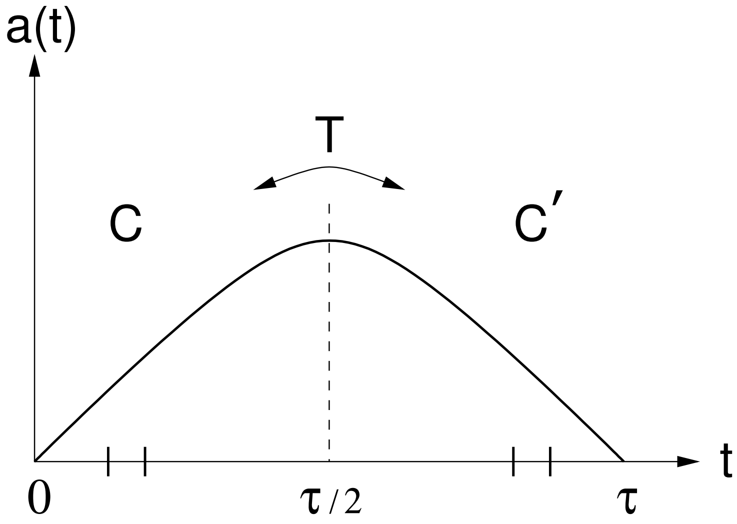

Fig. 5. KS universe with clocks C and C'.

Large-scale topology

Therefore, the physics near the initial singularity and the physics near the final singularity would be different, which may be part of the answer to Penrose's question.

Of course, the effect discussed gives only a *direction in time* and the main dynamics of the big-bang singularity still needs to be explained.

Compare with spontaneous magnetization in ferromagnets, where a small impurity or boundary effect determines the *direction in space* of the magnetization in the domain considered but the dynamics is really driven by the spin interactions.

Small-scale topology

4. SMALL-SCALE TOPOLOGY

4.1 CPT ANOMALY FOR A SINGLE WORMHOLE

Consider an orientable three-space N_3 with a wormhole constructed by removing two identical open balls from $\mathbb{R}^3$ and identifying their surfaces. These balls have diameter b and centers separated by a distance d in $\mathbb{R}^3$.

Simplest case has width b of the wormhole “mouth” put to zero. The three-space N_3 is then $\mathbb{R}^3$ with two points identified.

Denote these points by $\vec{x} = \pm(d/2)\,\hat{e}$, for some unit vector $\hat{e}$.

Define a dipole-like function $\Phi(\vec{x})$ and corresponding coordinate $\eta \equiv \Phi(\vec{x})$:

$$\Phi(\vec{x}) \equiv \arctan\left(\frac{d}{|\vec{x} - (d/2)\,\hat{e}|} - \frac{d}{|\vec{x} + (d/2)\,\hat{e}|} \right) .$$

Small-scale topology

It suffices to establish the existence of the CPT anomaly for one particular class of gauge fields, indicated by a prime.

For the electromagnetic component a'_μ, the anomalous contribution to the effective action is then proportional to

$$\int_N \mathrm{d}^4x \; \frac{\alpha\,\Phi(x)}{\pi} \; \epsilon^{\kappa\lambda\mu\nu}\, f'_{\kappa\lambda}(x)\, f'_{\mu\nu}(x)\,,$$

with field strength $f'_{\mu\nu}(x) \equiv \partial_\mu a'_\nu(x) - \partial_\nu a'_\mu(x)$ and fine-structure constant $\alpha \approx 1/137$. The general structure of this term is as before:

$$\int_{\mathbb{R}^4} \mathrm{d}^4x \; f_N(x;a] \; \epsilon^{\kappa\lambda\mu\nu}\, f_{\kappa\lambda}(x)\, f_{\mu\nu}(x)\,,$$

with integration domain extended to $\mathbb{R}^4$. The factor $f_N(x;a]$ is both a function of the spacetime coordinates x^μ and a gauge-invariant functional of the gauge field $a_\mu(x)$.

Small-scale topology (skip)

4.2 PHOTON MODEL AND DISPERSION RELATION

Introduce a random (time-independent) background field g to mimic the anomalous effects of a multiply connected (static) spacetime foam, generalizing the result for a single wormhole.

The photon model is defined by the action

$$I_{\text{photon}} = -\frac{1}{4}\int_{\mathbb{R}^4} \mathrm{d}^4x \,\Big(f_{\mu\nu}(x)\, f_{\kappa\lambda}(x)\, \eta^{\kappa\mu}\, \eta^{\lambda\nu} + g(x)\, f_{\kappa\lambda}(x)\, \widetilde{f}^{\kappa\lambda}(x) \Big)\,,$$

with Maxwell field strength tensor $f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$ and its dual $\widetilde{f}^{\kappa\lambda} \equiv \frac{1}{2}\,\epsilon^{\kappa\lambda\mu\nu}\, f_{\mu\nu}$, for Levi–Civita symbol $\epsilon^{\kappa\lambda\mu\nu}$.

Models of this type have been considered before, but only for coupling constants $g(x)$ varying smoothly over cosmological scales [11,12].

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[11] Carroll, Field & Jackiw, PRD 41 (1990) 1231.
[12] Kostelecký, Lehnert & Perry, PRD 68(2003)123511 [astro-ph/0212003].

Small-scale topology (skip)

Assumed properties of the background field $g(x)$:

1. g is time-independent, $g = g(\vec{x})$,
2. g has amplitude $A \ll 1$ (perhaps $A \sim \alpha$?),
3. average of $g(\vec{x})$ vanishes in the large volume limit,
4. $g(\vec{x})$ varies over length scales which are small compared to the considered wavelengths of the photon field a_μ ,
5. autocorrelation function of $g(\vec{x})$ is finite and isotropic, and drops off "fast enough" at large distances.

The modified Maxwell equations (in Lorentz gauge) read:

$$\Box\, a^\nu(x) = -\partial_\mu g(x)\, \widetilde{f}^{\mu\nu}(x)\,,$$

The photon dispersion relation is then found by expanding the solution to second order in g, under the assumption that the power spectrum of g vanishes for momenta $|\vec{q}| < q_{\text{low}}$ and that the photons have $|\vec{k}| < q_{\text{low}}/2$.

Small-scale topology (skip)

The result is

$$\omega^2 = c_{\text{ren}}^2\, k^2 - c_{\text{ren}}^2\, A^2\, l_\gamma^2\, k^4 + \mathrm{O}(A^4, k^6)\,,$$

with $k \equiv |\vec{k}|$, renormalized light velocity

$$c_{\text{ren}} \equiv c\,\sqrt{1 - A^2\,\gamma_1}\,,$$

and $g(\vec{x})$ amplitude A. The constants γ_1 and l_γ are given by

$$\gamma_1 = \frac{\pi}{18\,A^2}\,C(0)\,, \quad l_\gamma^2 = \frac{2\pi}{15\,A^2}\int_0^\infty \mathrm{d}x\; x\,C(x)\,,$$

in terms of the isotropic autocorrelation function ($x \equiv |\vec{x}|$)

$$C(x) = \widehat{C}(\vec{x}) \equiv \lim_{R\to\infty} \frac{1}{(4\pi/3)R^3} \int_{|\vec{y}|<R} \mathrm{d}^3\vec{y}\; g(\vec{y})\, g(\vec{y}+\vec{x})\,.$$

Small-scale topology (skip)

4.3 EXPERIMENTAL LIMITS

From the absence of Cherenkov-like processes ($p \to p\gamma$) for high-energy cosmic rays ($E \sim c\,k \approx 3 \times 10^{11}$ GeV) follows [13,14]:

$$l_\gamma < \left(1.0 \times 10^{-34}\,\text{cm}\right)\ (\alpha/A)\ , \quad \gamma_1 < \left(6 \times 10^{-19}\right)\ (\alpha/A)^2\ ,$$

Now define

$$l_\gamma \equiv l_{\text{wormhole}}\ \left(l_{\text{wormhole}}/l_{\text{separation}}\right)^{3/2}\ , \quad \gamma_1 \equiv \left(l_{\text{wormhole}}/l_{\text{separation}}\right)^3\ .$$

Here, the length l_{wormhole} would correspond to an appropriate characteristic dimension of an *individual* wormhole and length $l_{\text{separation}}$ would correspond to the average separation between *different* wormholes.

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[13] Gagnon & Moore, PRD 70 (2004) 065002 [hep-ph/0404196].
[14] K & Rupp, PRD 72 (2005) 017901 [hep-ph/0506071].

Small-scale topology

These astrophysics bounds give the following exclusion plot:

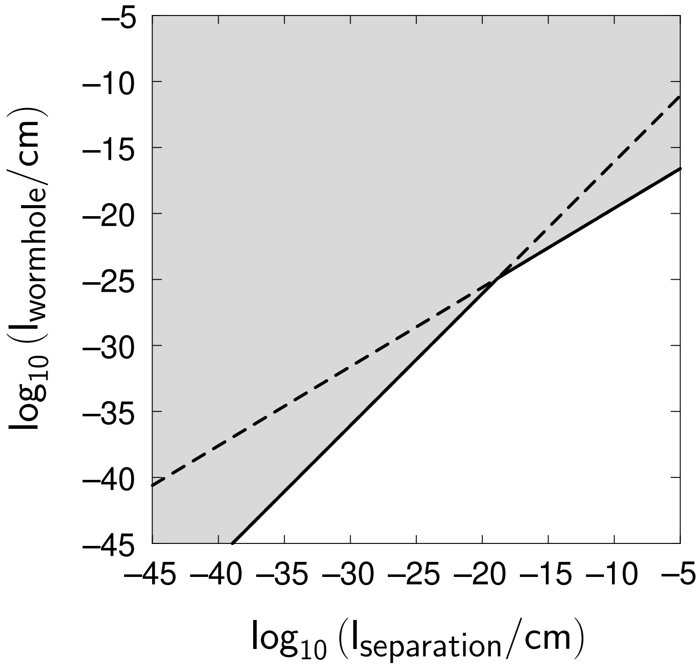

Fig. 6. Excluded region [shaded] for $A = \alpha \approx 1/137$.

Even stronger and more general bounds have been found recently ....

Small-scale topology

It is perhaps not unreasonable to expect some remnant "quantum-gravity" effect with both length scales l_{wormhole} and $l_{\text{separation}}$ of the order of the Planck length [Wheeler, 1957],

$$l_{\text{Planck}} \equiv \sqrt{G\,\hbar/c^3} \approx 1.6 \times 10^{-33}\ \text{cm}.$$

But this seems to be ruled out by the exclusion plot above.

Tentative conclusion:

a preferred-frame graininess of space with a single length scale l_{Planck} may be hard to reconcile with the current experimental bounds from cosmic-ray physics.

Summary

5. SUMMARY

The subtle role of topology on local properties of quantum field theory is well-known (e.g., the Casimir effect).

For certain chiral gauge theories, the interplay of UV and IR effects may also lead to:

Lorentz and CPT noninvariance,

even for flat spacetime manifolds, that is, without gravity.

The basic idea is quite simple, but, as always, there are subtleties . . .

Summary

Possible applications:

First, we considered nontrivial large-scale topology (for example, $\mathbb{R} \times S^1 \times S^1 \times S^1{}_{\text{PSS}}$), which may give rise to new effects in photon physics (e.g., vacuum birefringence [2,9] and photon splitting [15,16]).

Specifically, we have discussed the potential role of the CPT anomaly as one ingredient of an explanation for the very special initial conditions of our universe [cf. Penrose].

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[15] Adam & K, NPB 657 (2003) 214 [hep-th/0212028].
[16] Kaufhold & K, NPB 734 (2006) 1 [hep-th/0508074].

Summary

Second, we turned to nontrivial small-scale topology of spacetime, which may be probed by the CPT anomaly.

Simplest case would be a static multiply-connected spacetime foam consisting of many randomly-oriented, localized defects (e.g., wormholes).

With certain assumptions, the modified photon dispersion relation was calculated in the long-wavelength limit.

Experimental data from cosmic-ray physics then give upper bounds on typical length scales of a (static) spacetime foam.

But perhaps more important than these applications is the general idea:

spacetime topology affects the second-quantized vacuum and the fundamental symmetries of chiral gauge theory (Lorentz and CPT invariance), which, in turn, allows us to probe certain properties of spacetime.